Quantum Cryptanalysis of Post-Quantum Cryptography

Thijs Laarhoven, Michele Mosca, Joop van de Pol

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SIAM AG'13, Fort Collins, USA
(August 3, 2013)

Solving the Shortest Vector Problem in Lattices Faster Using Quantum Search

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Outline

Lattices

What is a lattice?

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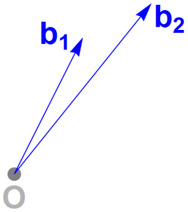

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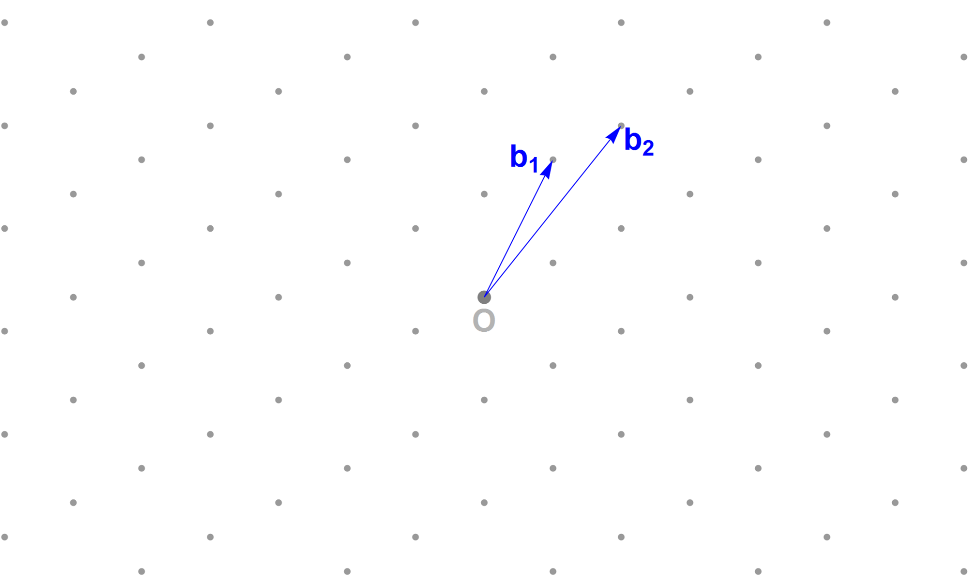
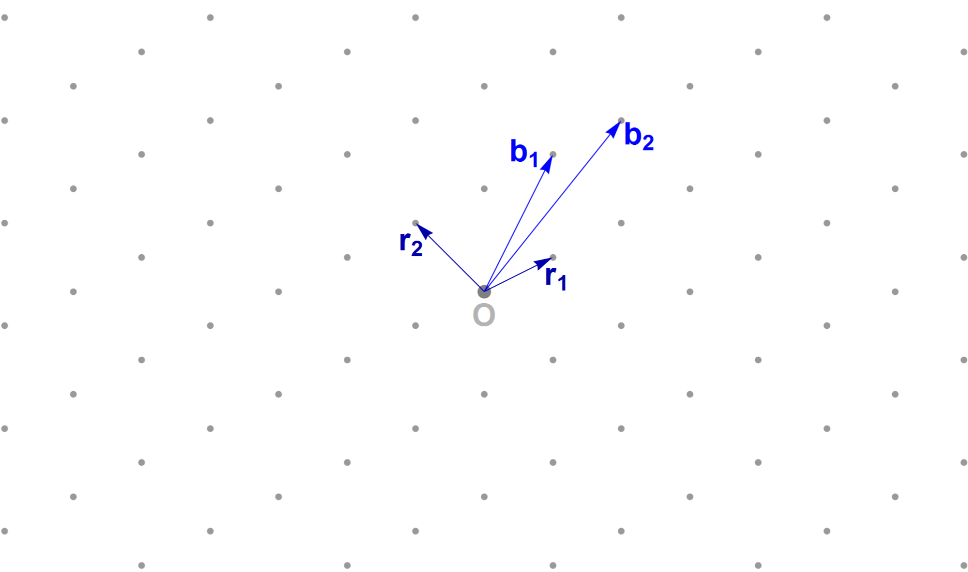


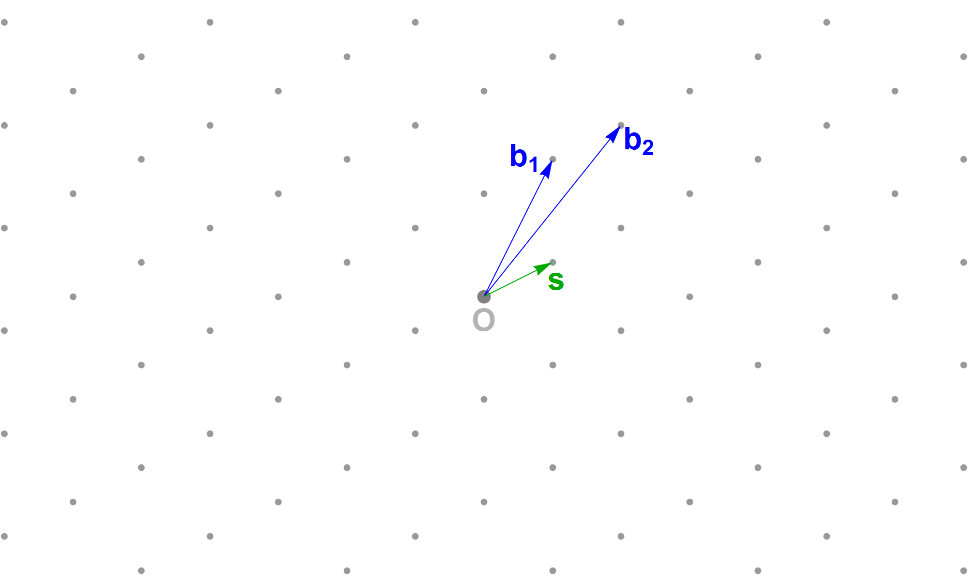
Lattices

Lattice Basis Reduction



Lattices

Shortest Vector Problem (SVP)



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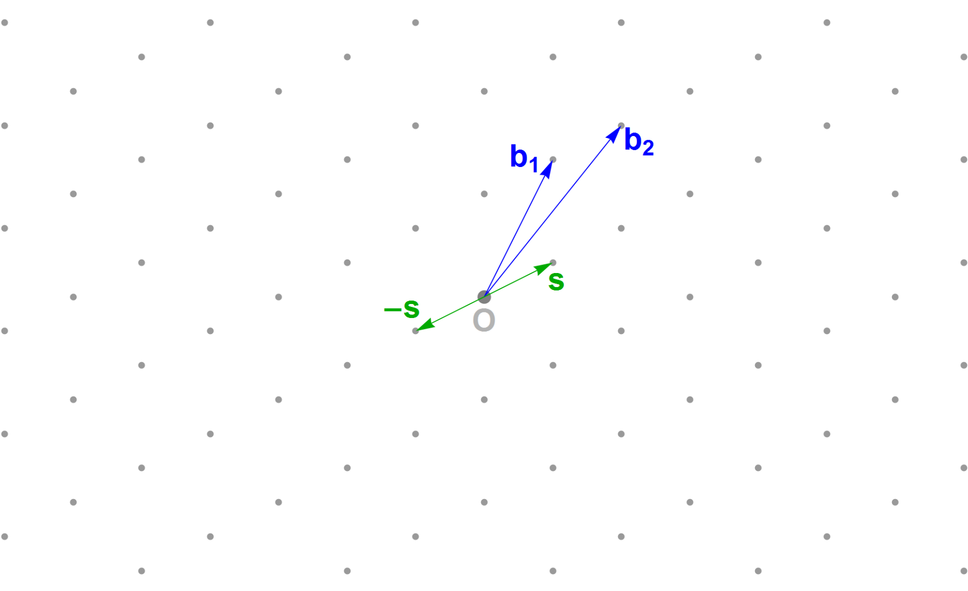

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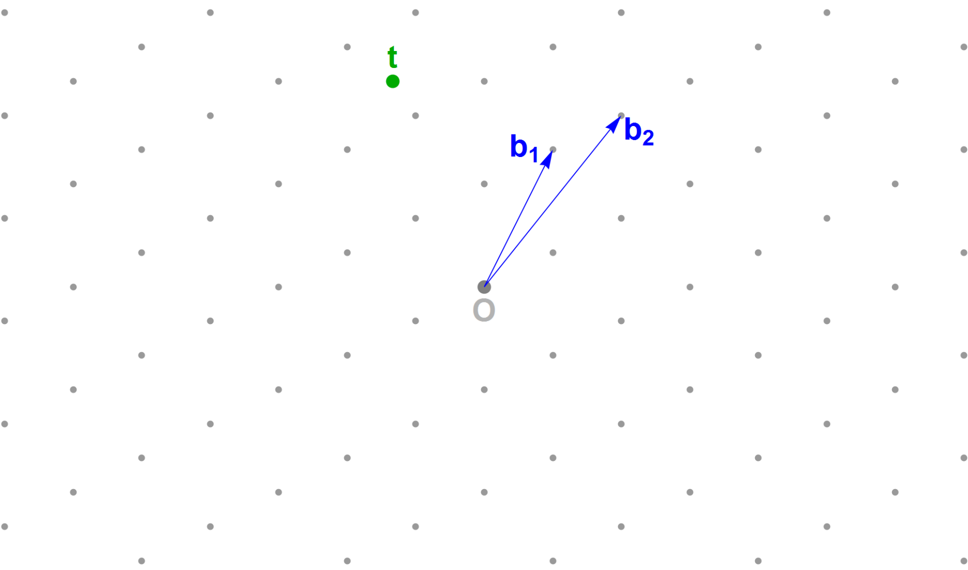

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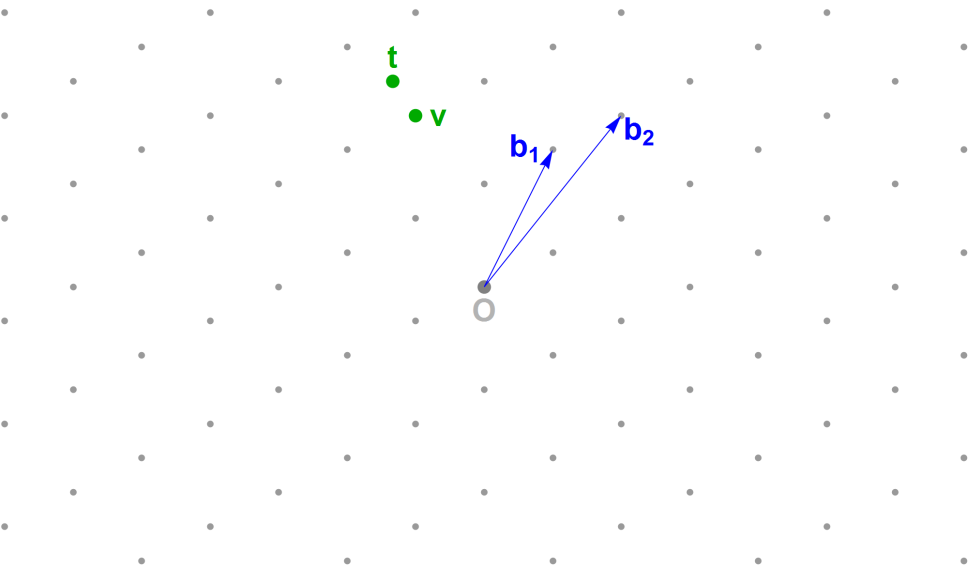

Quantum Search

Classical form

Problem: Given a list L of size N, and a function $f : L \to \{0, 1\}$ such that there is exactly one element $e \in L$ with $f(e) = 1$. Find this element e.

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- Classical search: $\Theta(N)$ time
- Quantum search: $\Theta(\sqrt{N})$ time [Gro96]

Quantum Search

General form

Problem: Given a list L of size N, and a function $f : L \to \{0, 1\}$ such that there are $c = O(1)$ elements $e \in L$ with $f(e) = 1$. Find one such element e.

- Classical search: $\Theta(N/c)$ time
- Quantum search: $\Theta(\sqrt{N/c})$ time [Gro96]

Applications

(Why do we care?)

- “Constructive cryptography”: Lattice-based cryptosystems
  - Based on hard lattice problems (SVP, CVP)
  - NTRU cryptosystem [HPS98]
  - Fully Homomorphic Encryption [Gen09]
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How (quantum-)hard are hard lattice problems such as SVP?

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Studied since the early '80s [Poh81, Kan83, FP85, ..., GNR10]

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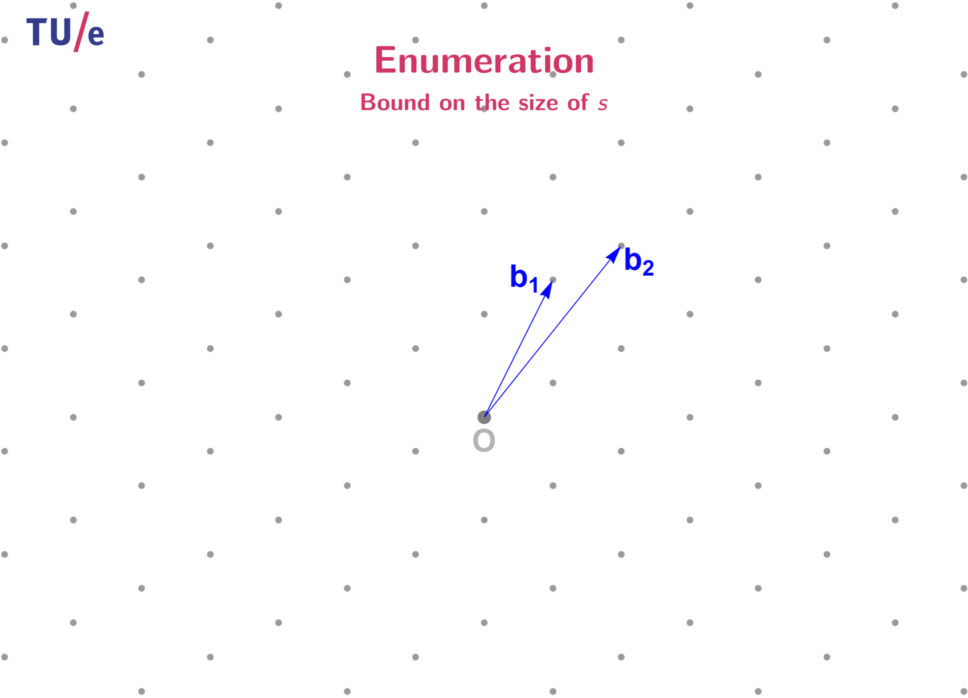
Bound on the size of s



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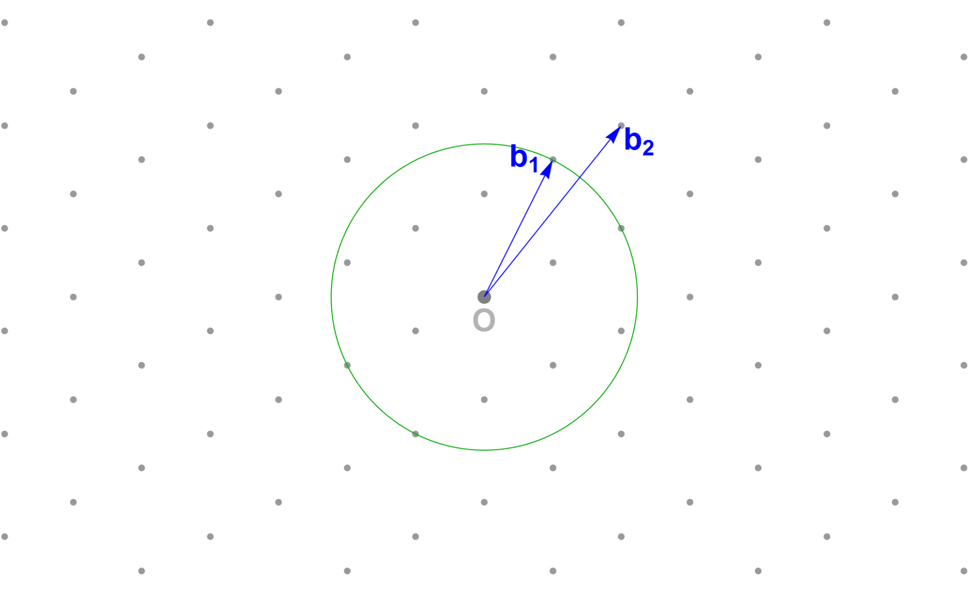

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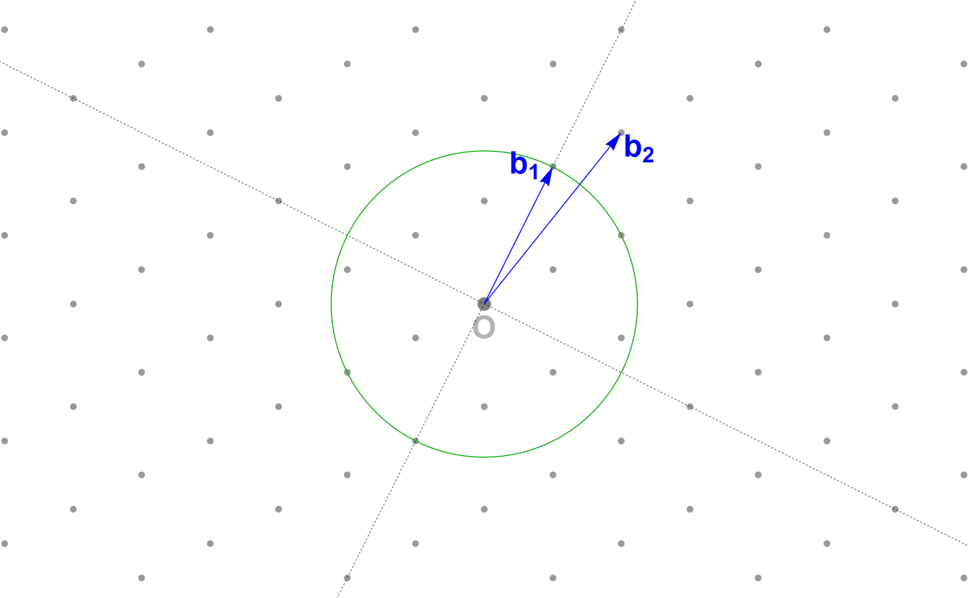
Possible coefficients of b_2



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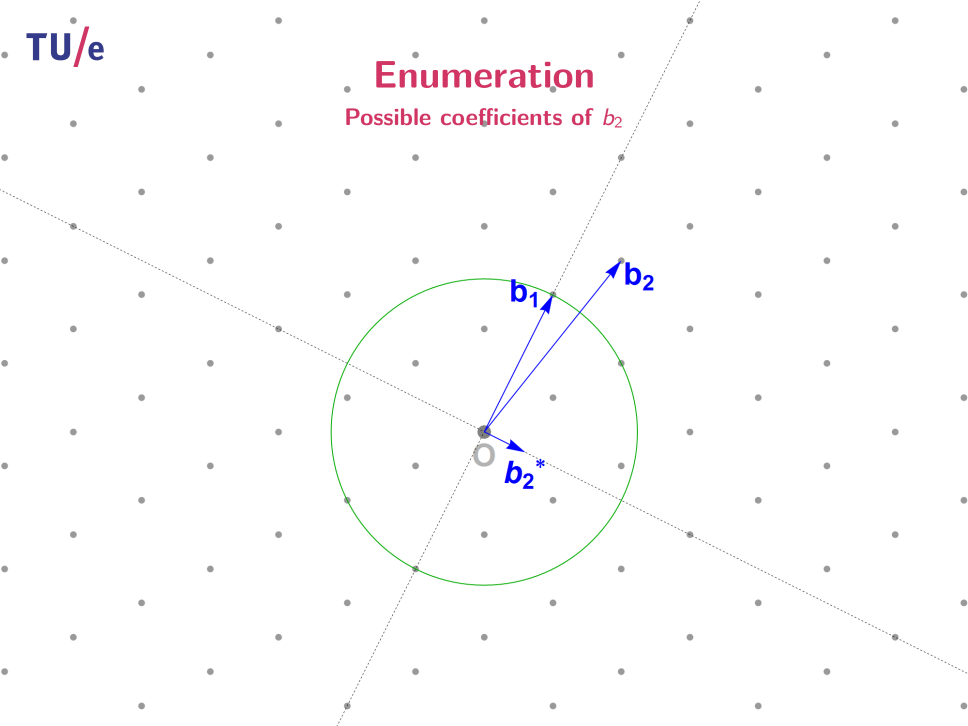

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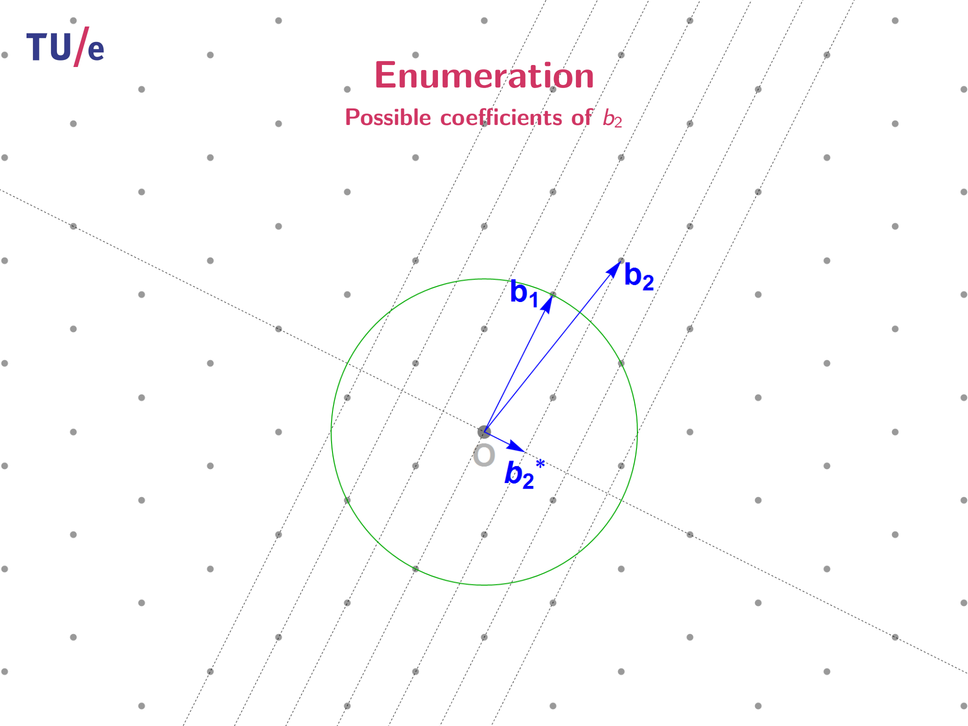

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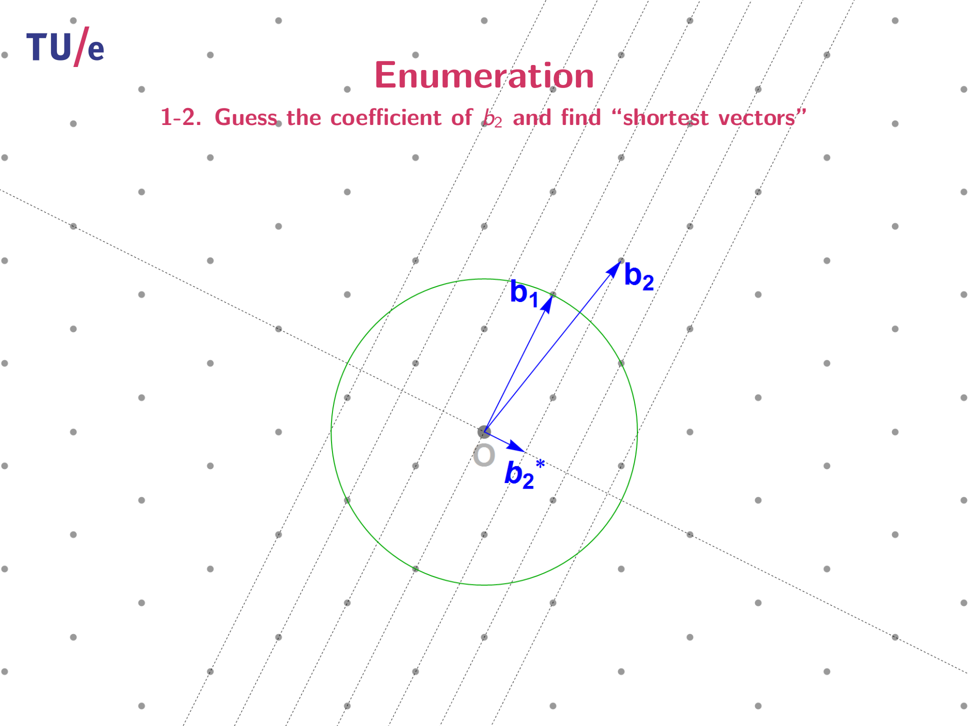

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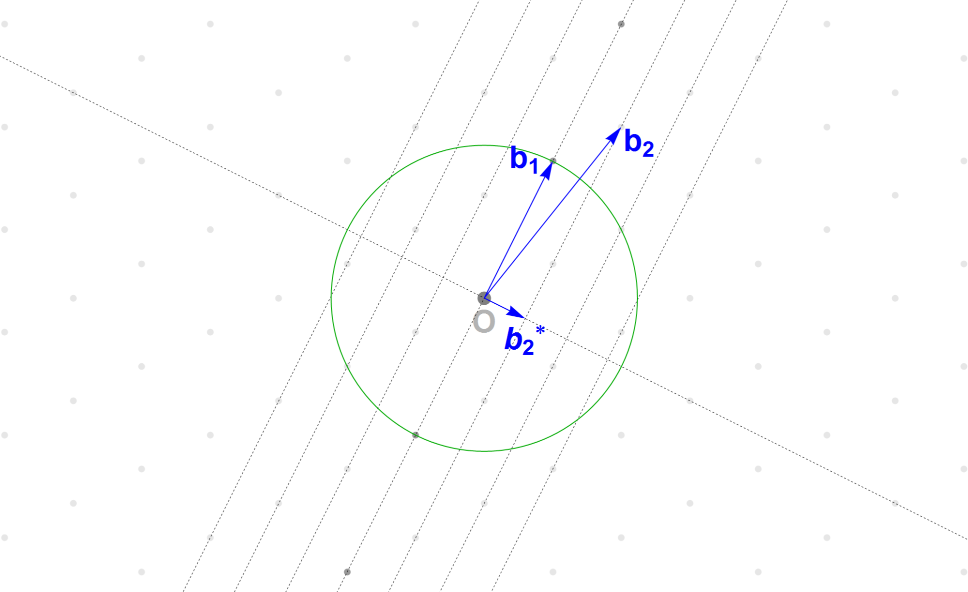

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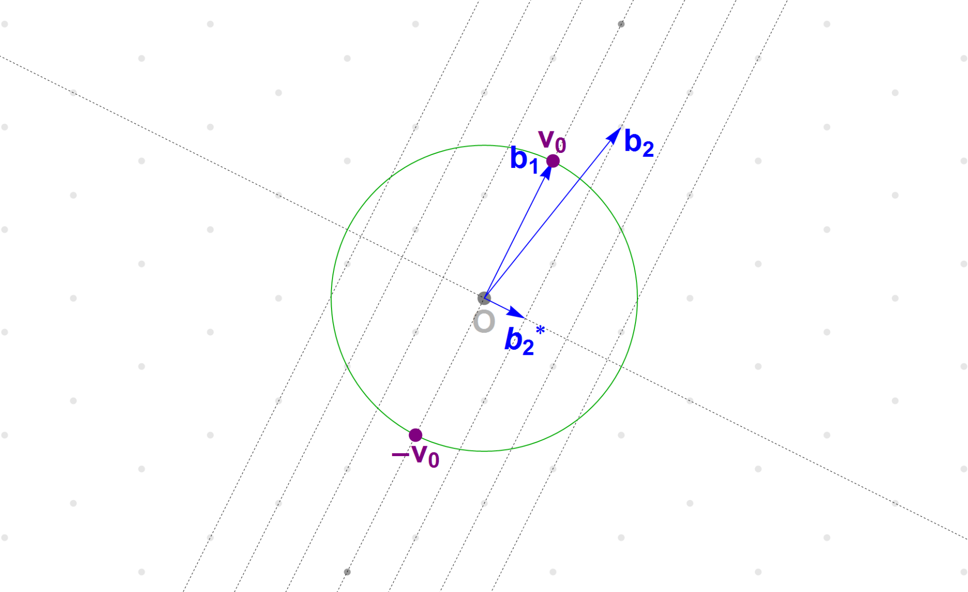

TU/e

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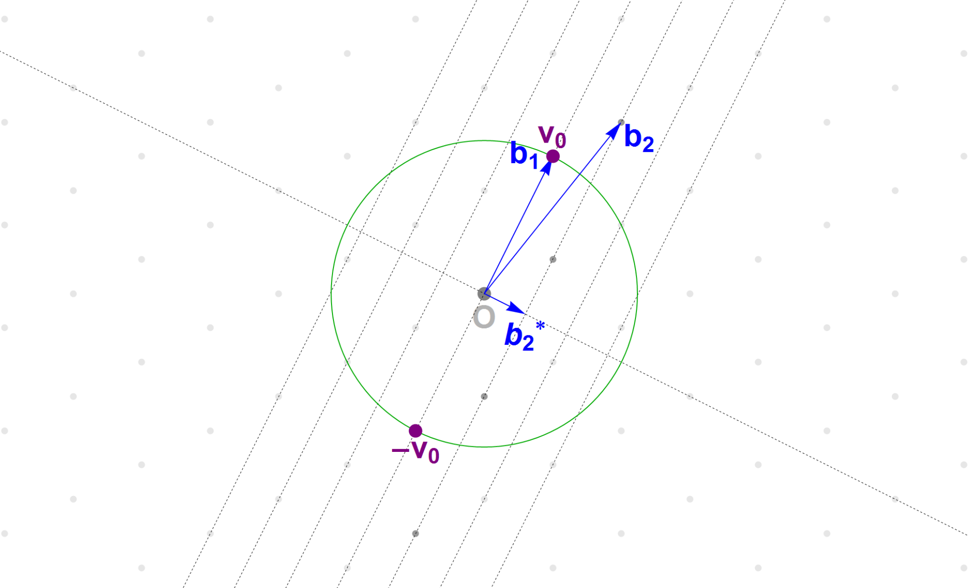

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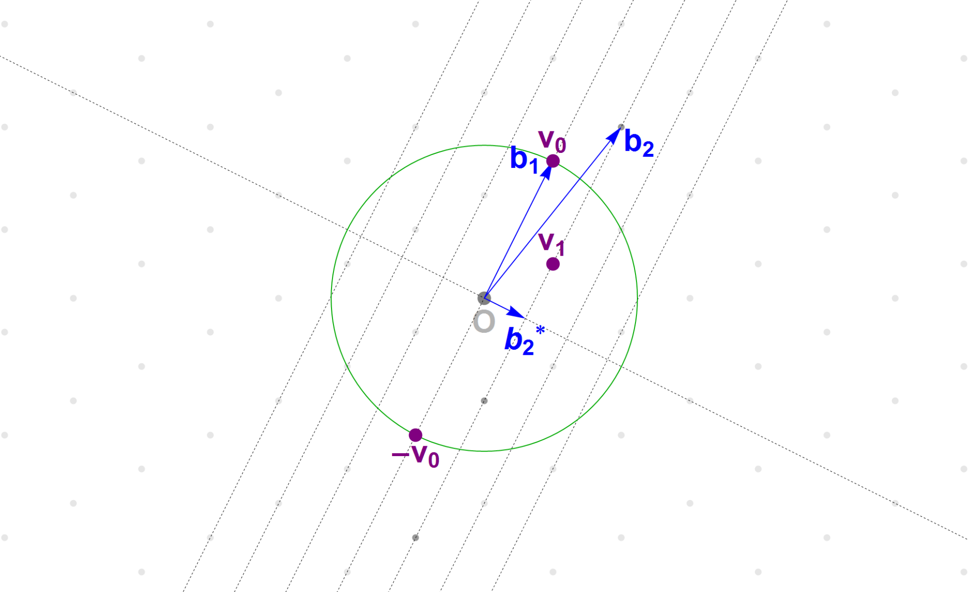

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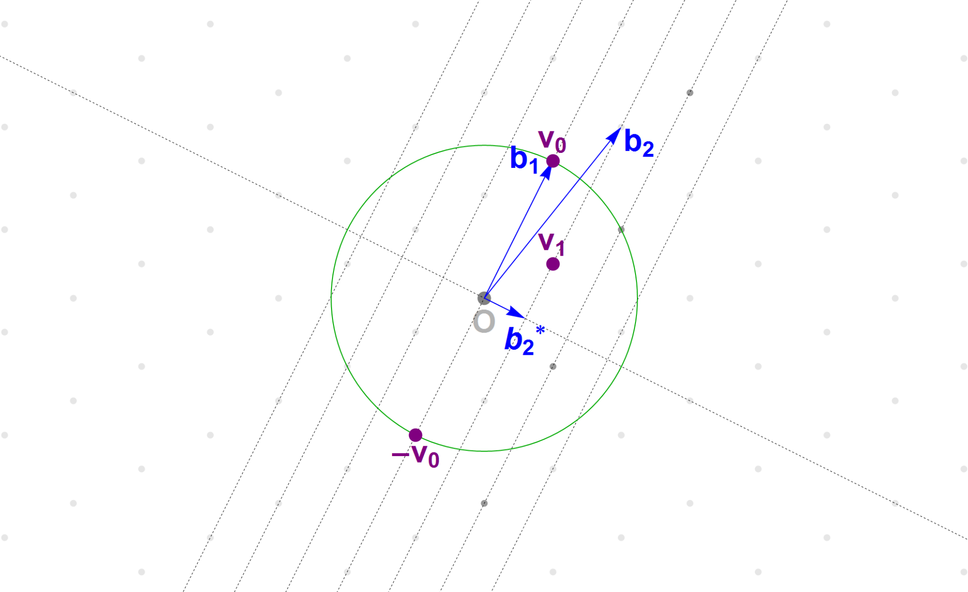

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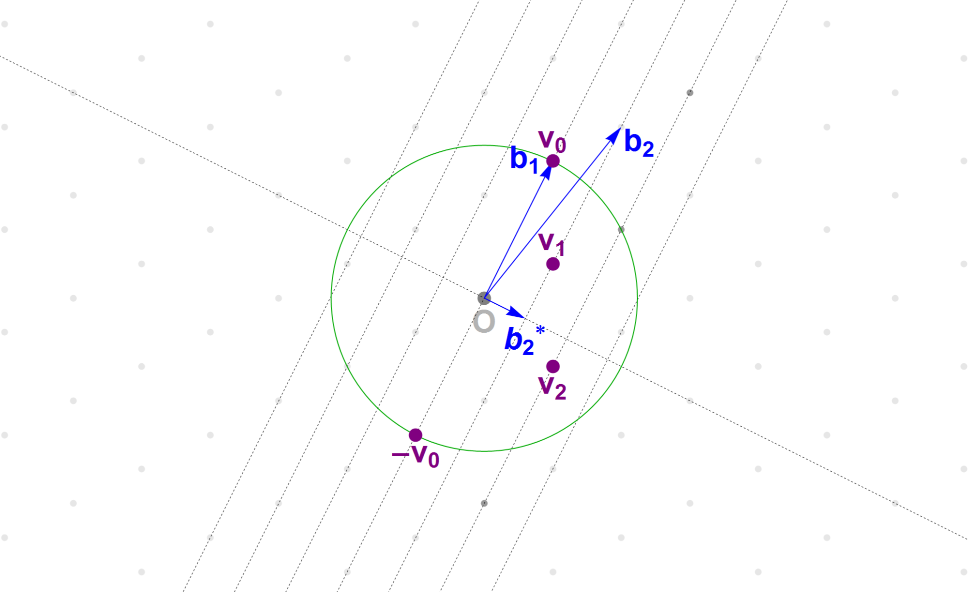

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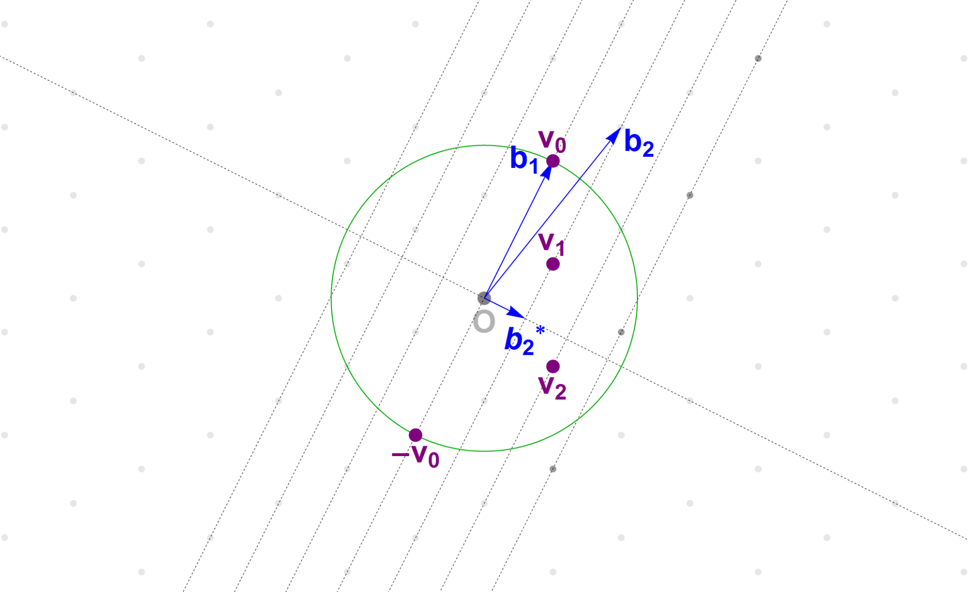

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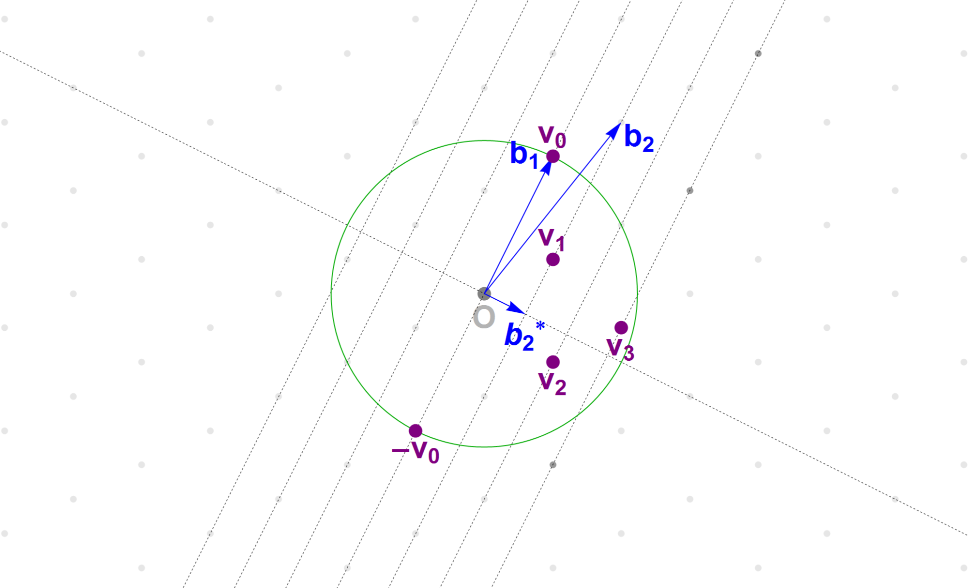

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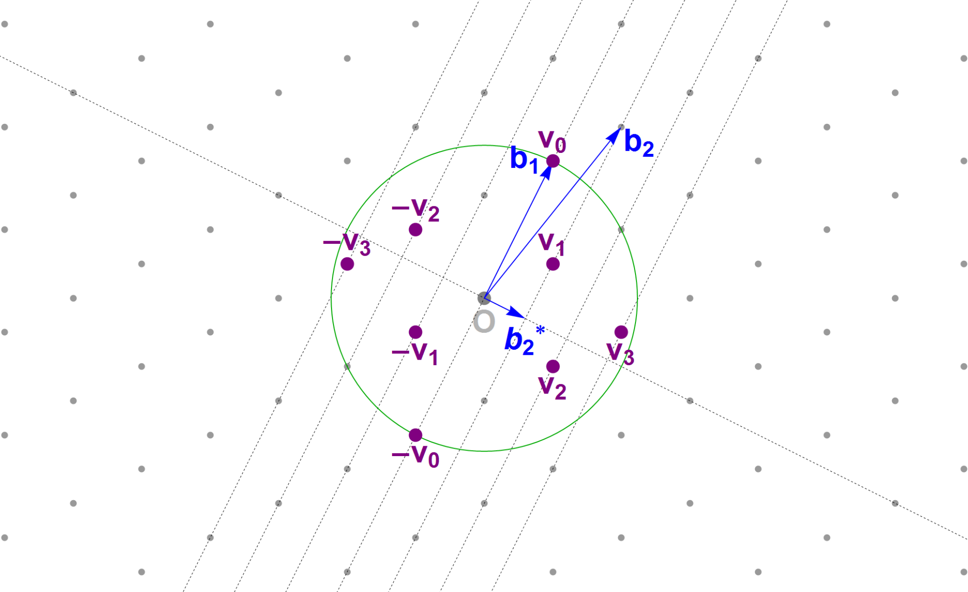

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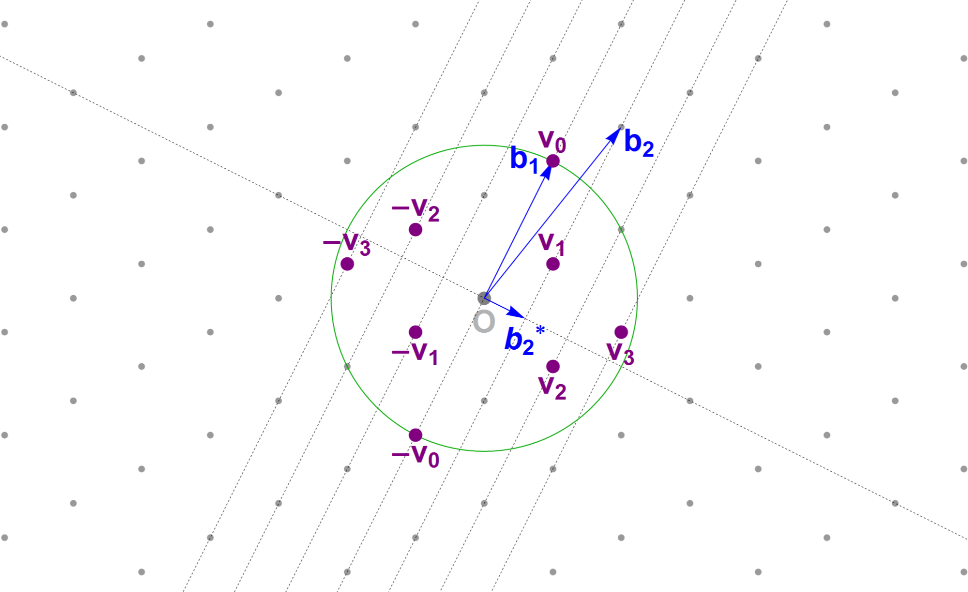

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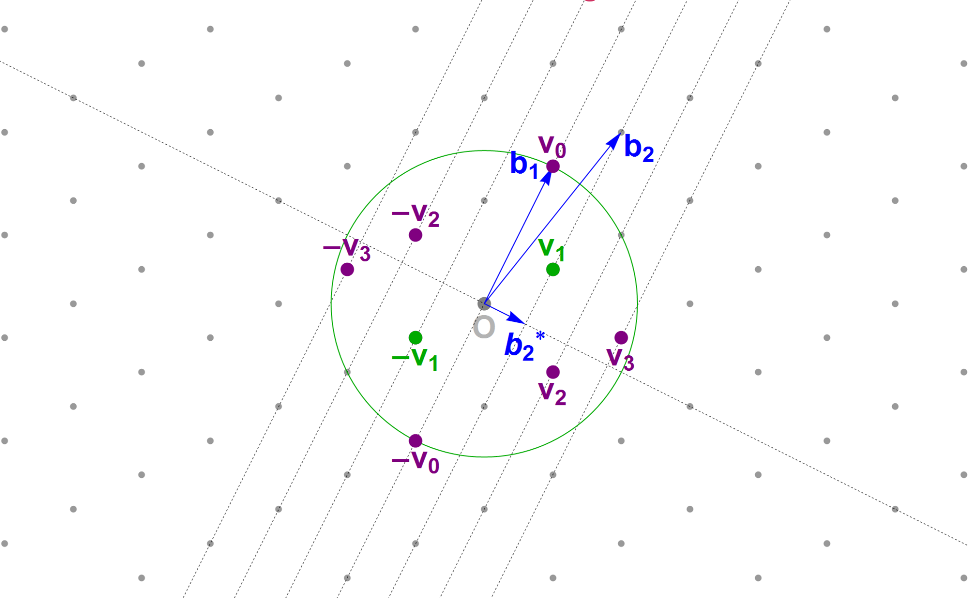

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Asymptotically suboptimal time complexity

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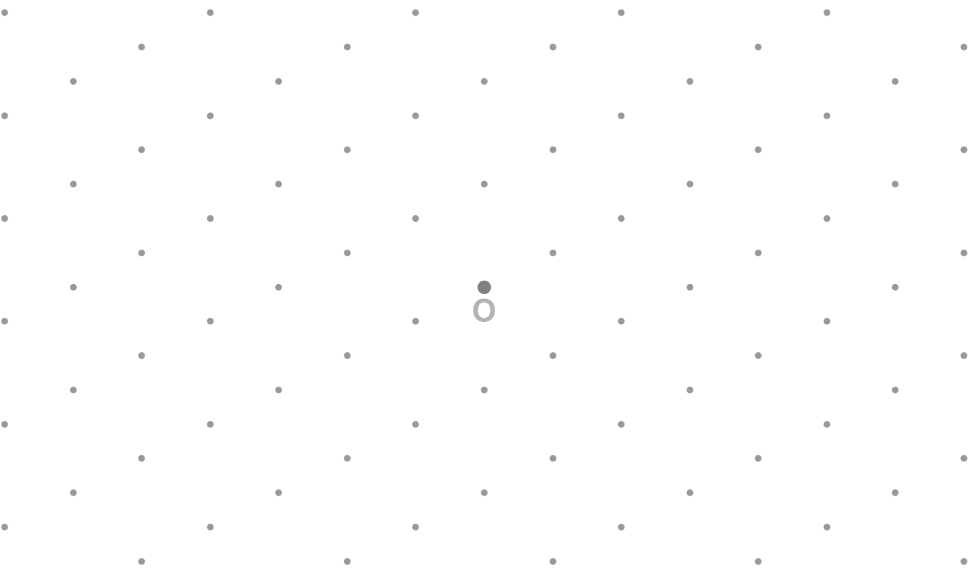

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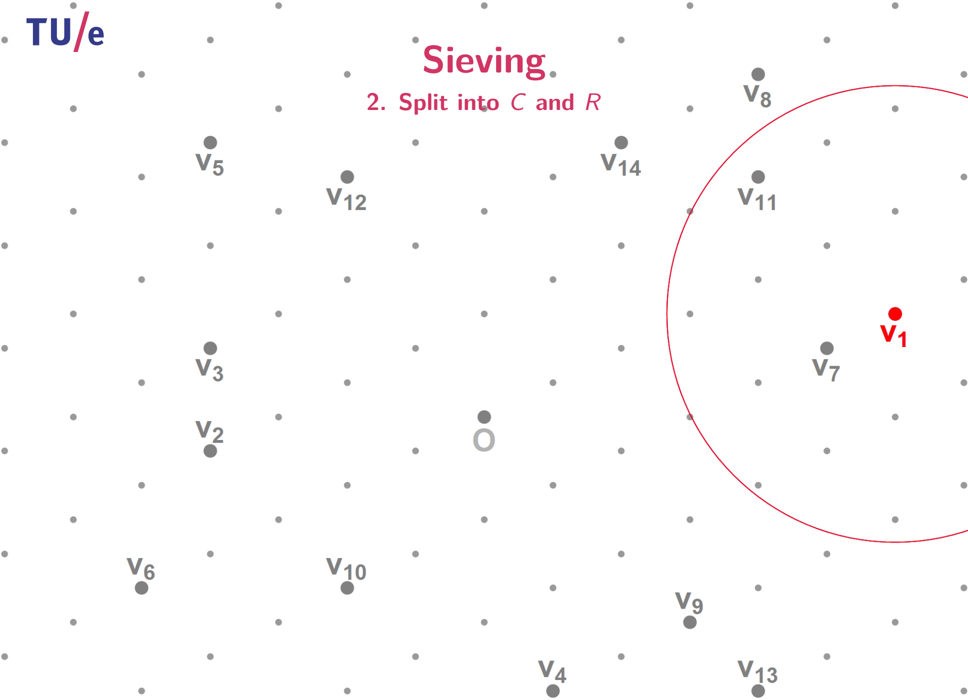

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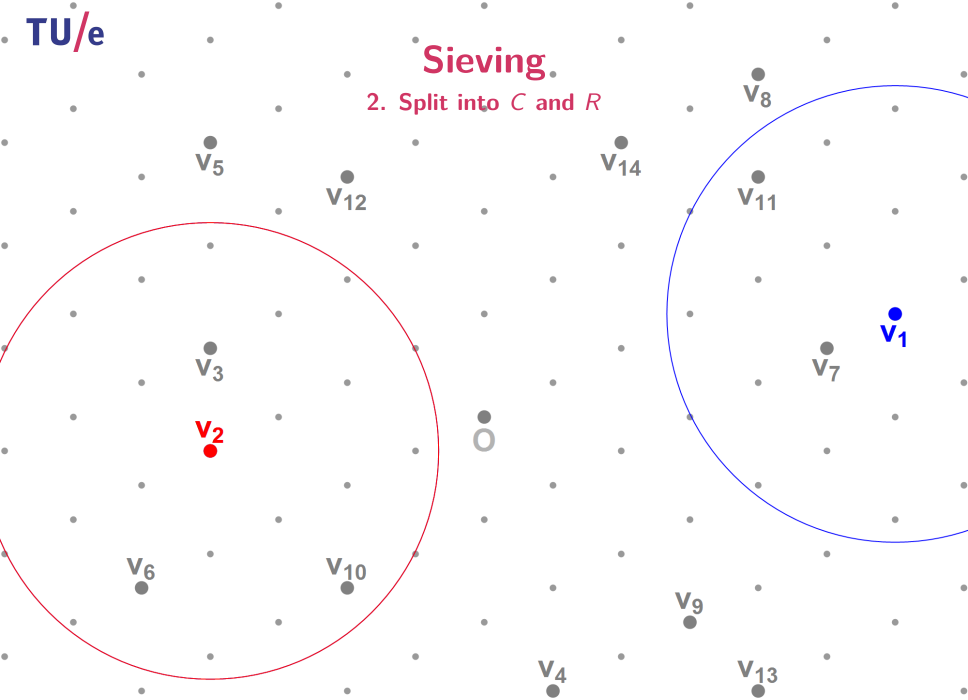

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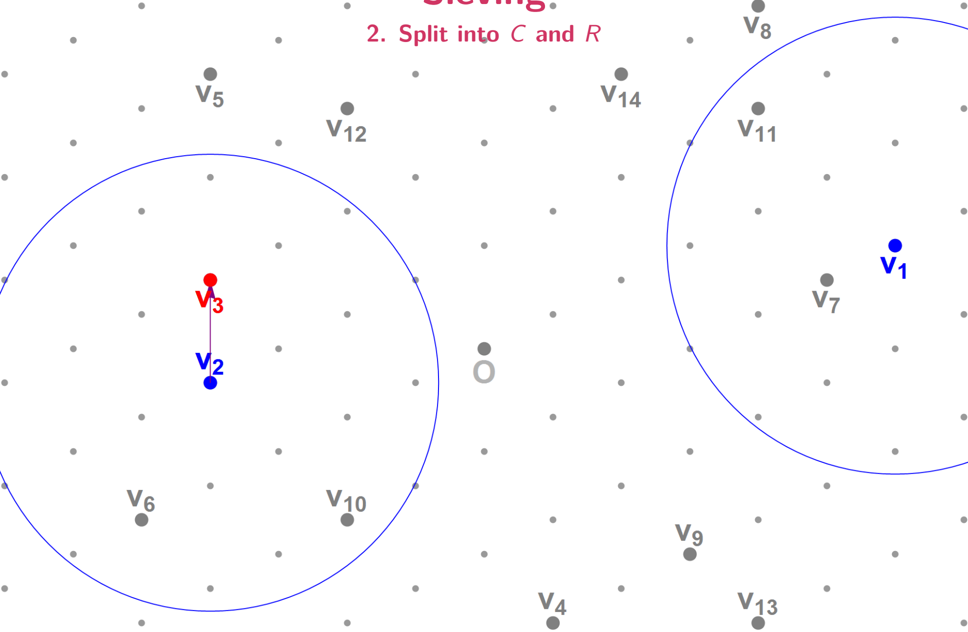

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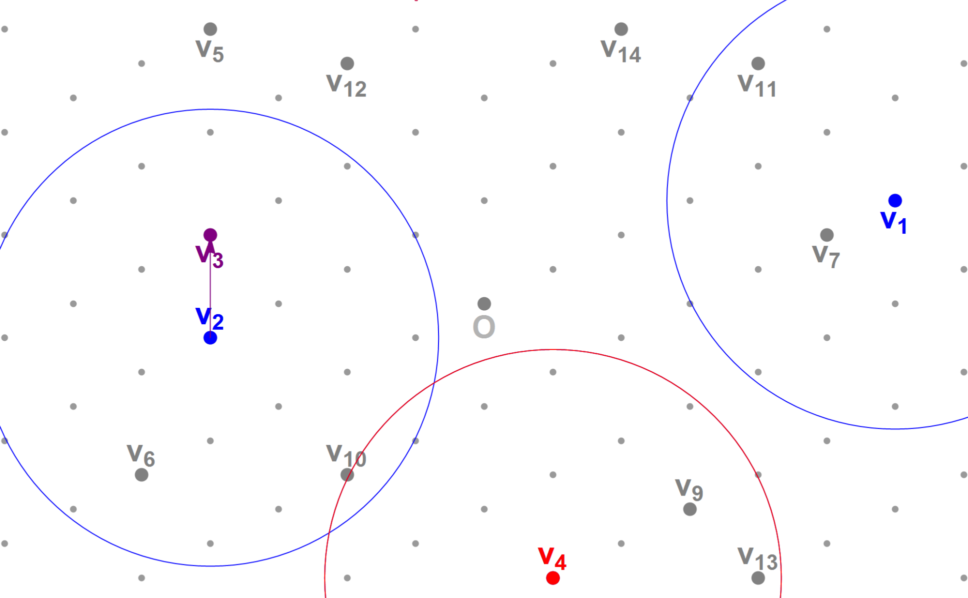

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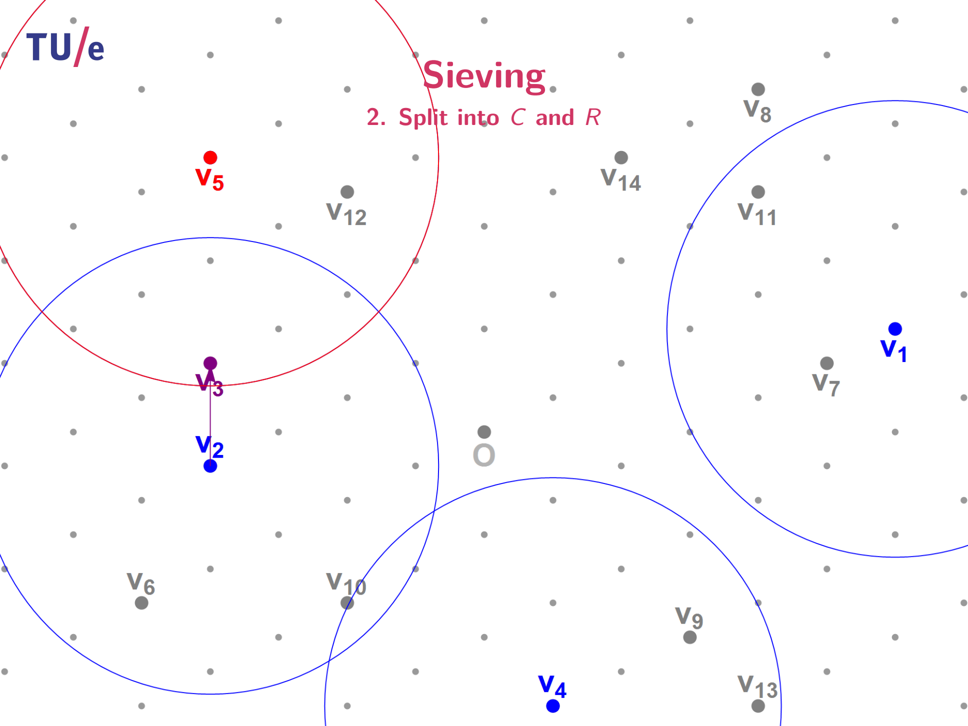

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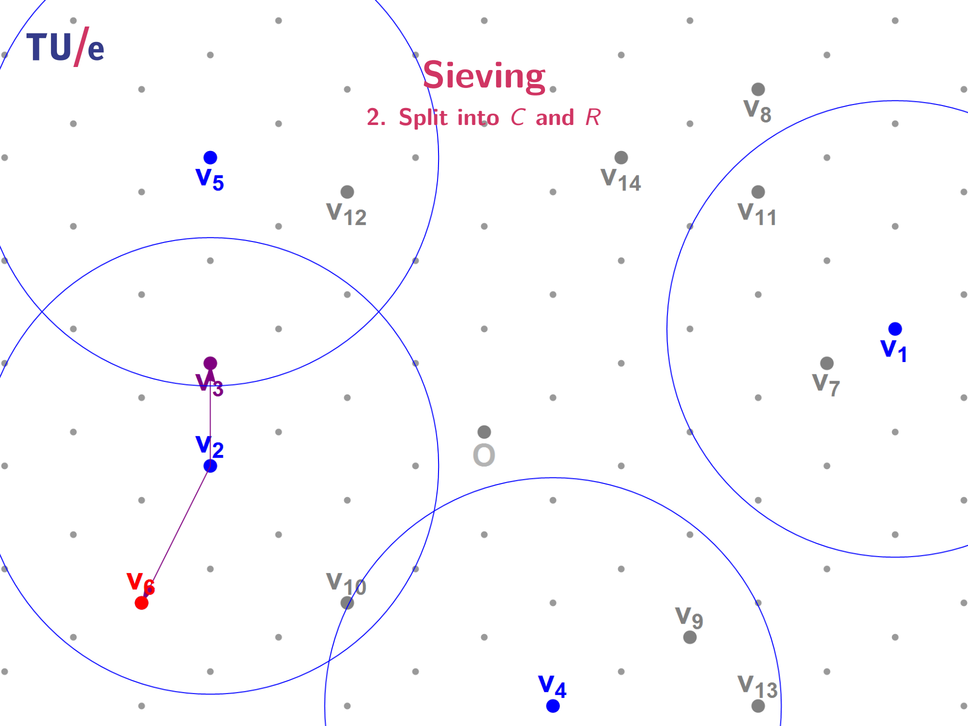

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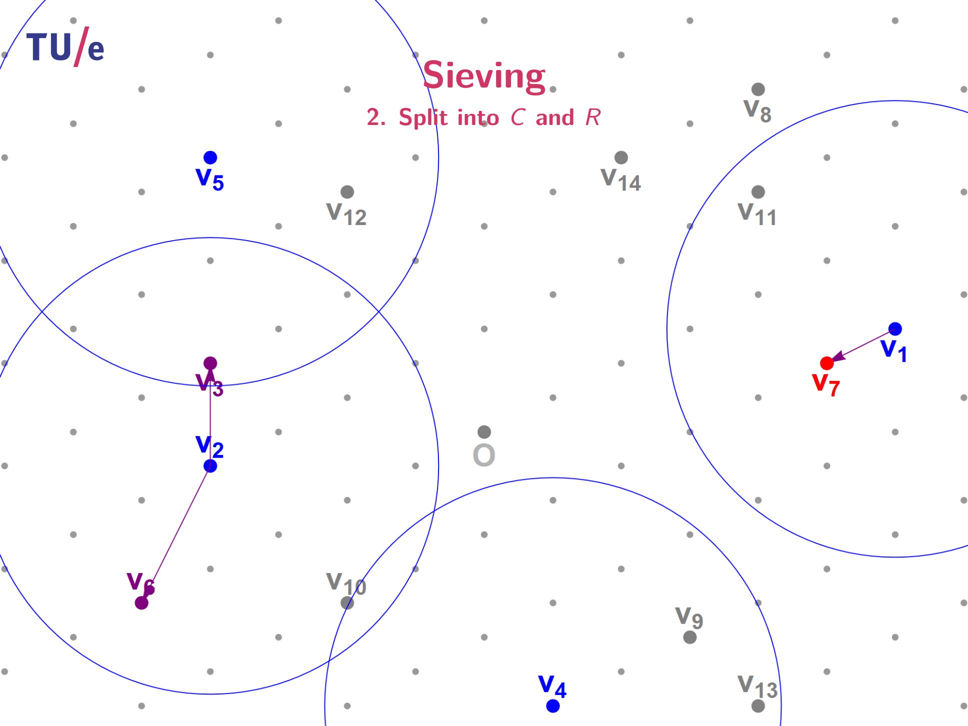

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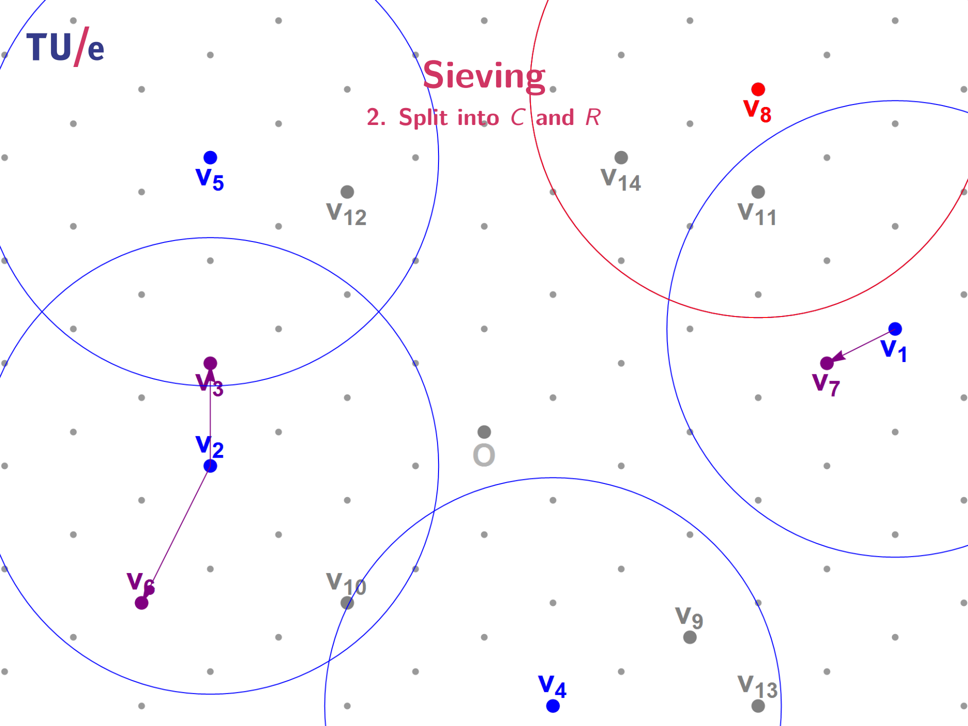

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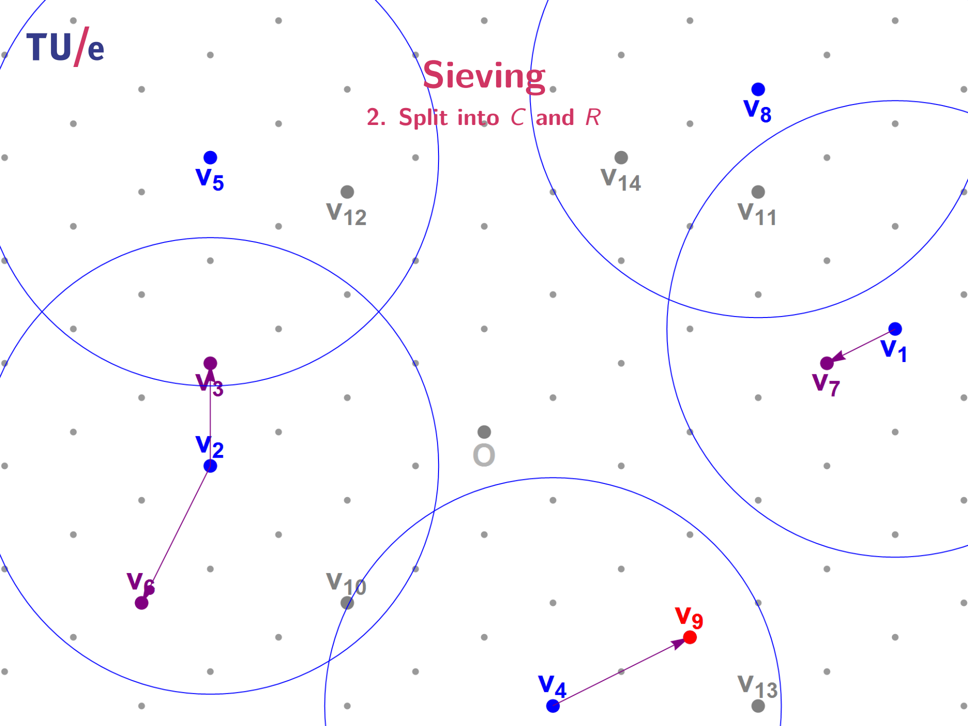

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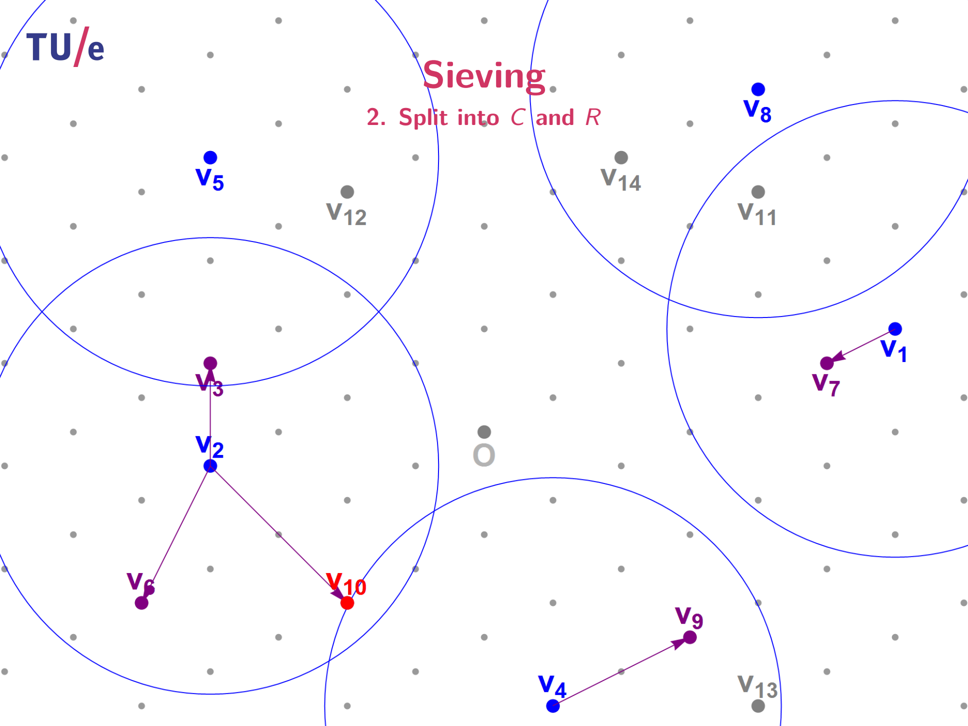

TU/e

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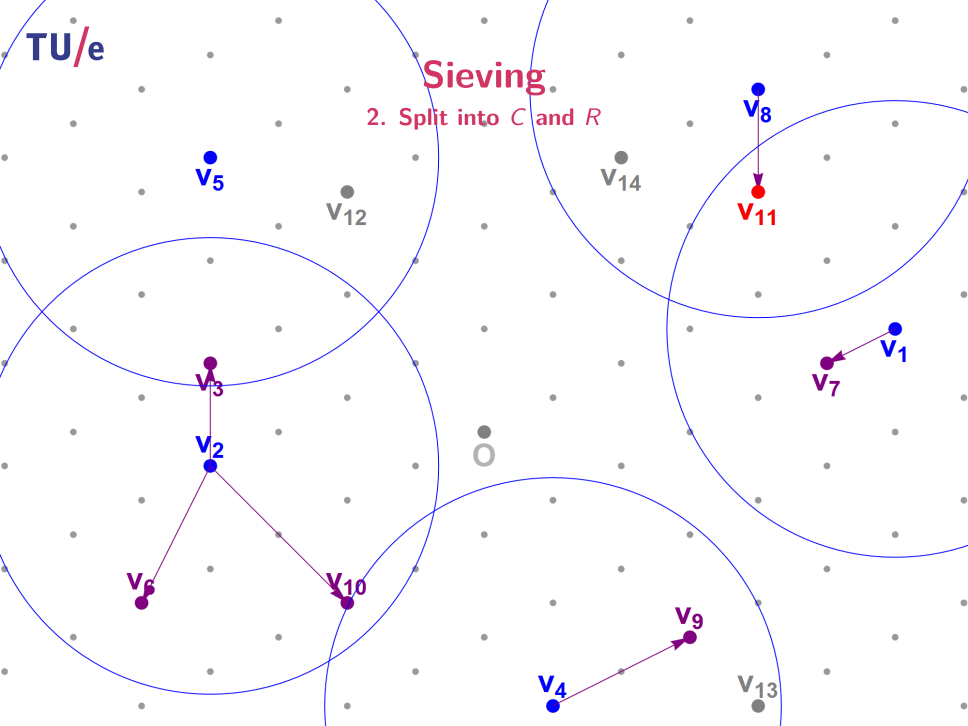

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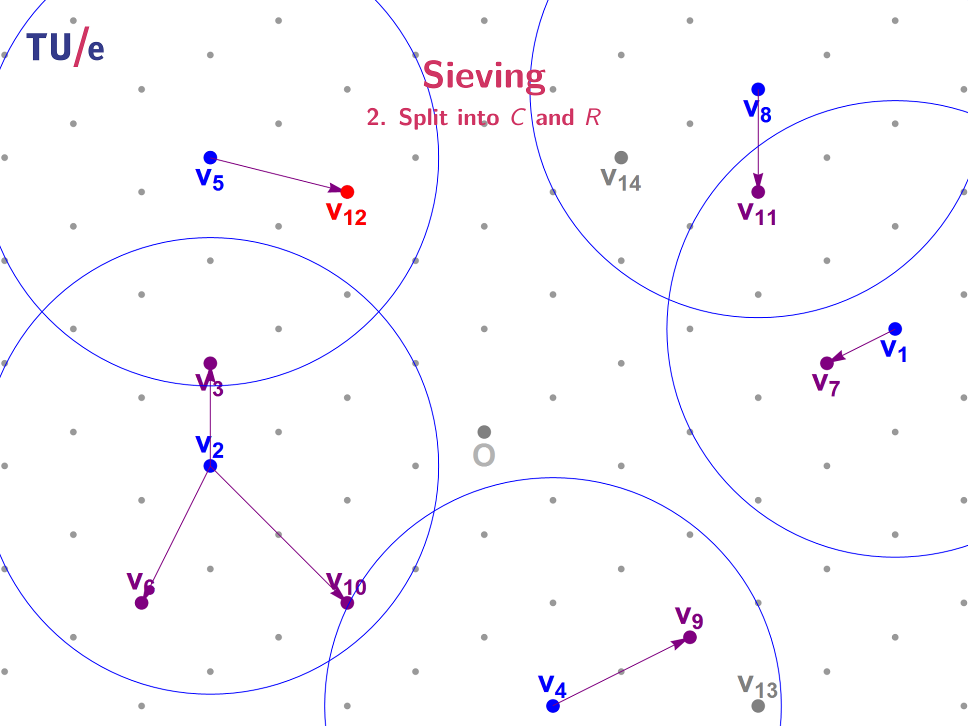

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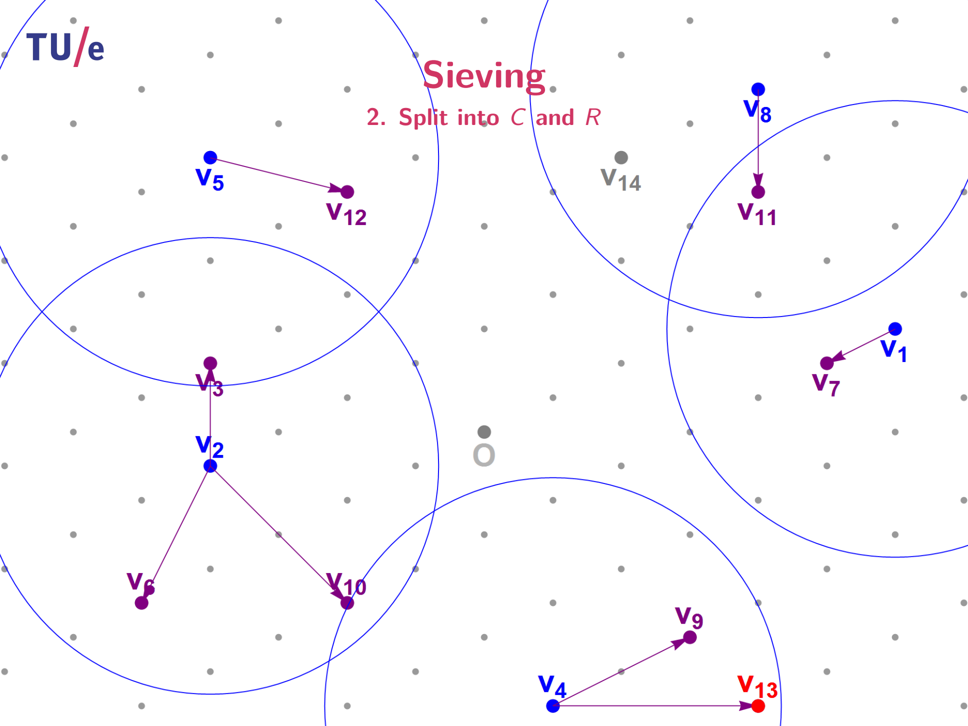

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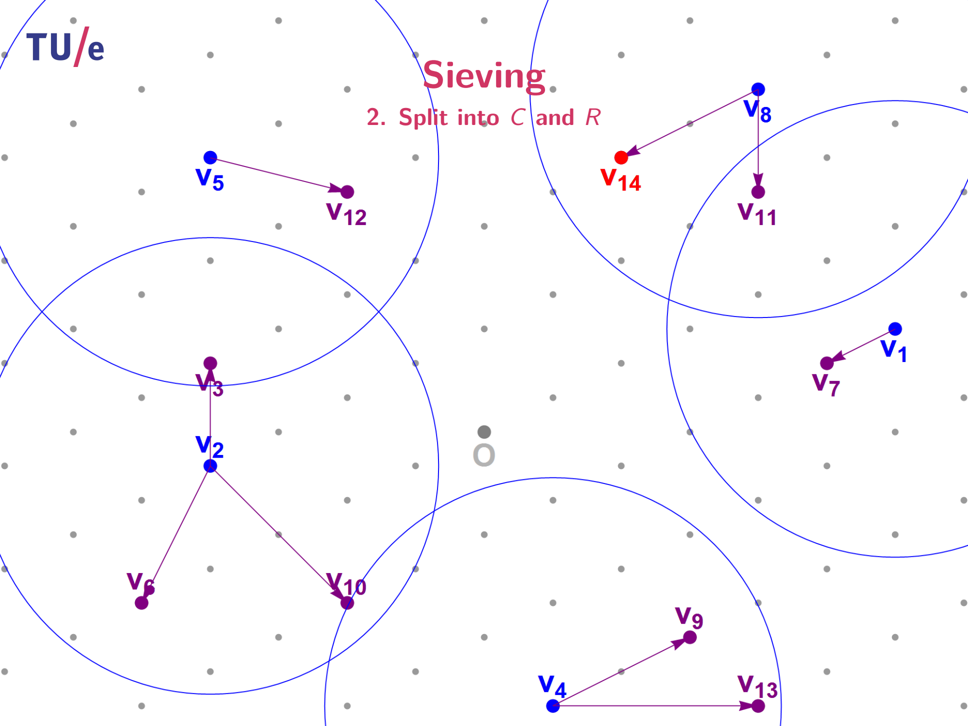

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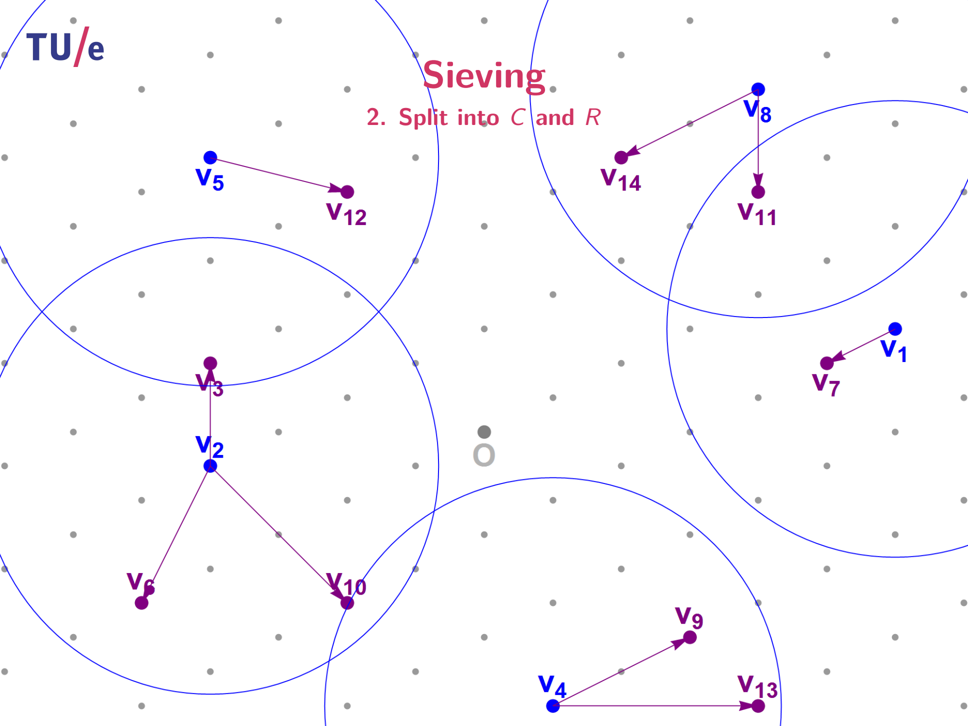

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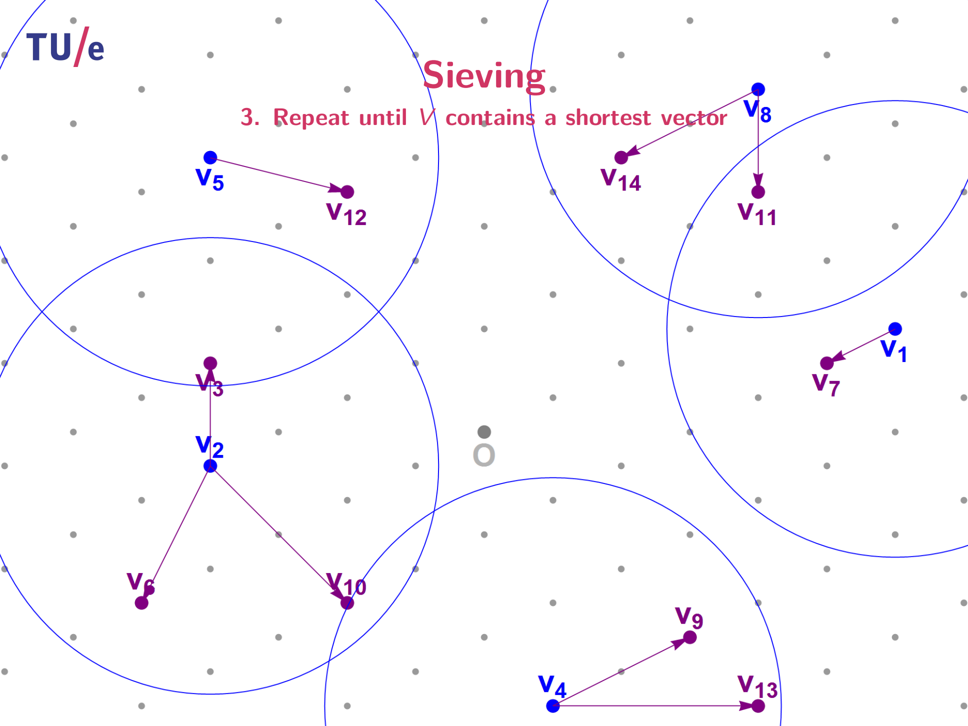
3. Repeat until V contains a shortest vector



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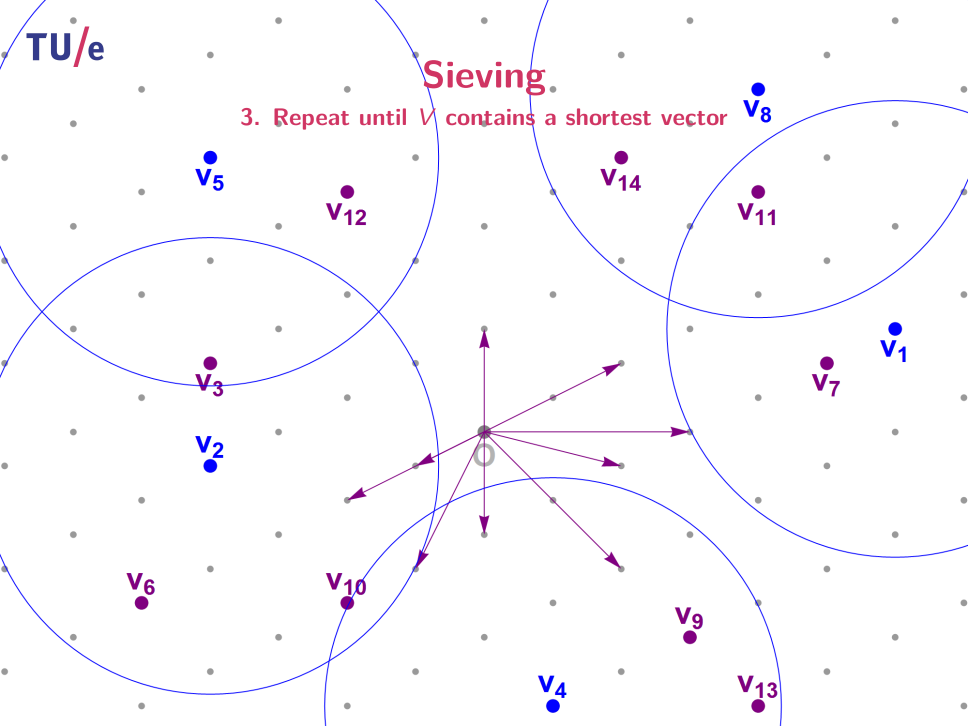

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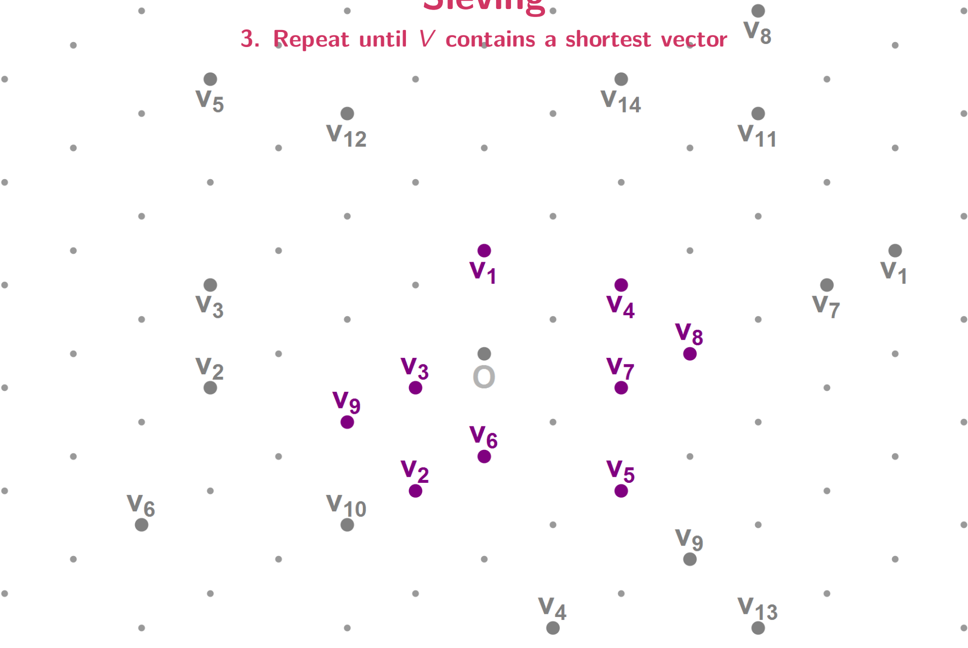

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- Space: $|V|, |C|, |R| \leq 2^{\alpha n}$ for some α

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- Space: $|V|, |C|, |R| \leq 2^{\alpha n}$ for some α
- Classical Time: $\approx 2^{\alpha n} \cdot 2^{\alpha n} = 2^{2\alpha n}$

Sieving

Studied since 2001 [AKS01, Reg04, NV08, . . ., HPS11]

1. Generate a long list V of random lattice vectors
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   - Set $C = \emptyset$ and $R = \emptyset$
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     - If $\|v - c\|$ is "small", add $v - c$ to R
3. Set $V = R$ and repeat until V contains a shortest vector

Complexity?

- Space: $|V|, |C|, |R| \leq 2^{\alpha n}$ for some α
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- Space: $|V|, |C|, |R| \leq 2^{\alpha n}$ for some α
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Complexity?

- Space: $|V|, |C|, |R| \leq 2^{\alpha n}$ for some α
- Classical Time: $\approx 2^{\alpha n} \cdot 2^{\alpha n} = 2^{2\alpha n}$
- Quantum Time: $\approx 2^{\alpha n} \cdot \sqrt{2^{\alpha n}} = 2^{\frac{3}{2}\alpha n}$
- Quantum speed-up: $\approx 25\%$ in the exponent

Saturation

Studied since 2009 [MV10, PS09, Sch11, IKMT13]

1. Generate a long list V of random lattice vectors

Saturation

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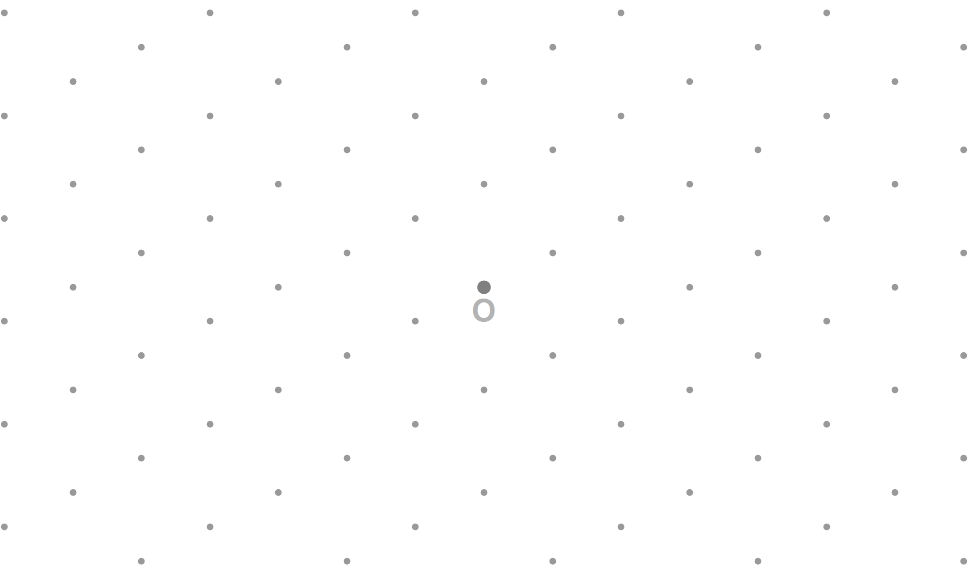
3. Search C for a shortest vector

Saturation

1. Generate random lattice vectors



Saturation

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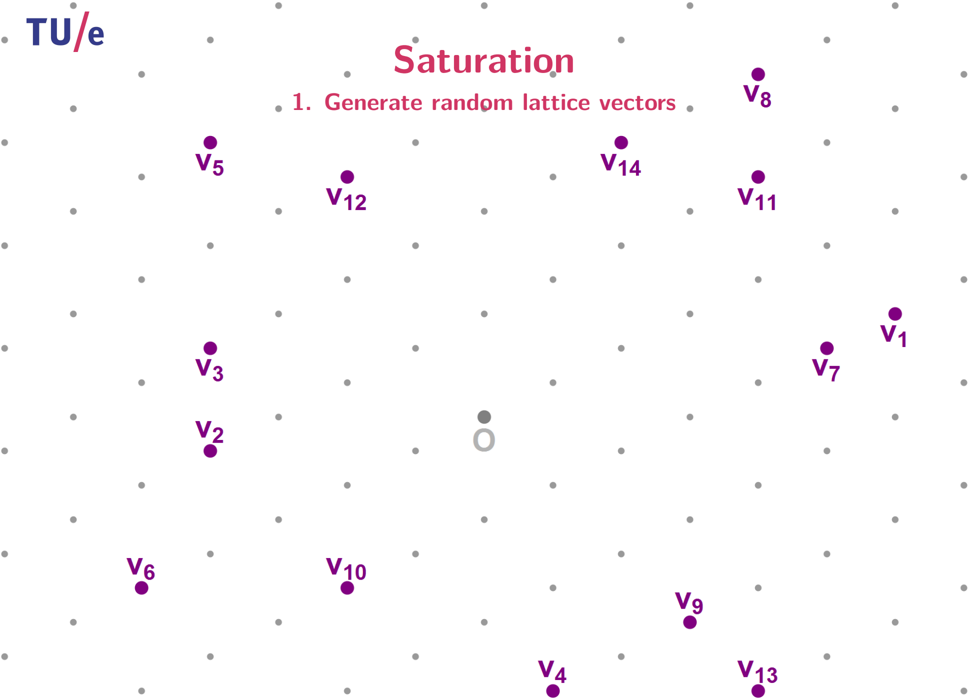
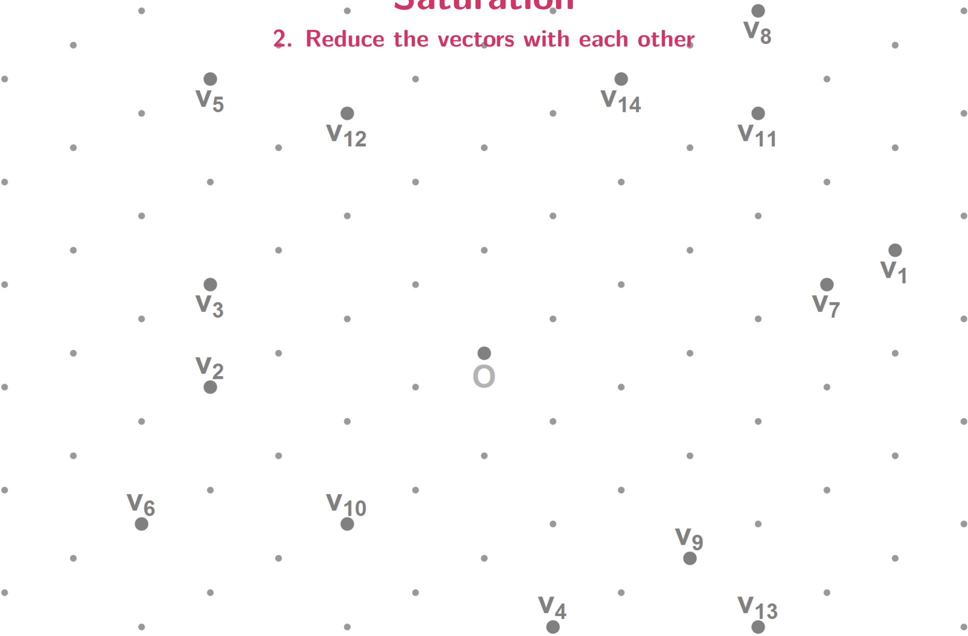


Saturation

2. Reduce the vectors with each other



Saturation

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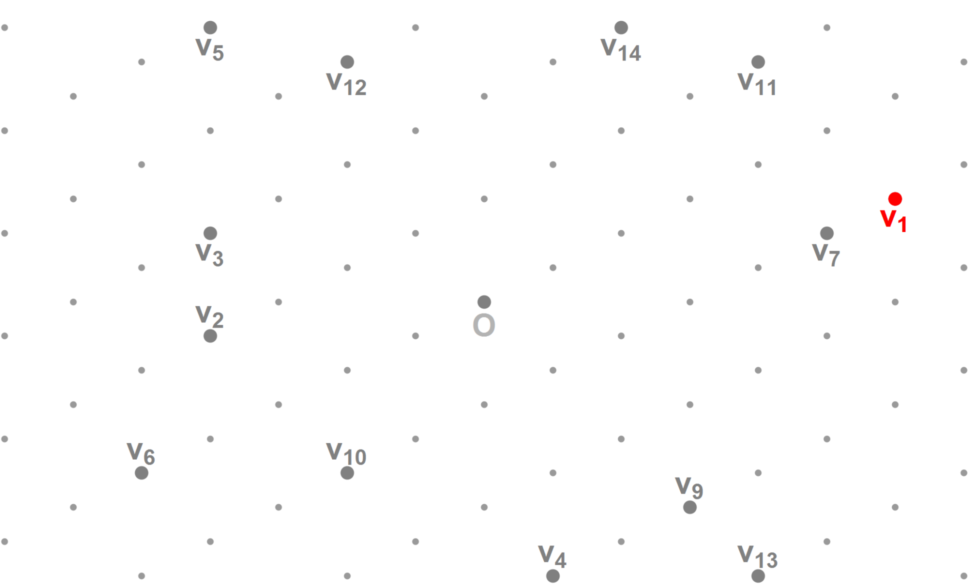

Saturation

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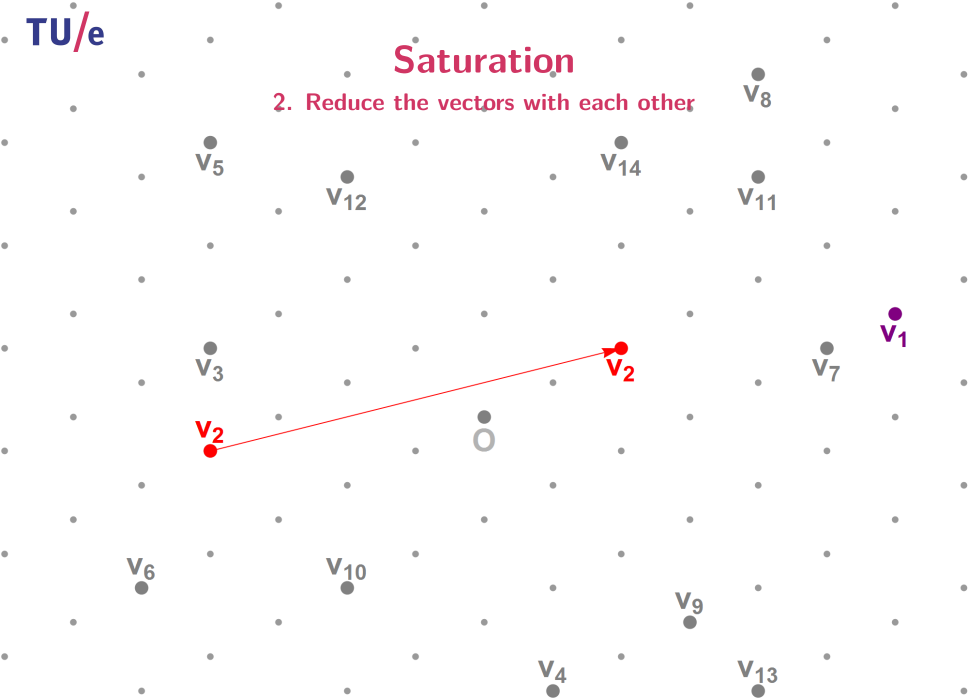
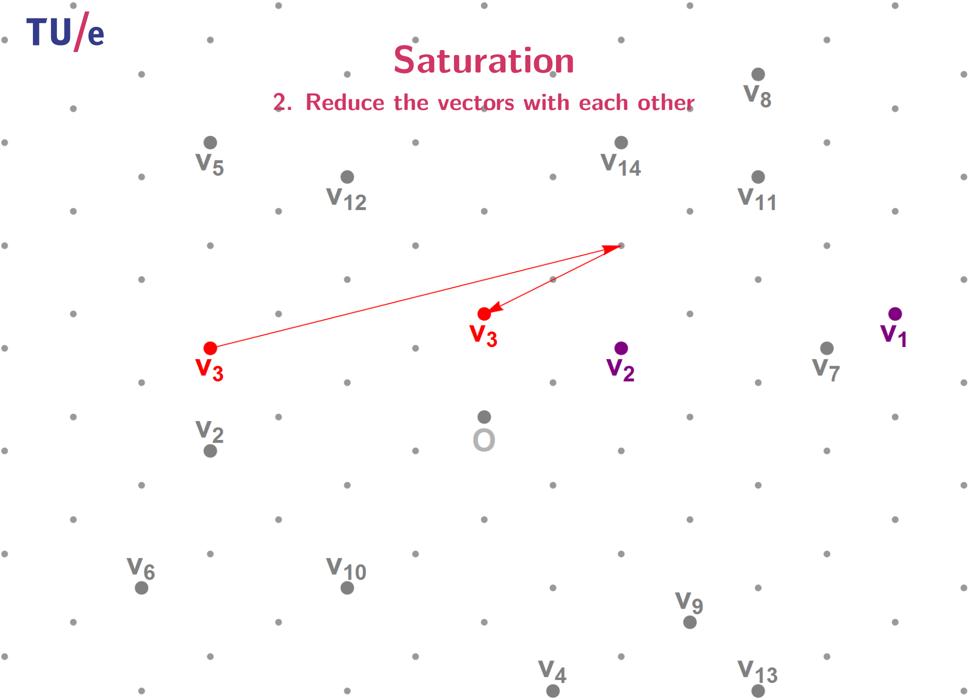


Saturation

2. Reduce the vectors with each other



Saturation

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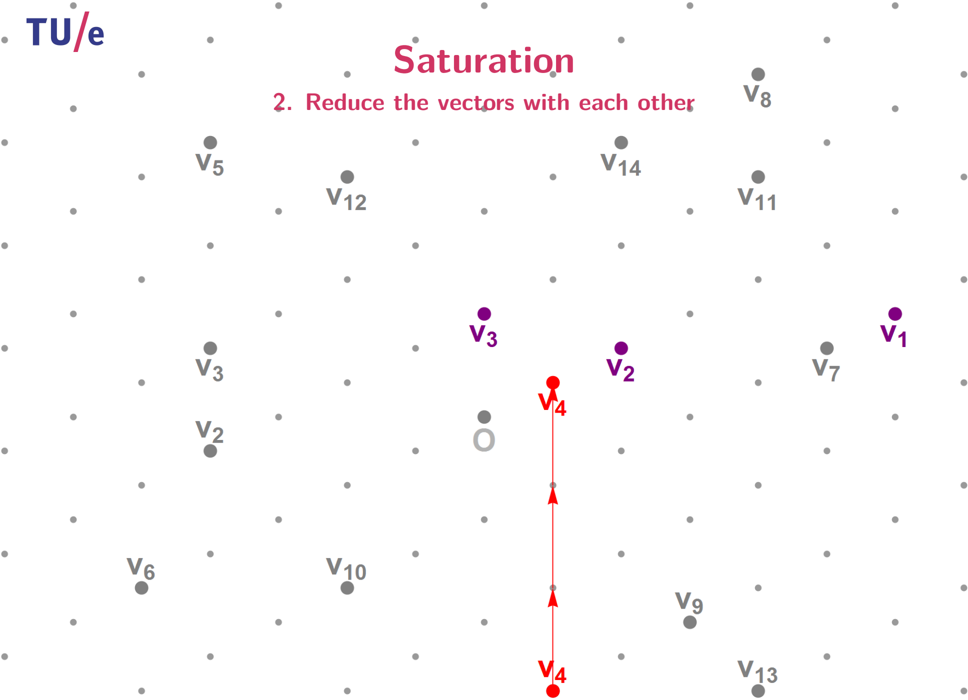

Saturation

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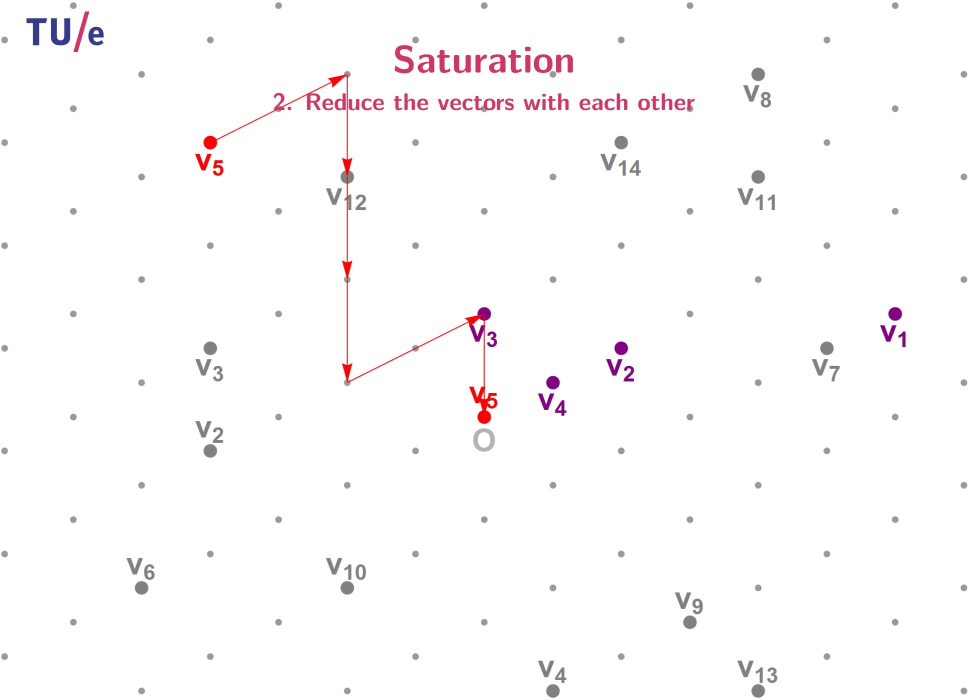

Saturation

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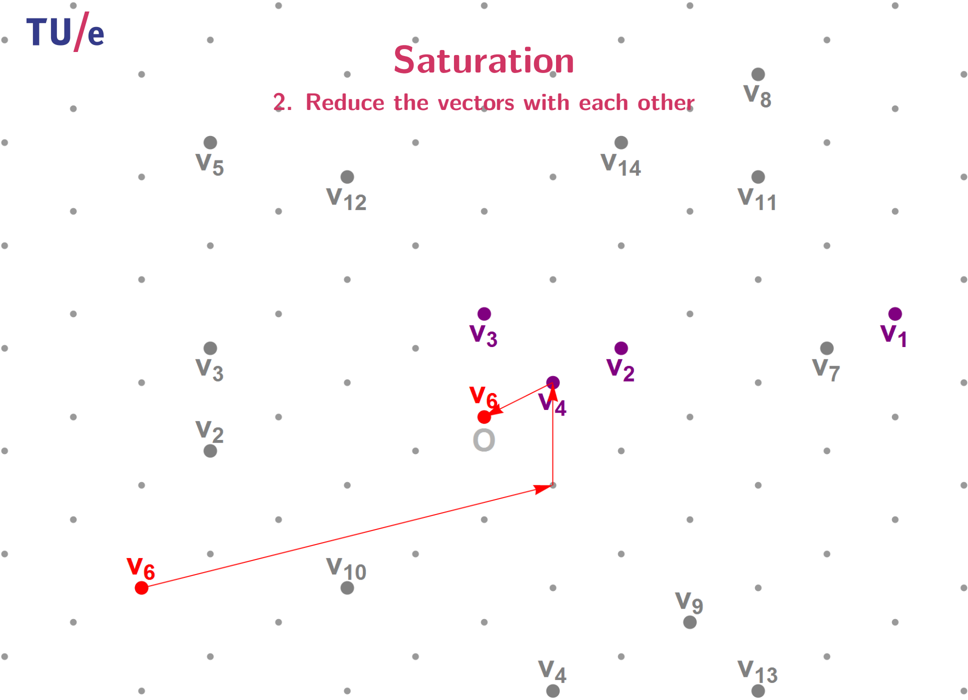
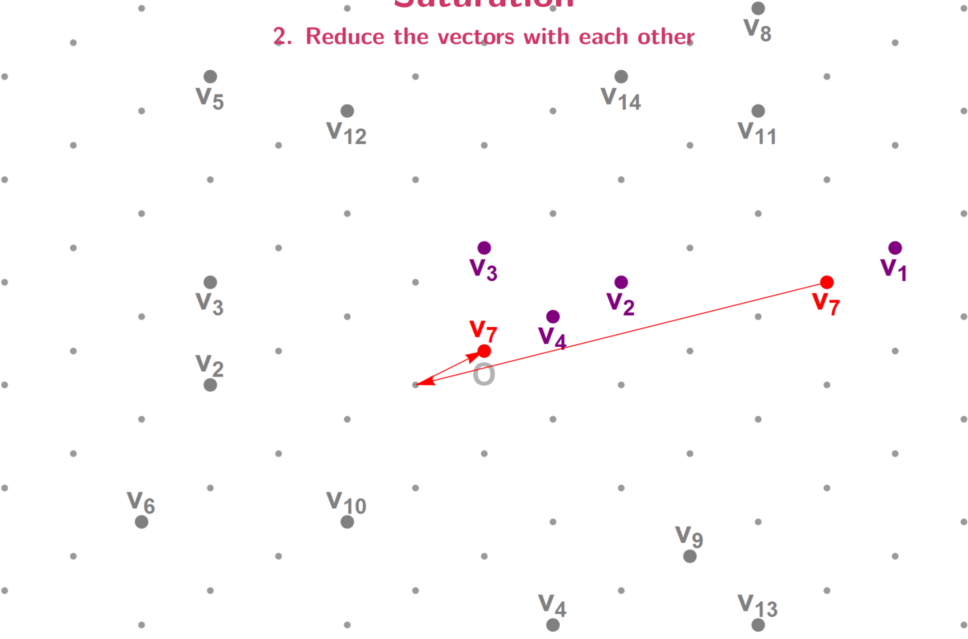


TU/e

Saturation

2. Reduce the vectors with each other



Saturation

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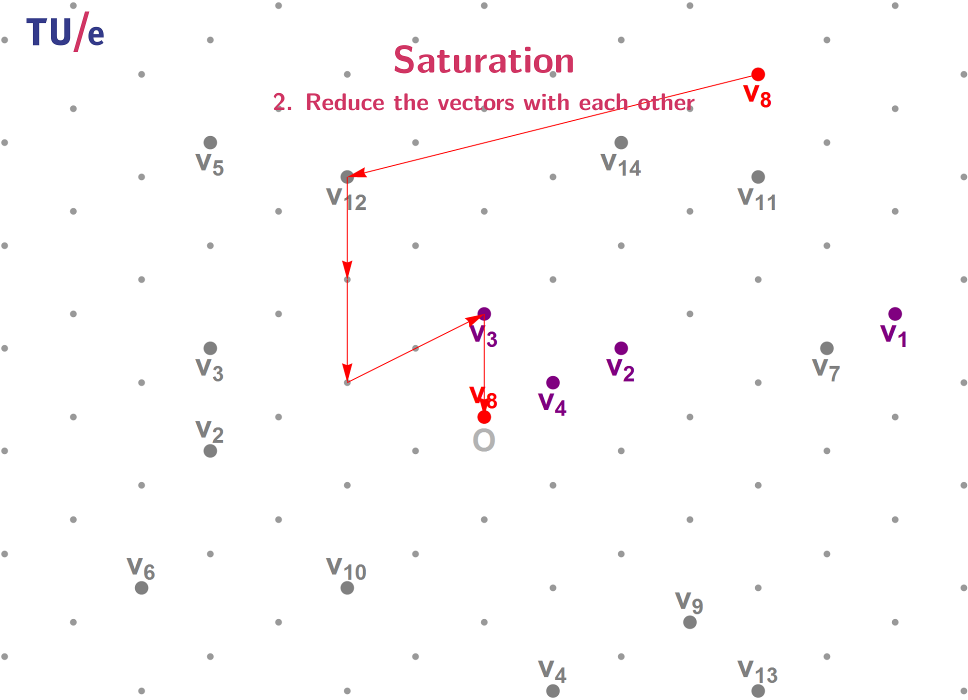

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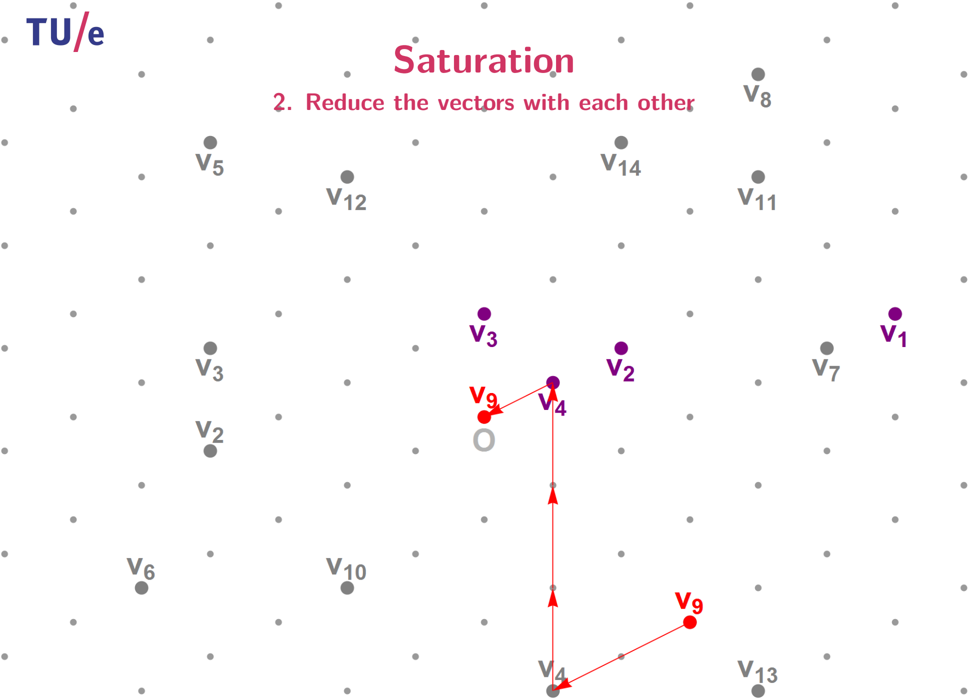

TU/e

Saturation

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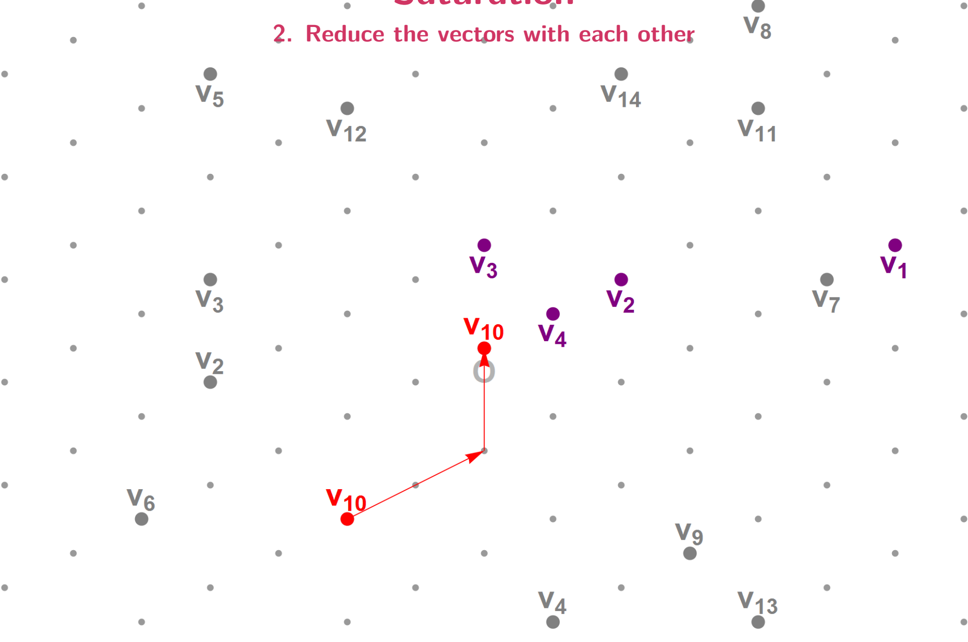

TU/e

Saturation

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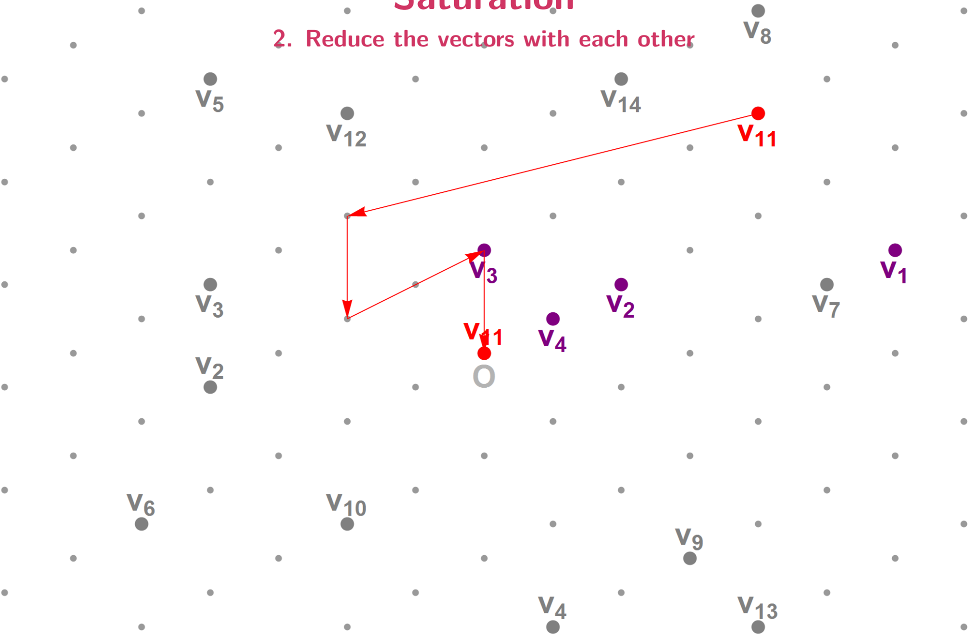

Saturation

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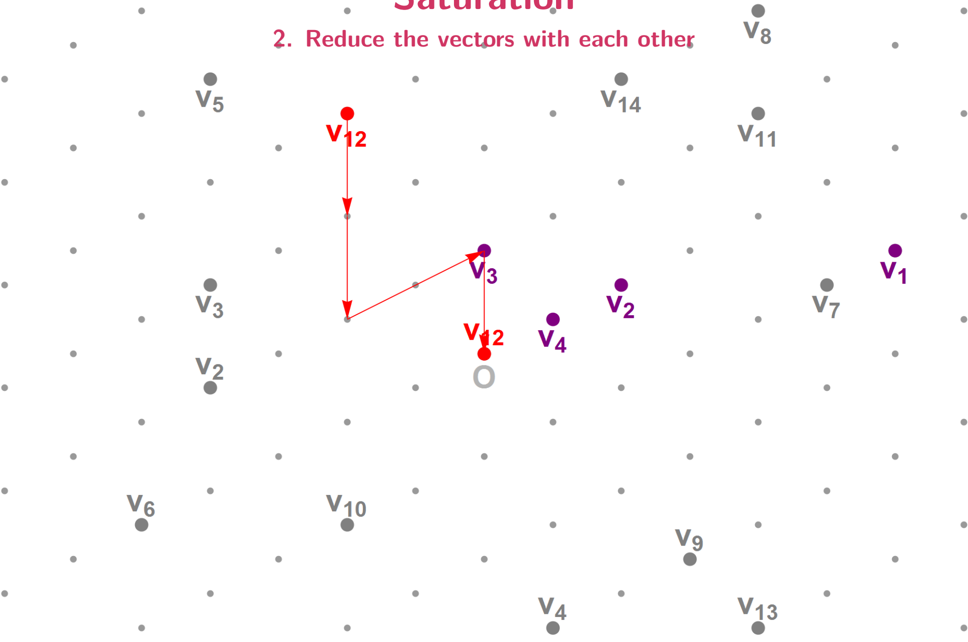

Saturation

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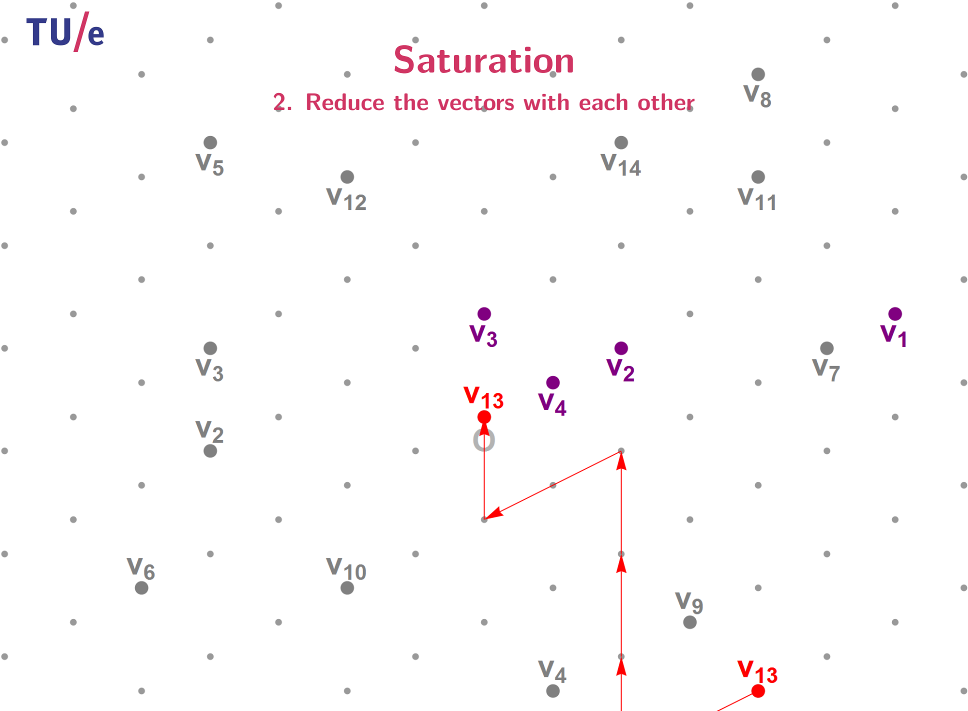

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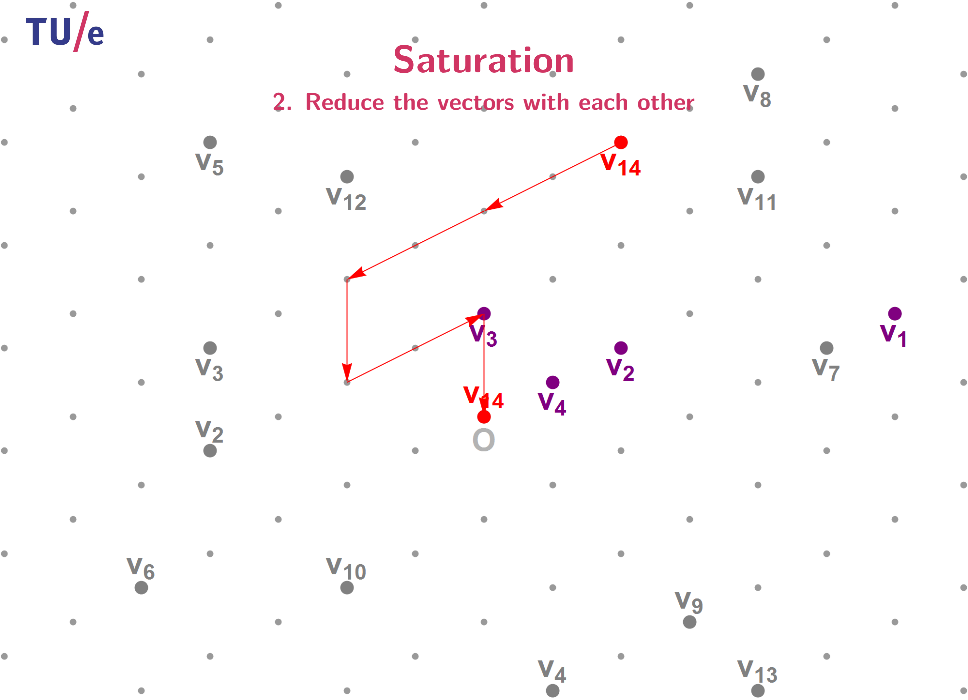

Saturation

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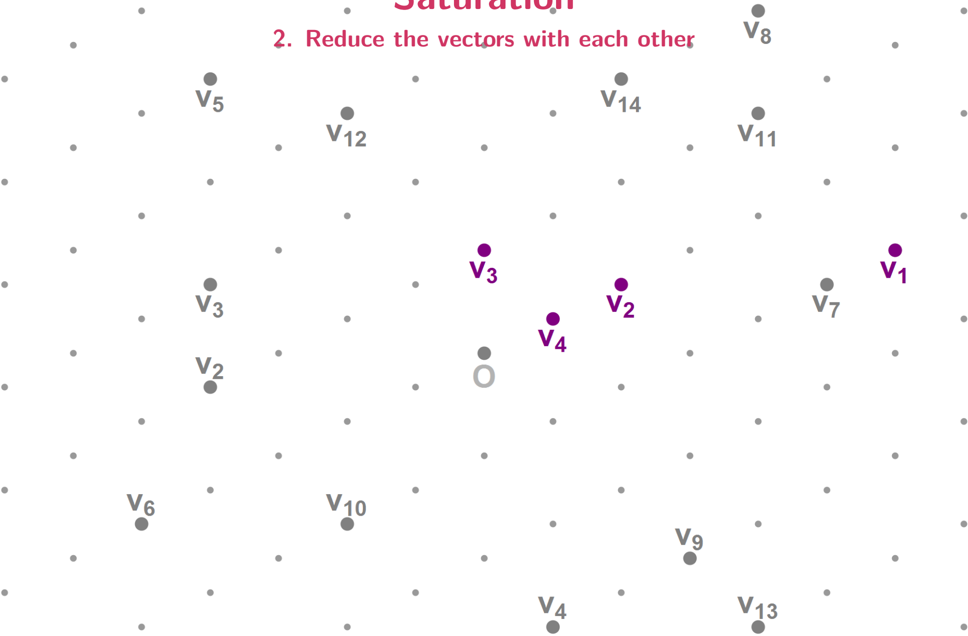
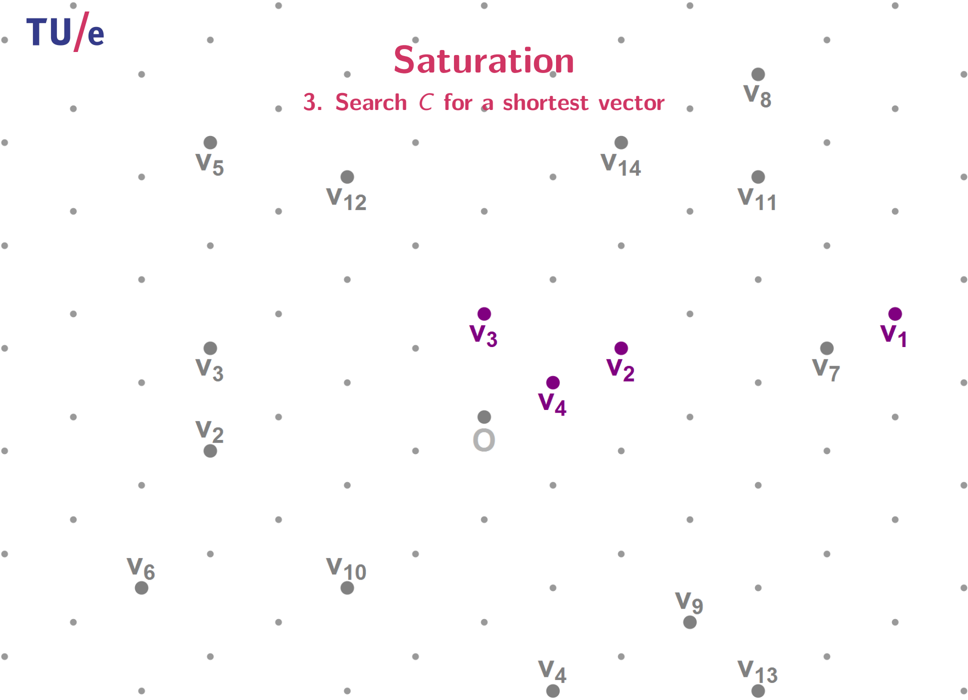


Saturation

3. Search C for a shortest vector



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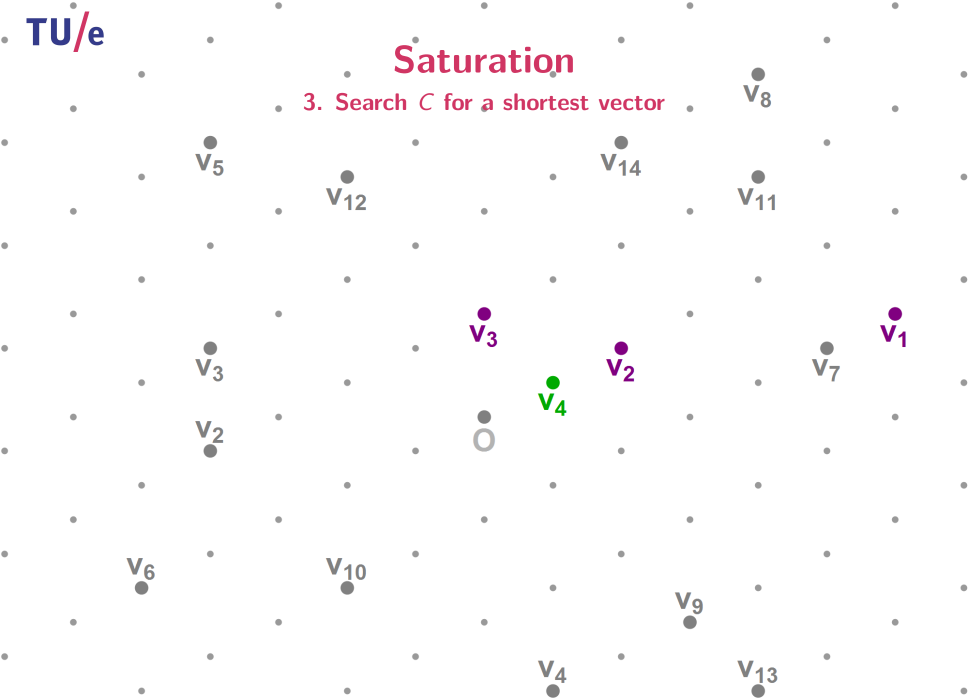

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Complexity?

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- Quantum Time:

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- Quantum speed-up: $\approx 25\%$ in the exponent

Overview

Provable results (large n asymptotics)

Table: Complexities of SVP algorithms in logarithmic leading order terms:

Algorithm	Classical Time	Classical Space	Quantum Time	Quantum Space
Enum. [Kan83]	$O(n \log n)$	$O(\log n)$	$O(n \log n)$	$O(\log n)$
Sieving [PS09]	$2.65n$	$1.33n$	$2.65n$	$1.33n$
Saturation [PS09]	$2.47n$	$1.24n$	$2.47n$	$1.24n$
Voronoi cell [MV10]	$2.00n$	$1.00n$	$2.00n$	$1.00n$

TU/e

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Provable results (large n asymptotics)

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	Classical		Quantum	
Algorithm	Time	Space	Time	Space
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Sieving [PS09]	$2.65n$	$1.33n$	$2.65n$	$1.33n$
Saturation [LMP13]	$2.47n$	$1.24n$	$\mathbf{1.80n}$	$\mathbf{1.29n}$
Voronoi cell [MV10]	$2.00n$	$1.00n$	$2.00n$	$1.00n$

Overview

Heuristic/Experimental results ($n \approx 100$)

Table: Complexities of SVP algorithms in logarithmic leading order terms:

	Classical		Quantum	
Algorithm	Time	Space	Time	Space
Enum. [GNR10]	$O(n \log n)$	$O(\log n)$	$O(n \log n)$	$O(\log n)$
Sieving [NV08]	$0.42n$	$0.21n$	$0.42n$	$0.21n$
Saturation [MV10]	$0.52n$	$0.21n$	$0.52n$	$0.21n$
Voronoi cell [MV10]	$2.00n$	$1.00n$	$2.00n$	$1.00n$

TU/e

Overview

Heuristic/Experimental results ($n \approx 100$)

Table: Complexities of SVP algorithms in logarithmic leading order terms:

	Classical		Quantum	
Algorithm	Time	Space	Time	Space
Enum. [GNR10]	$O(n \log n)$	$O(\log n)$	$O(n \log n)$	$O(\log n)$
Sieving [LMP13]	$0.42n$	$0.21n$	$\mathbf{0.32n}$	$\mathbf{0.21n}$
Saturation [LMP13]	$0.52n$	$0.21n$	$\mathbf{0.39n}$	$\mathbf{0.21n}$
Voronoi cell [MV10]	$2.00n$	$1.00n$	$2.00n$	$1.00n$

Conclusion

Using Grover search speeds up some SVP algorithms

- Faster sieving algorithms (exponent: $\approx -25\%$)
- Faster saturation algorithms (exponent: $\approx -25\%$)

Open quantum-problems

- Quantum speed-ups for other methods?
- Use other quantum algorithms?
- Build a quantum computer?

Questions