Sieving for shortest vectors in lattices using locality-sensitive hashing

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Microsoft Research, Redmond (WA), USA
(June 22, 2015)

Lattices

What is a lattice?

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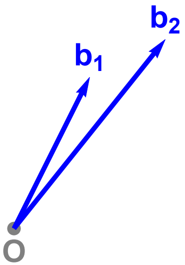

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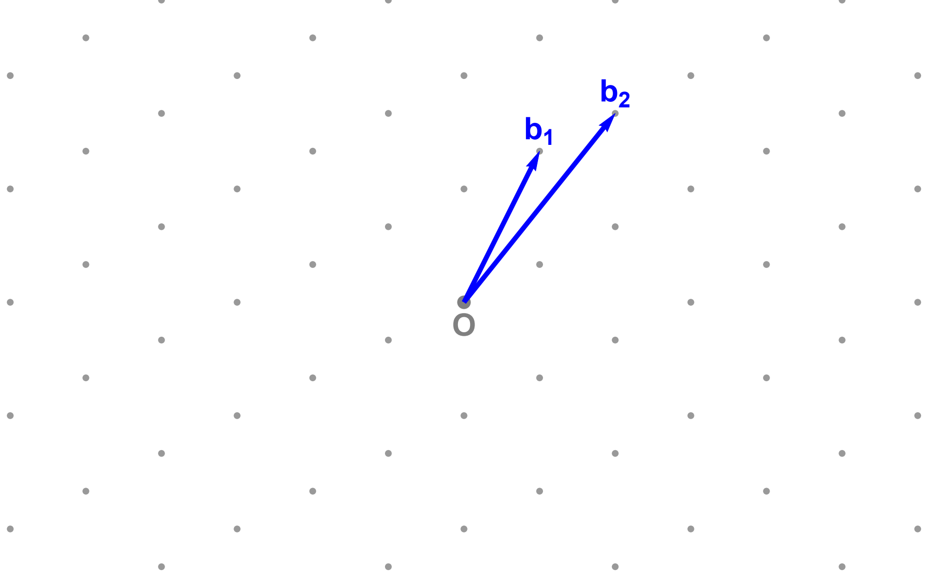
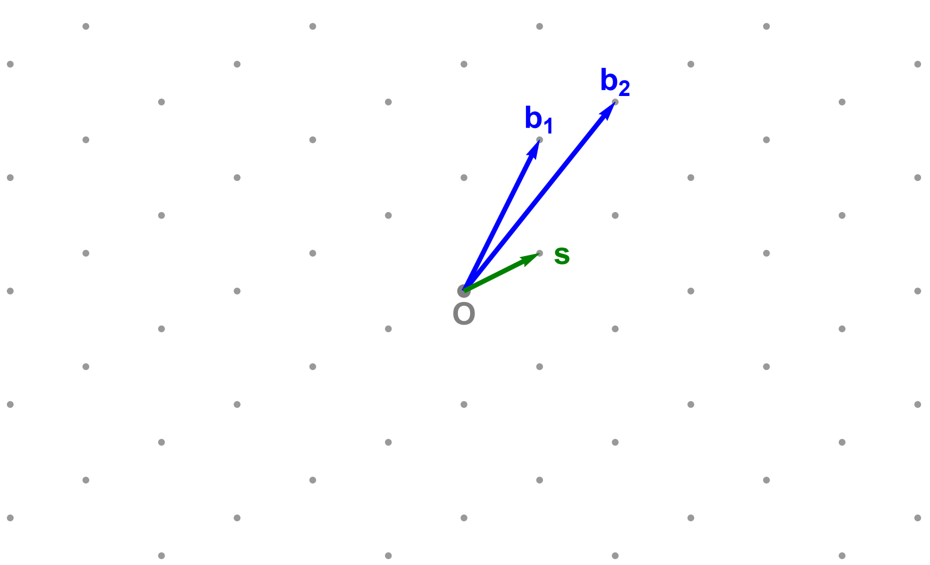


Lattices

Shortest Vector Problem (SVP)



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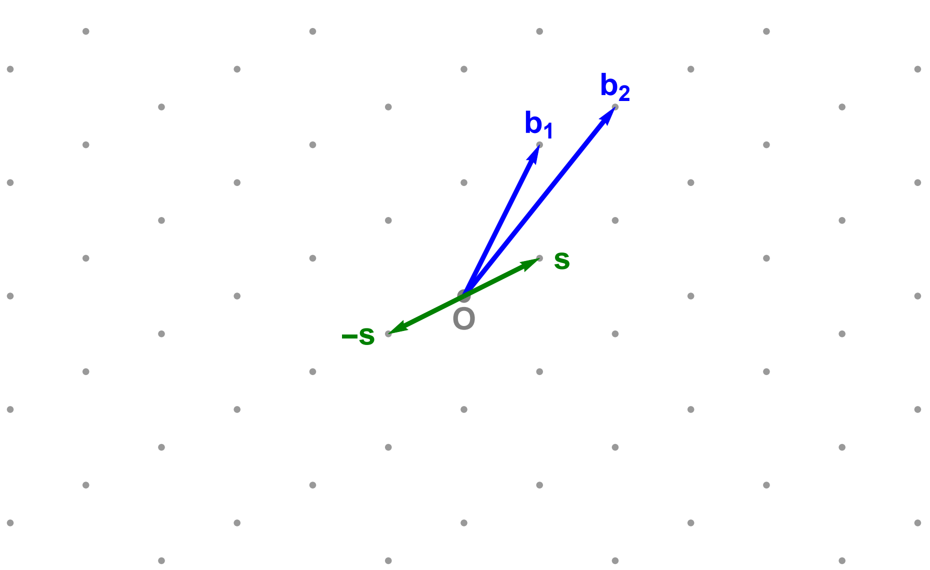

Lattices

SVP algorithms

	Algorithm	$\log_2(\text{Time})$	$\log_2(\text{Space})$
Provable SVP	Enumeration [Poh81, Kan83, ..., GNR10]	$\Omega(n \log n)$	$O(\log n)$
	AKS-sieve [AKS01, NV08, MV10, HPS11]	$3.398n$	$1.985n$
	ListSieve [MV10, MDB14]	$3.199n$	$1.327n$
	AKS-sieve-birthday [PS09, HPS11]	$2.648n$	$1.324n$
	ListSieve-birthday [PS09]	$2.465n$	$1.233n$
	Voronoi cell algorithm [MV10b]	$2.000n$	$1.000n$
	Discrete Gaussian sampling [ADRS15, ADS15]	$1.000n$	$1.000n$
Heuristic SVP	NV-sieve [NV08]	$0.415n$	$0.208n$
	GaussSieve [MV10, ..., IKMT14, BNvdP14]	$0.415n$?	$0.208n$
	Two-level sieve [WLTB11]	$0.384n$	$0.256n$
	Three-level sieve [ZPH13]	$0.3778n$	$0.283n$
	Decomposition approach [BGJ14]	$0.3774n$	$0.293n$
	HashSieve [Laa15, MLB15]	$0.337n$	$0.208n$*
	Another NNS sieve [BGJ15]	$0.311n$	$0.208n$
	SphereSieve [LdW15]	$0.298n$	$0.208n$
	CrossPolytopeSieve [BL15]	$0.298n$	$0.208n$*

TU/e

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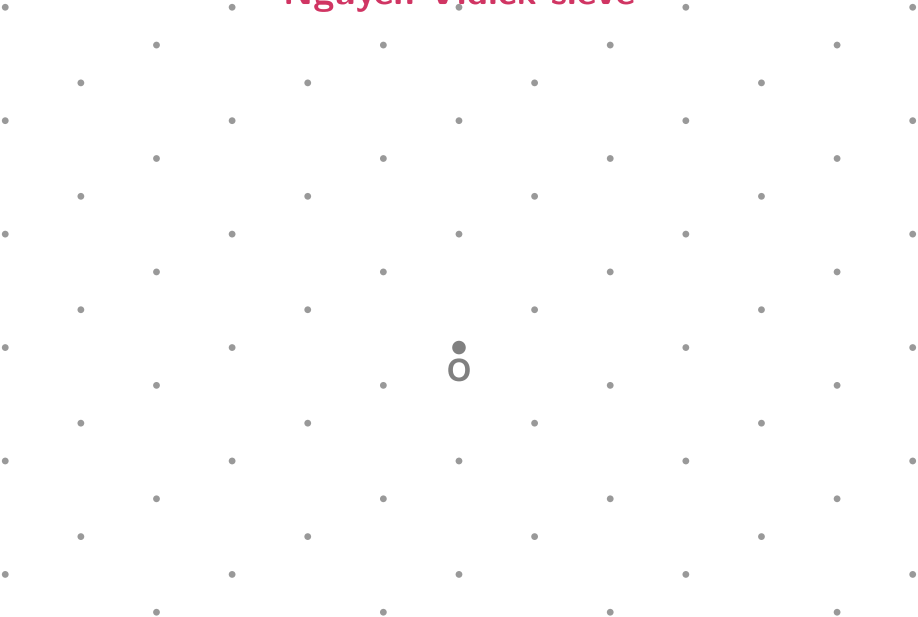
Nguyen-Vidick sieve



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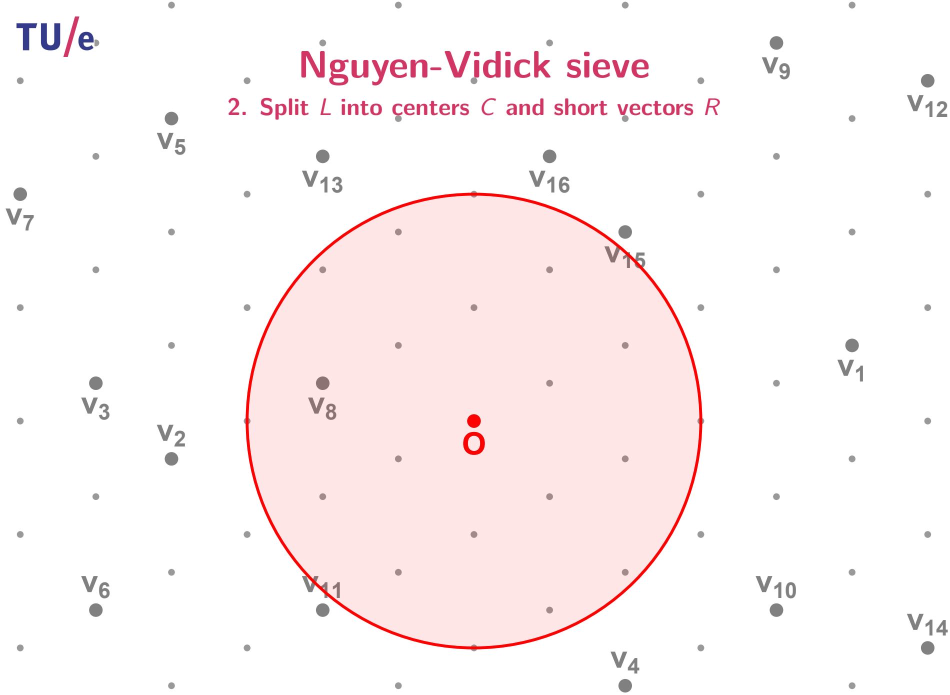

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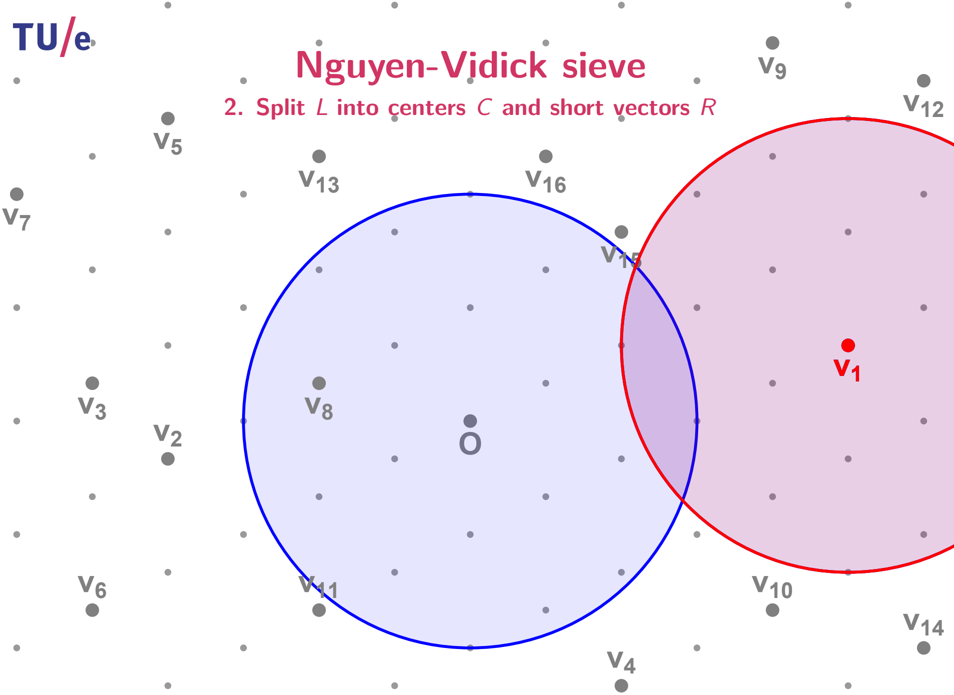

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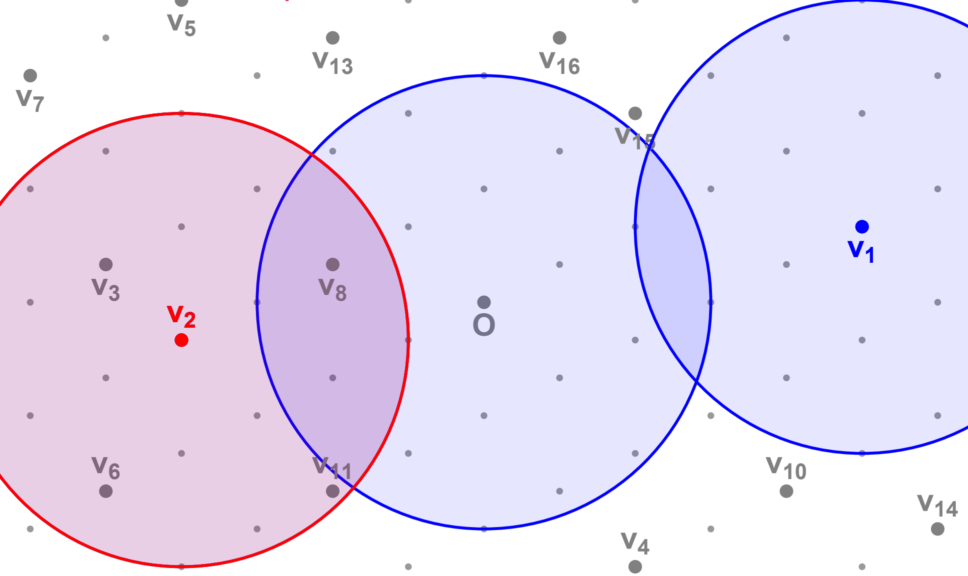

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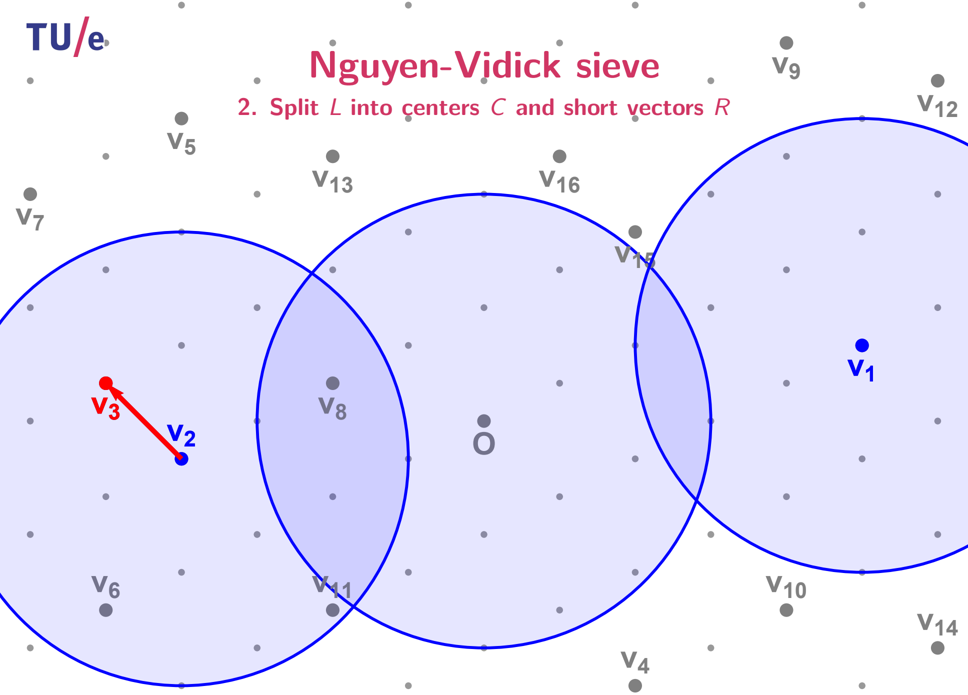

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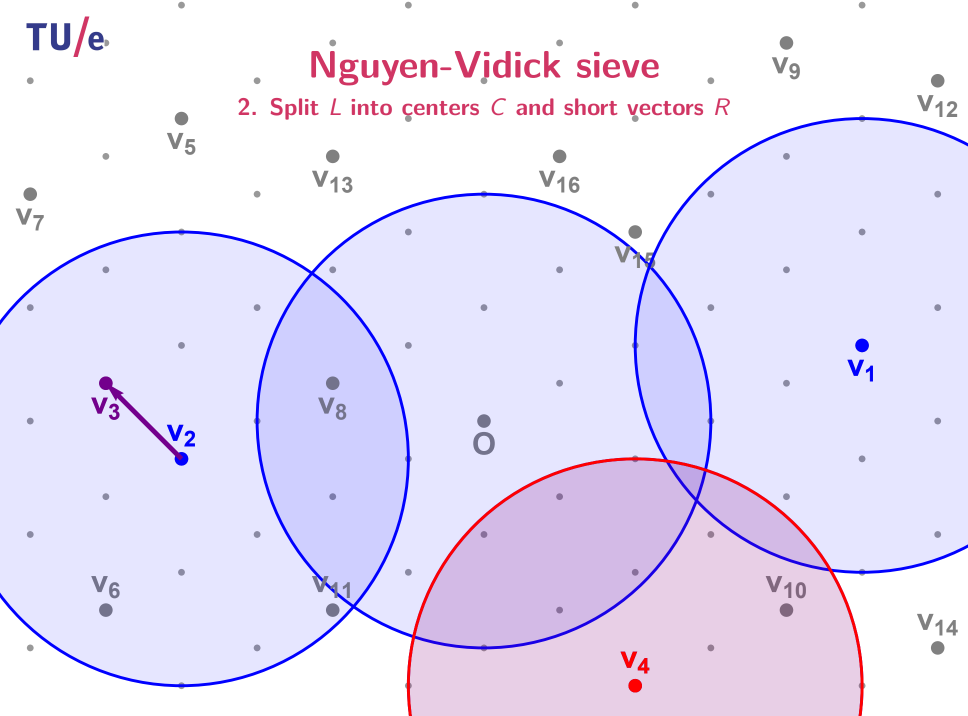

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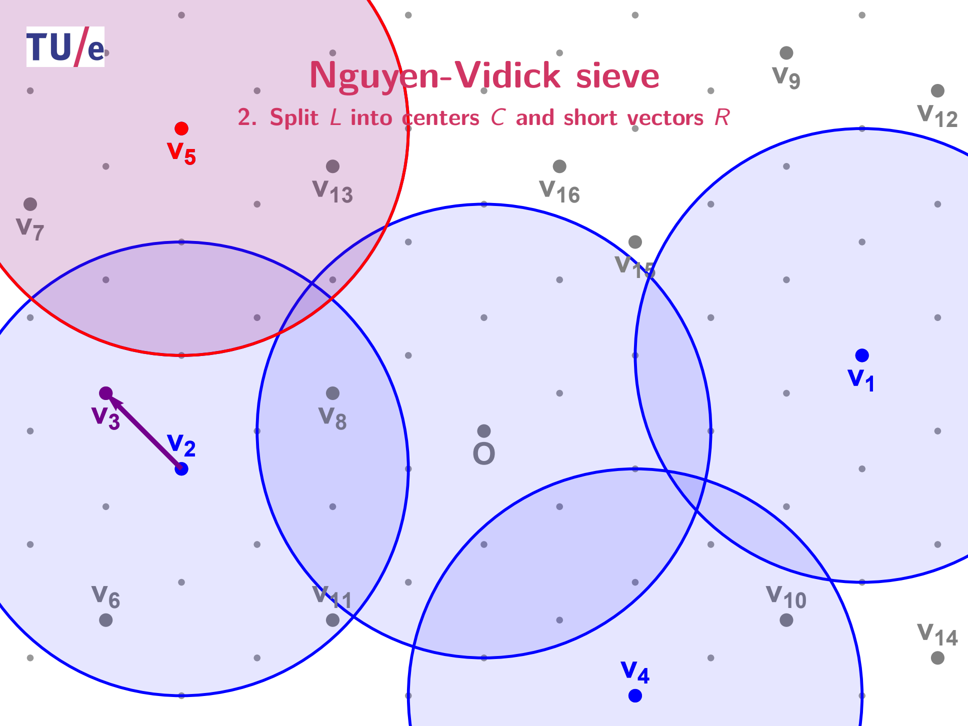

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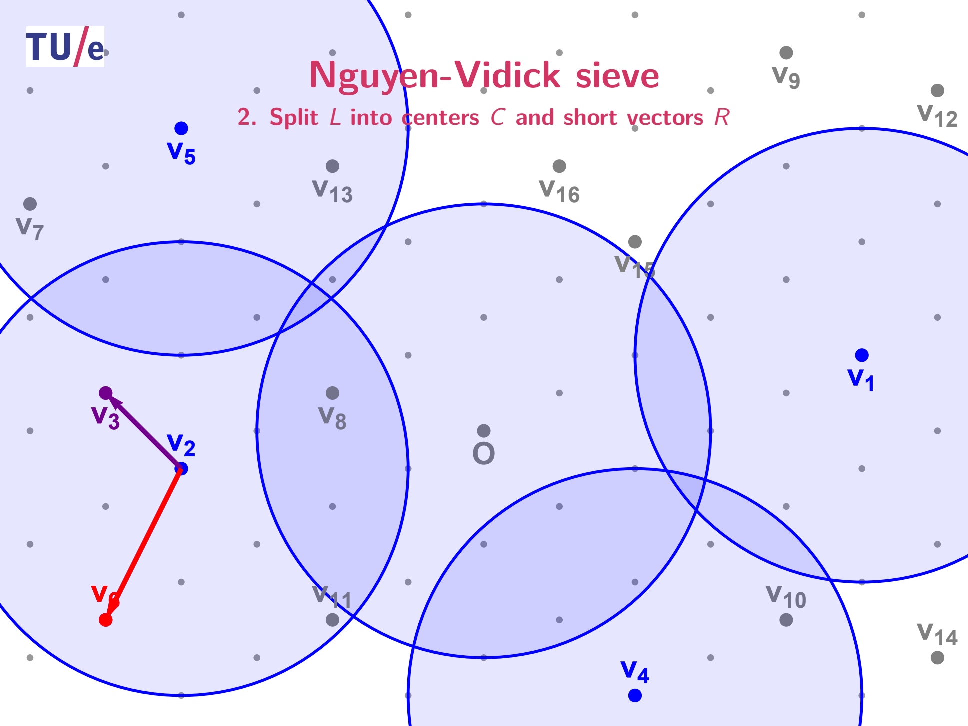

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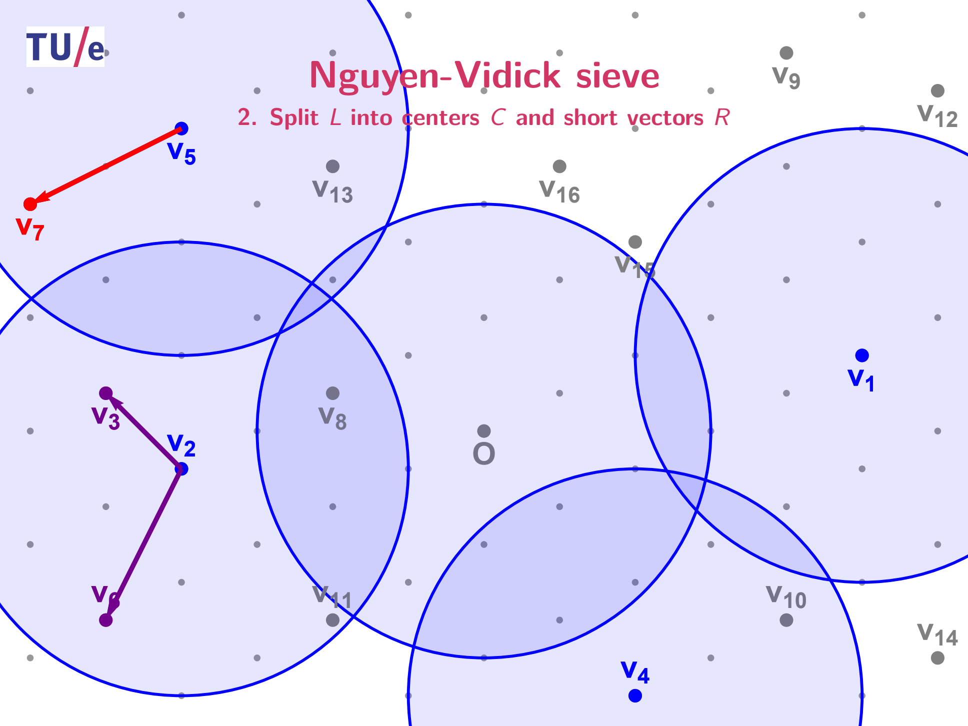

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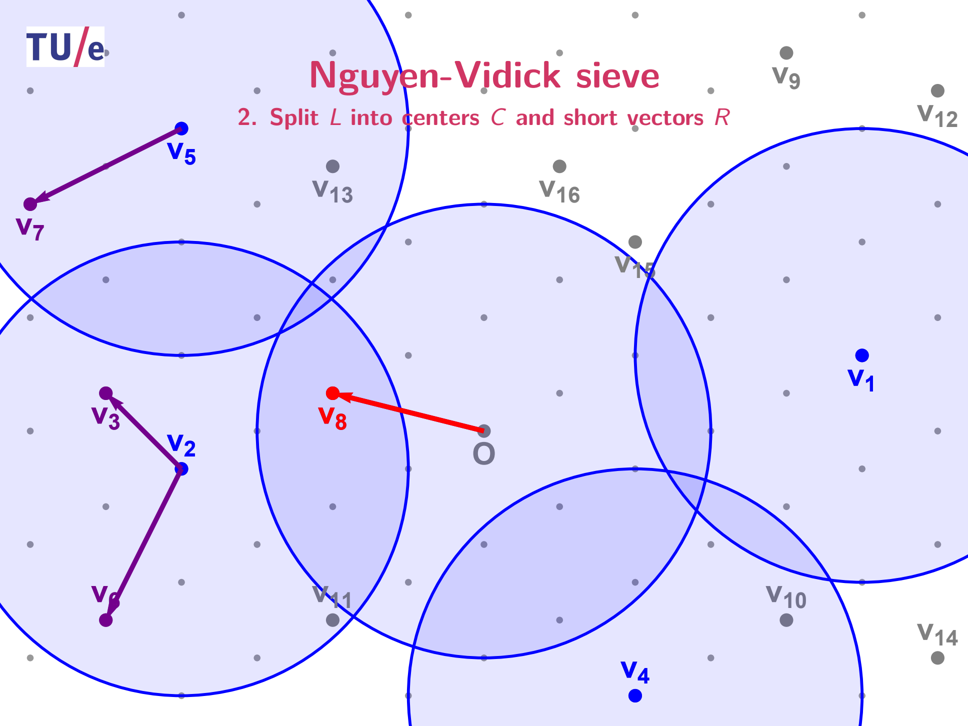

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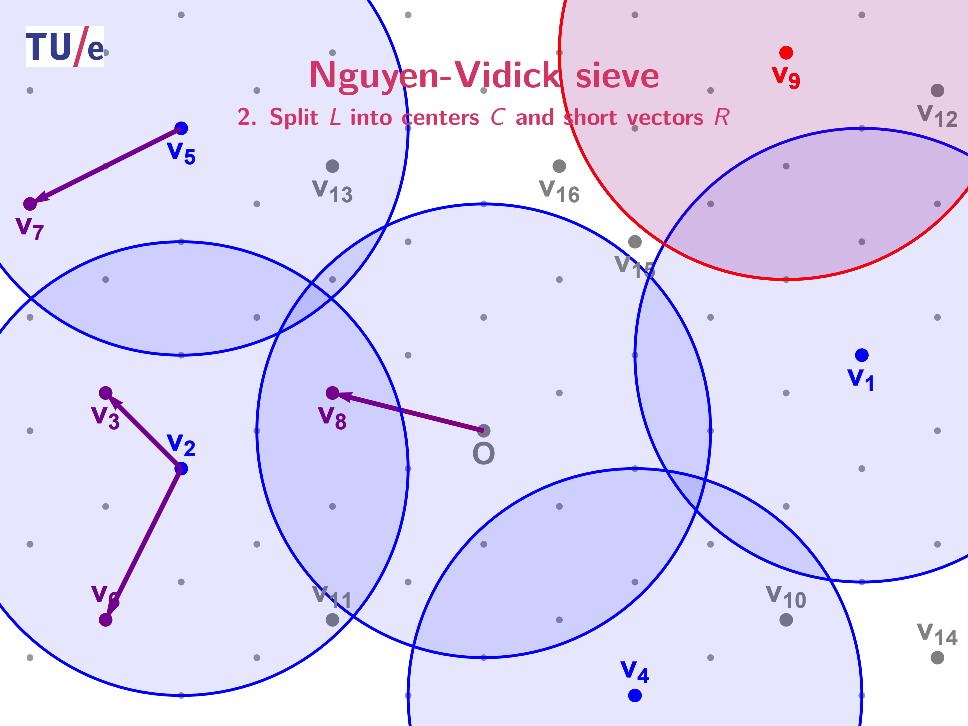

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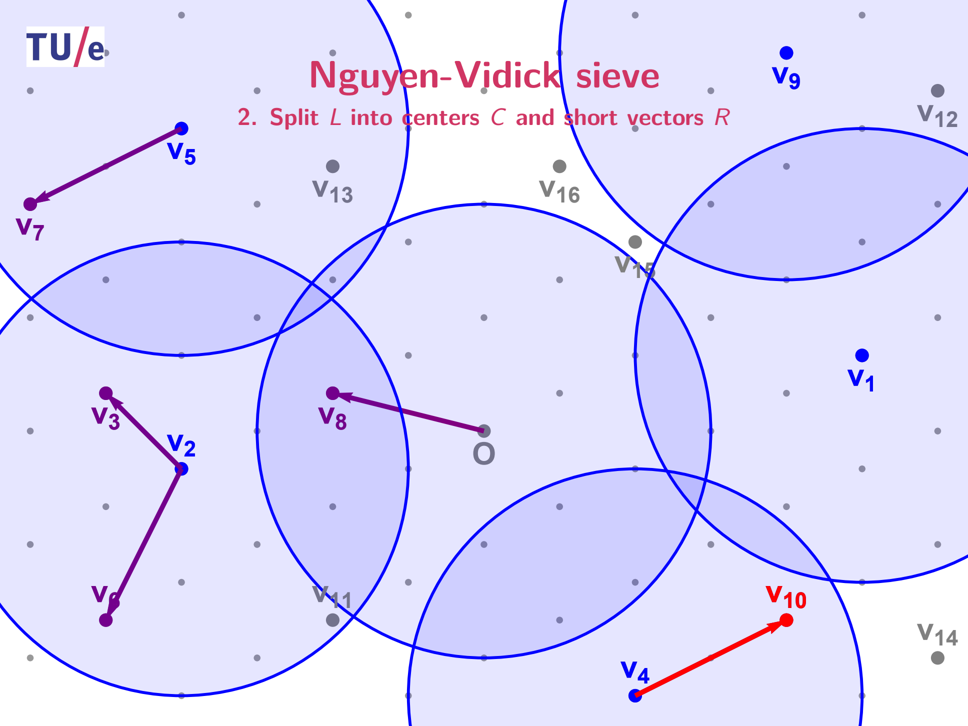

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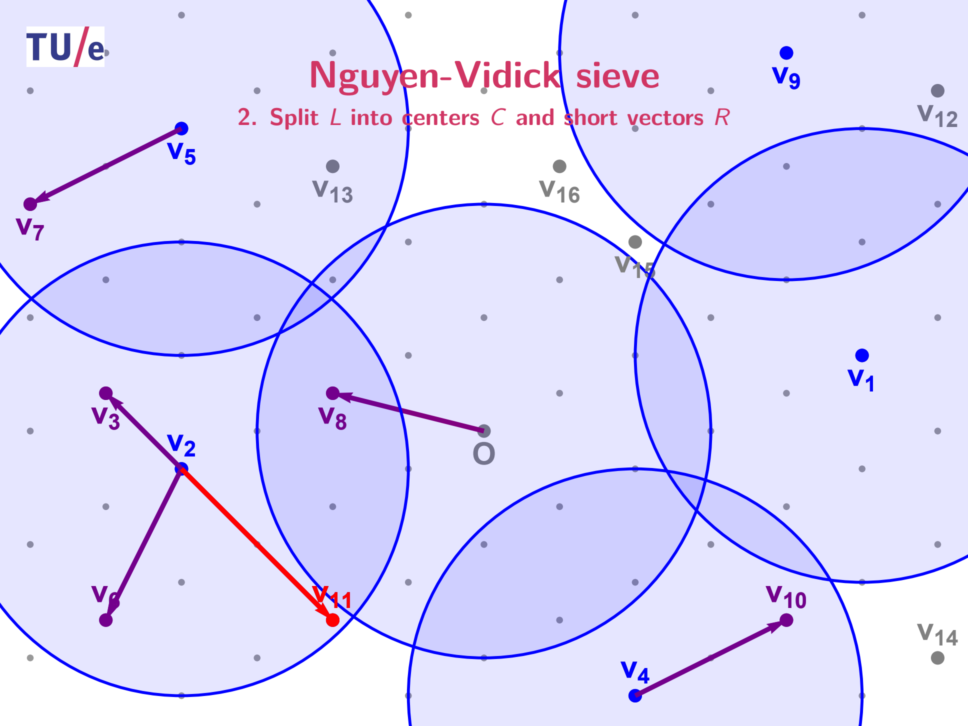

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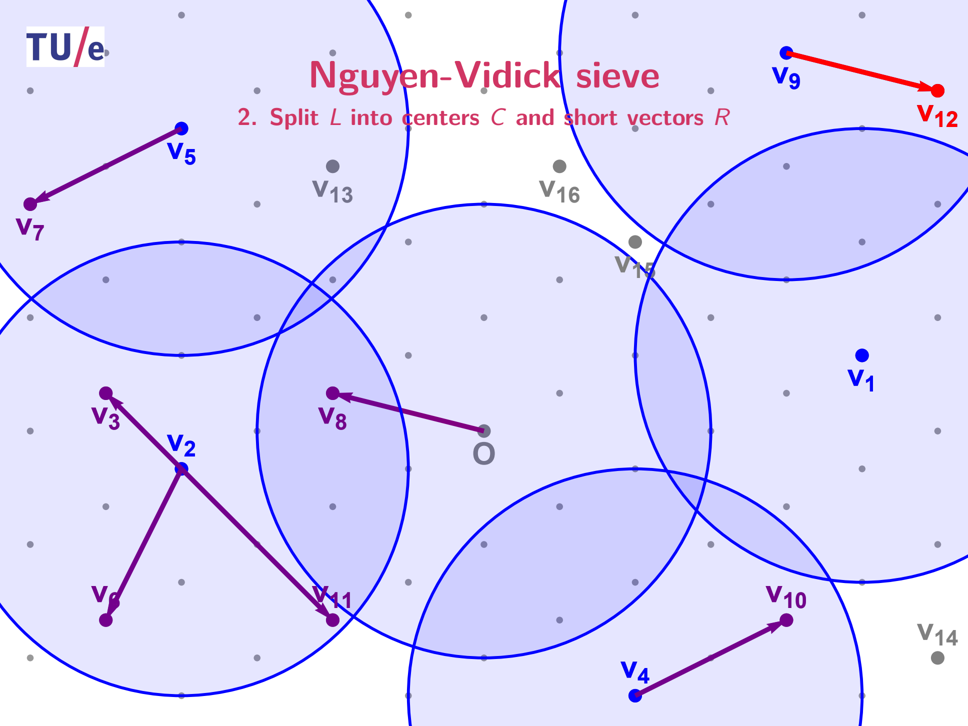

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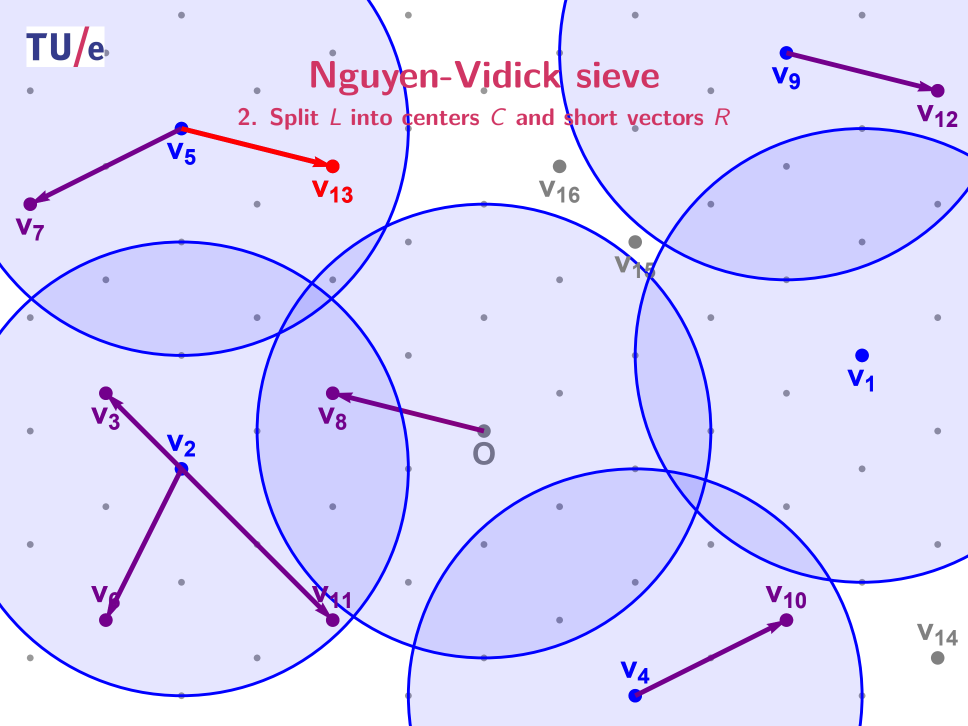

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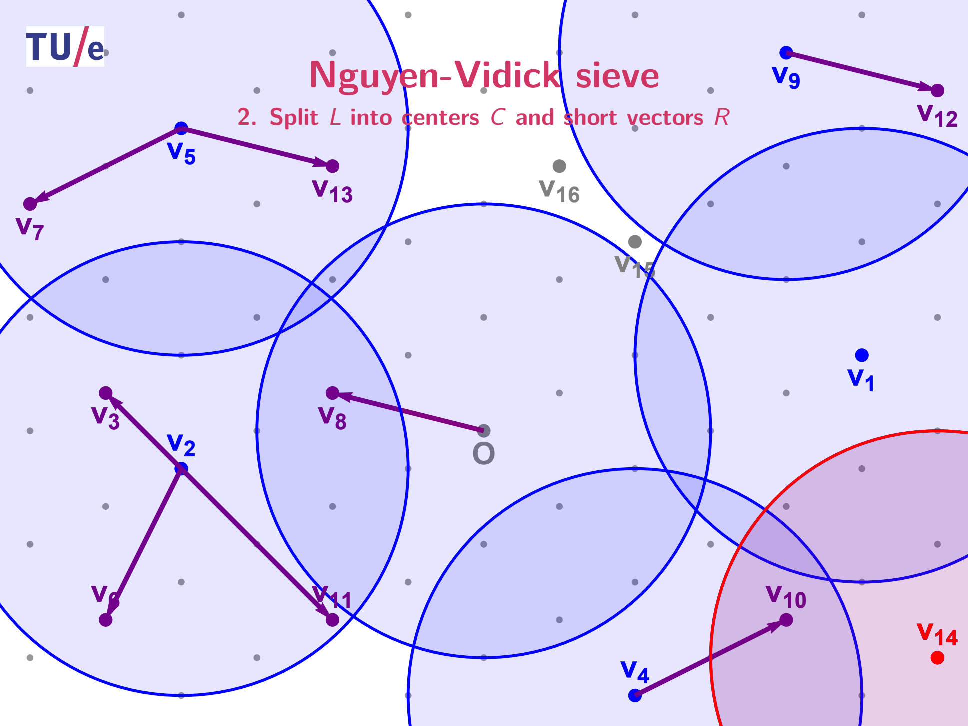

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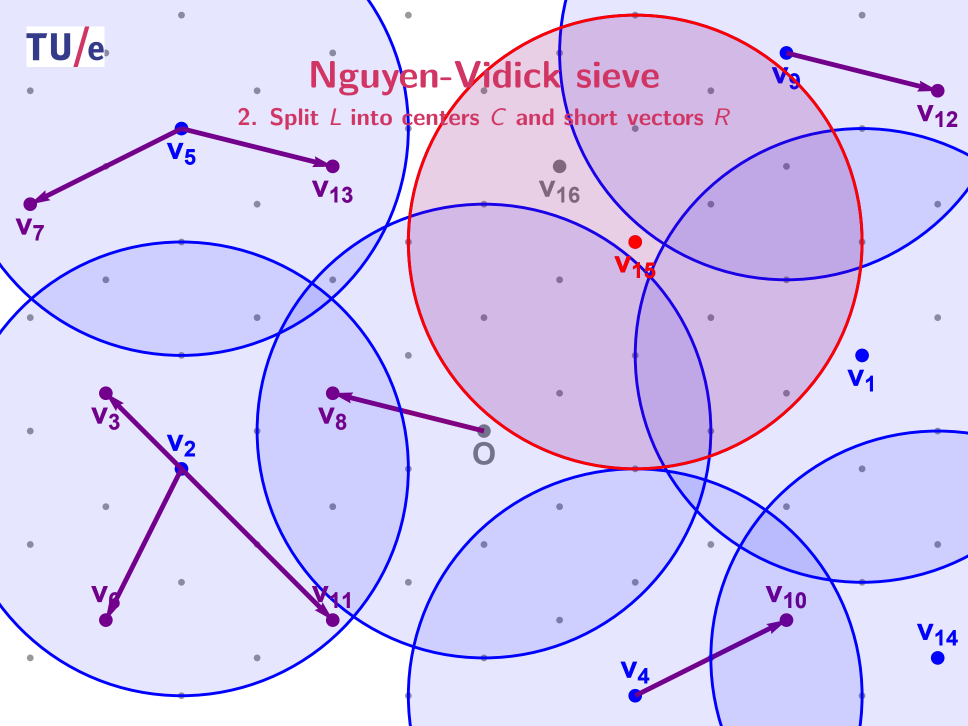

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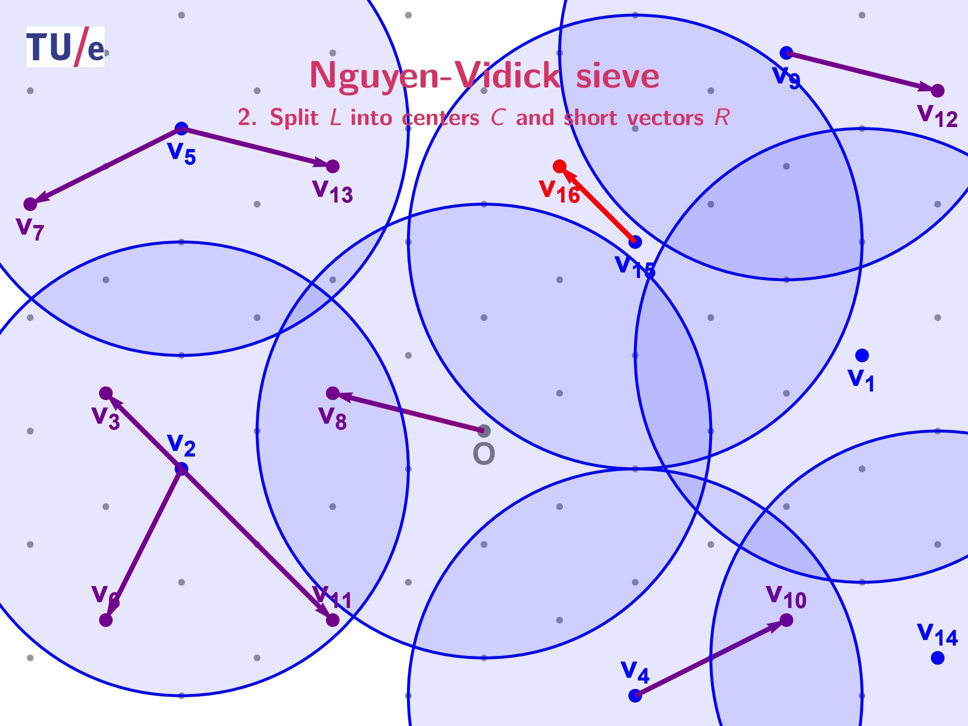

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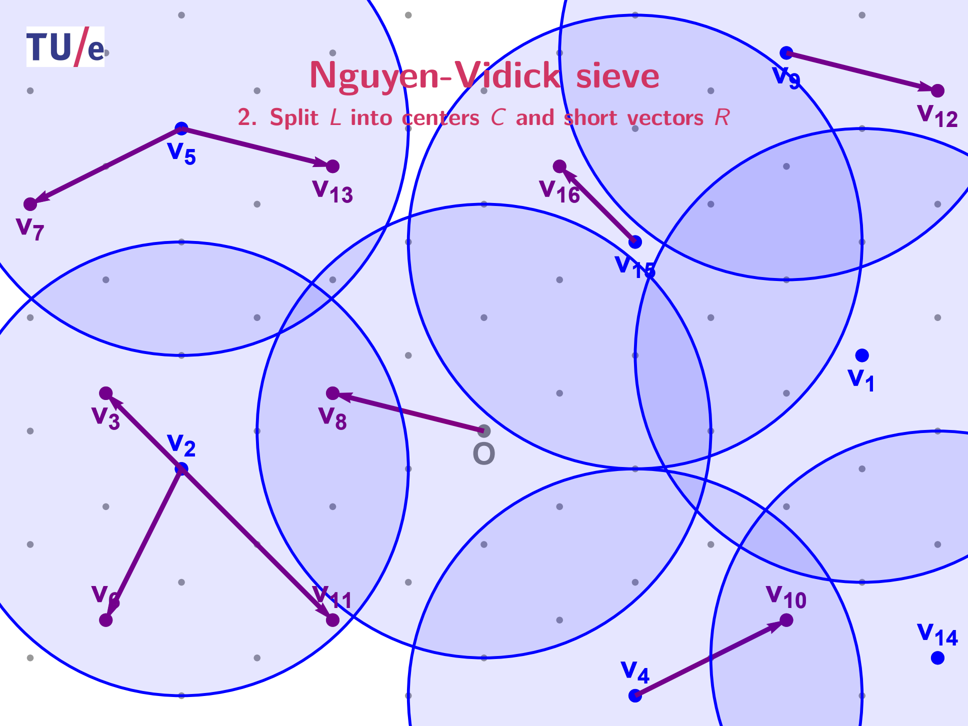

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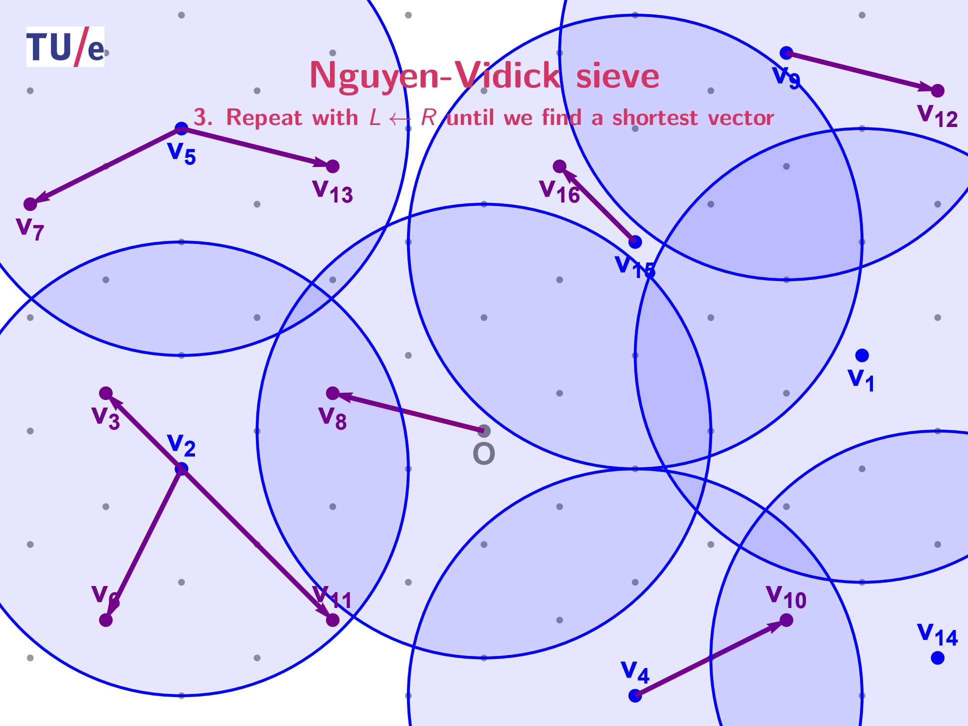

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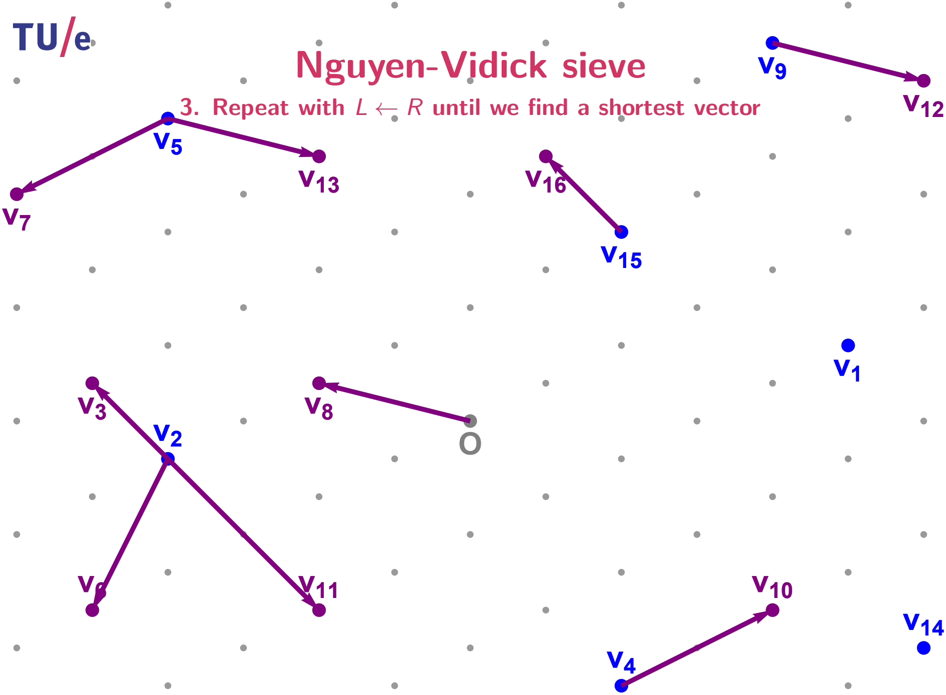

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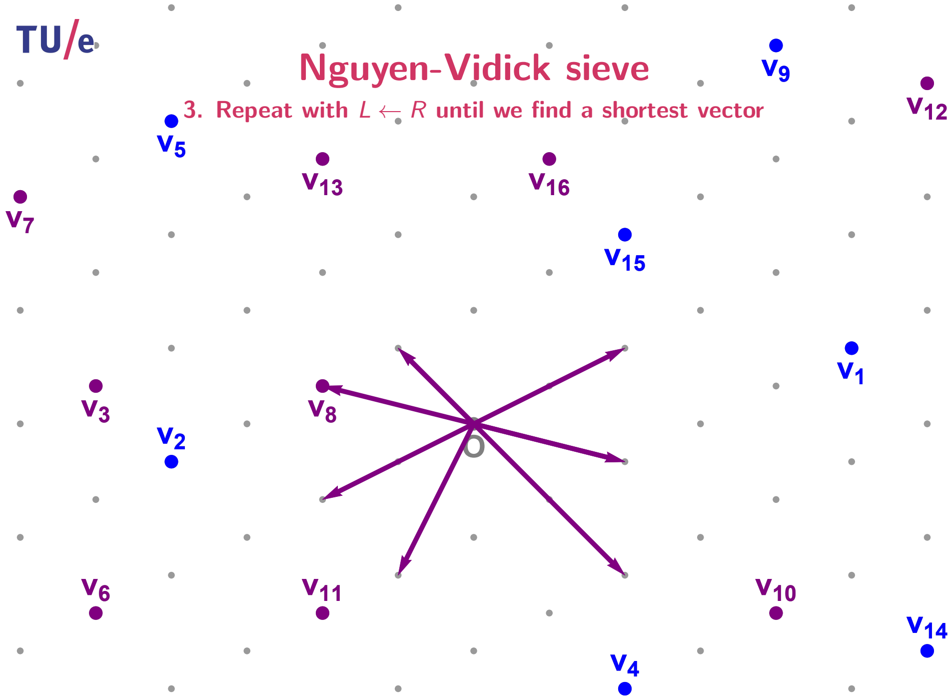

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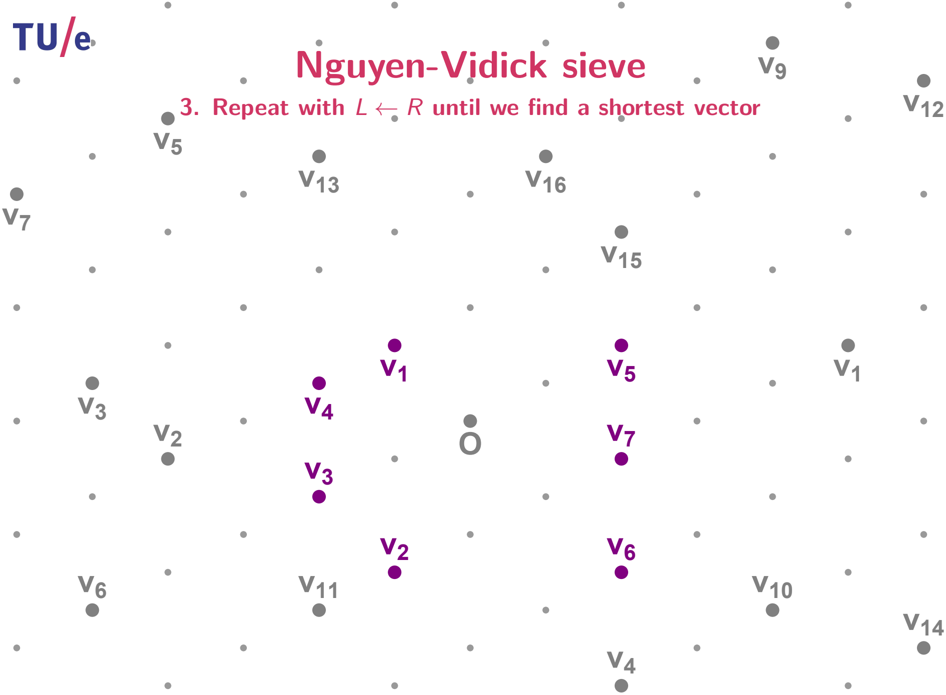

Nguyen-Vidick sieve

Overview

Nguyen-Vidick sieve

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- Space complexity: $\sqrt{4/3}^n \approx 2^{0.21n+o(n)}$ vectors
  - Need $\sqrt{4/3}^n$ vectors to cover all corners of $\mathbb{R}^n$

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  - Comparing a target vector to all centers: $2^{0.21n+o(n)}$
  - Repeating this for each list vector: $2^{0.21n+o(n)}$
  - Repeating the whole sieving procedure: $\mathrm{poly}(n)$

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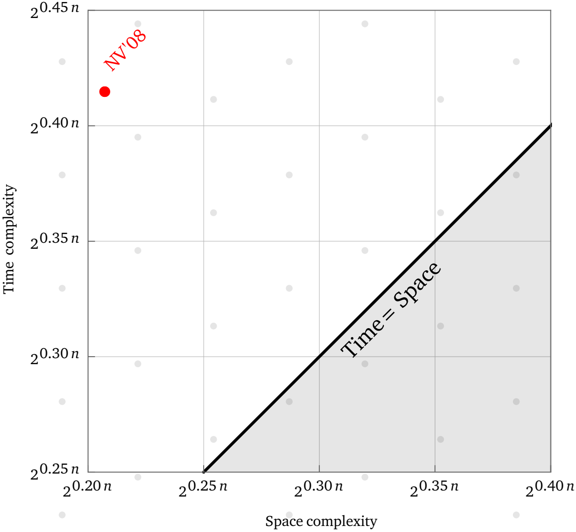
Heuristic (Nguyen and Vidick, J. Math. Crypt. '08)

The NV-sieve runs in time $2^{0.42n+o(n)}$ and space $2^{0.21n+o(n)}$.

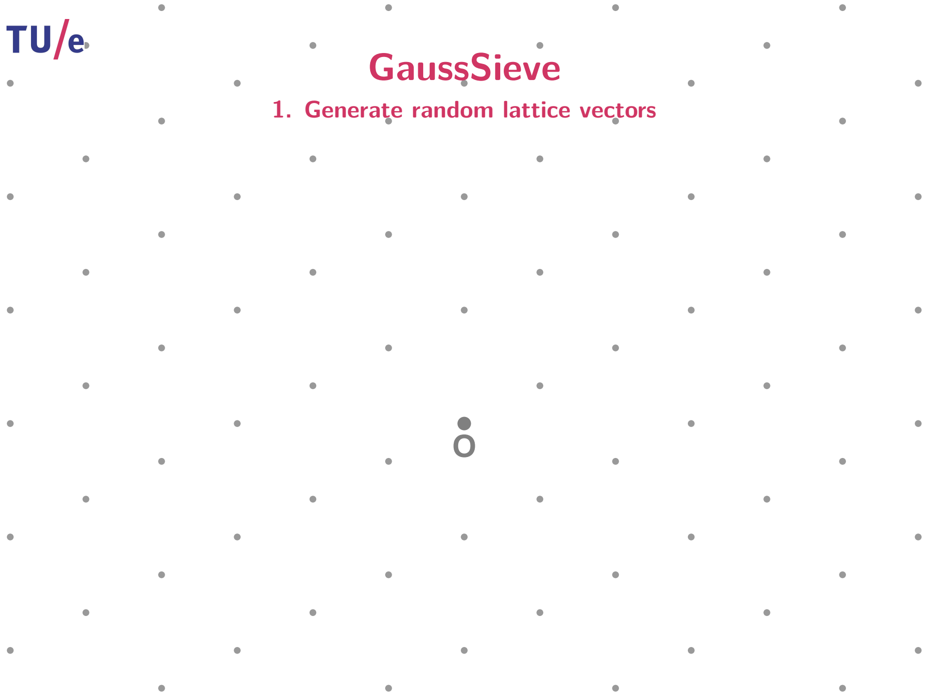
Nguyen-Vidick sieve

Space/time trade-off



GaussSieve

1. Generate random lattice vectors



GaussSieve

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GaussSieve

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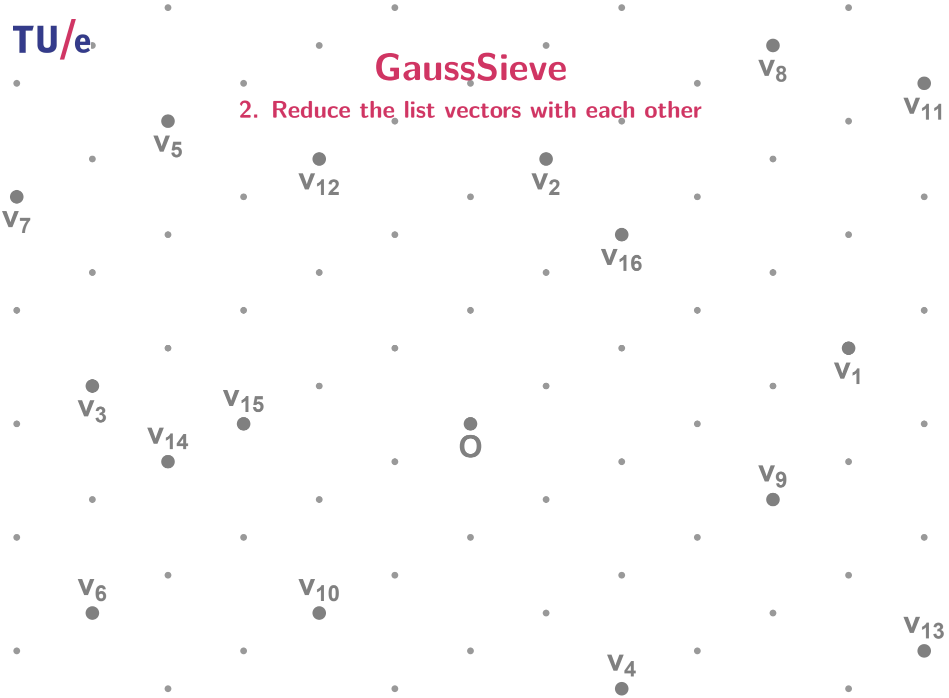

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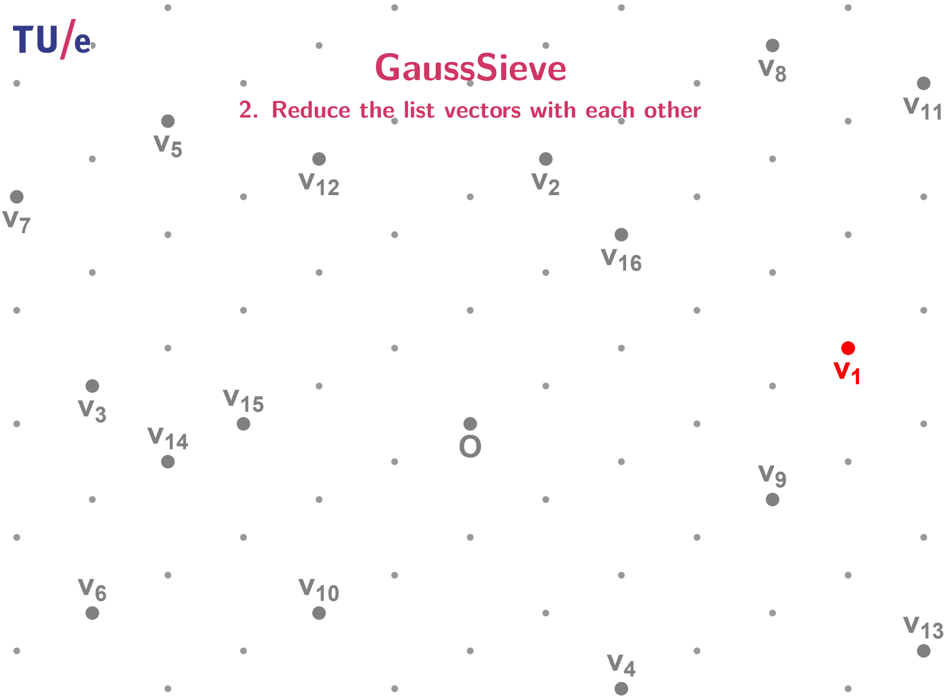

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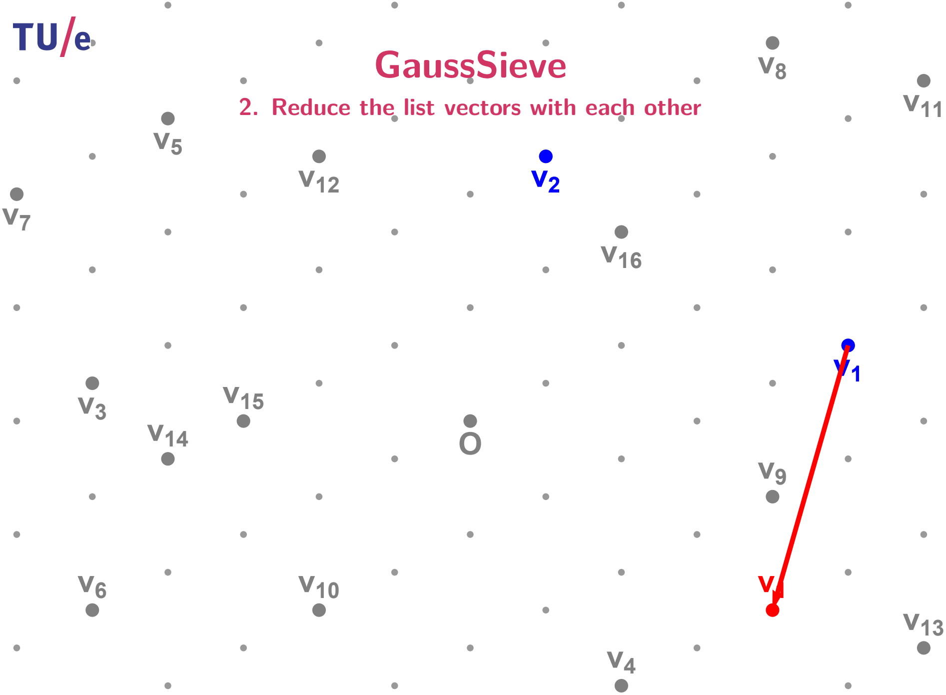

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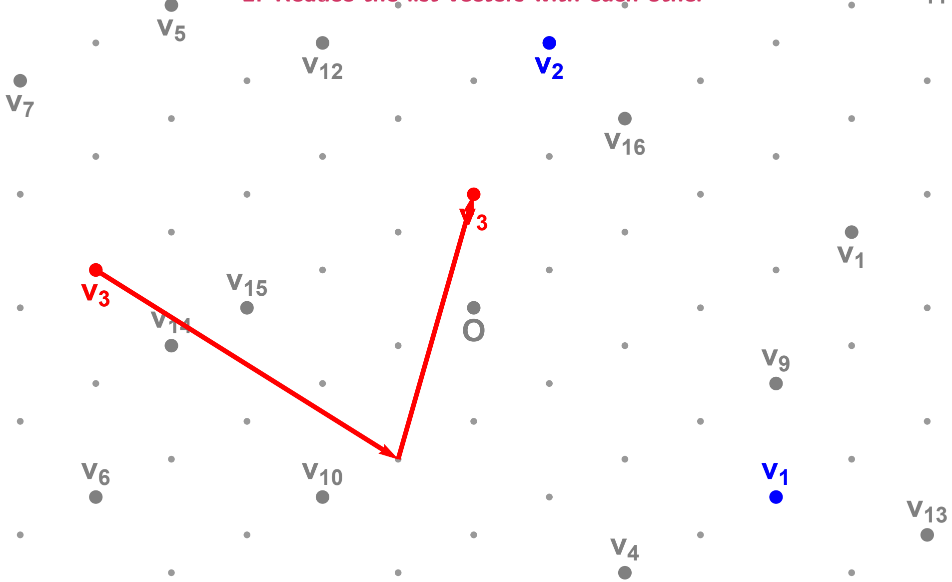

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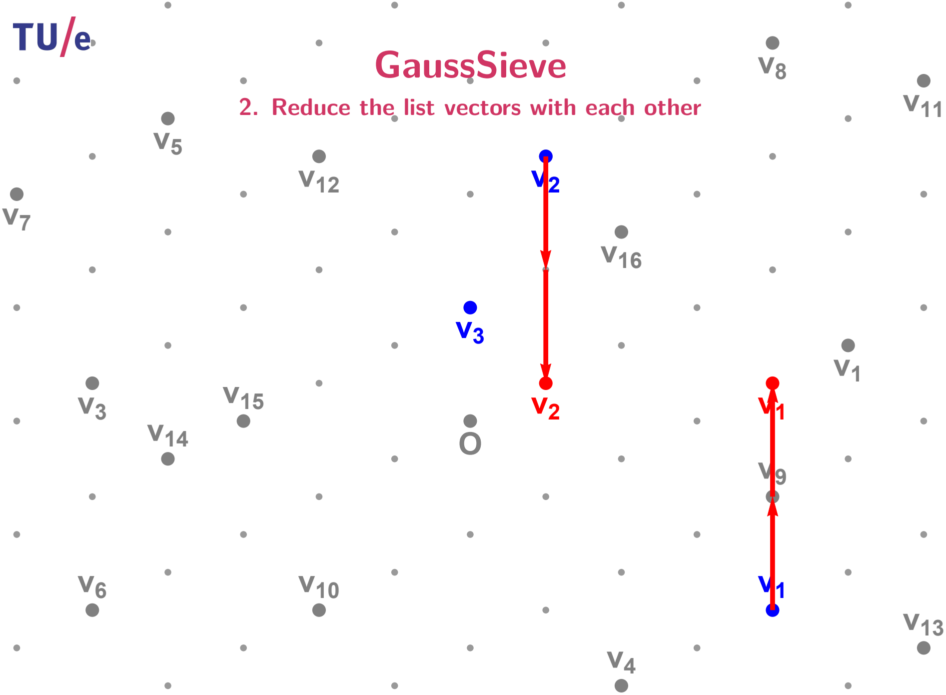

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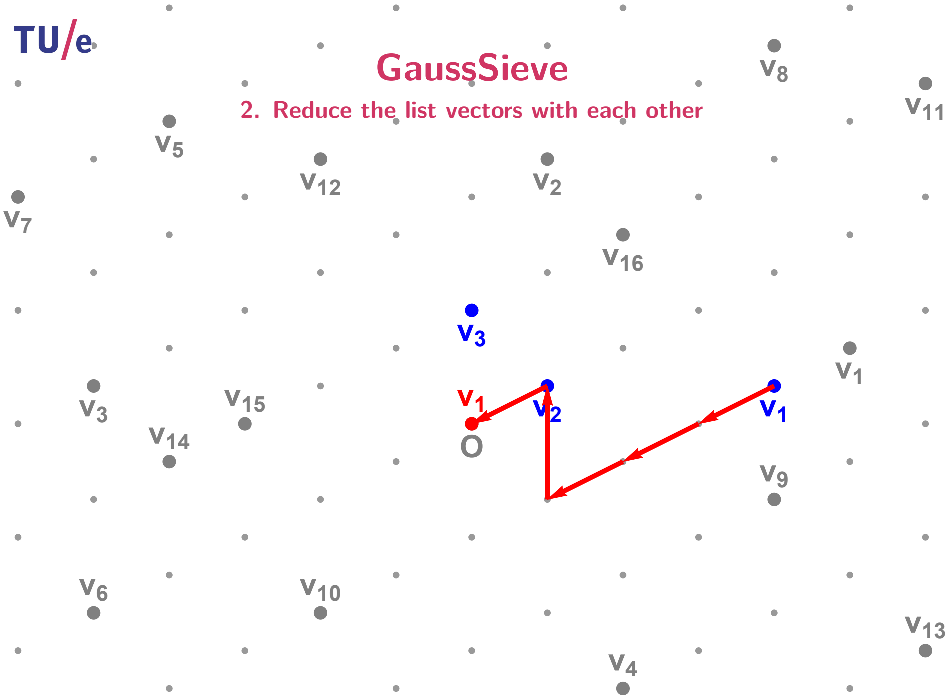

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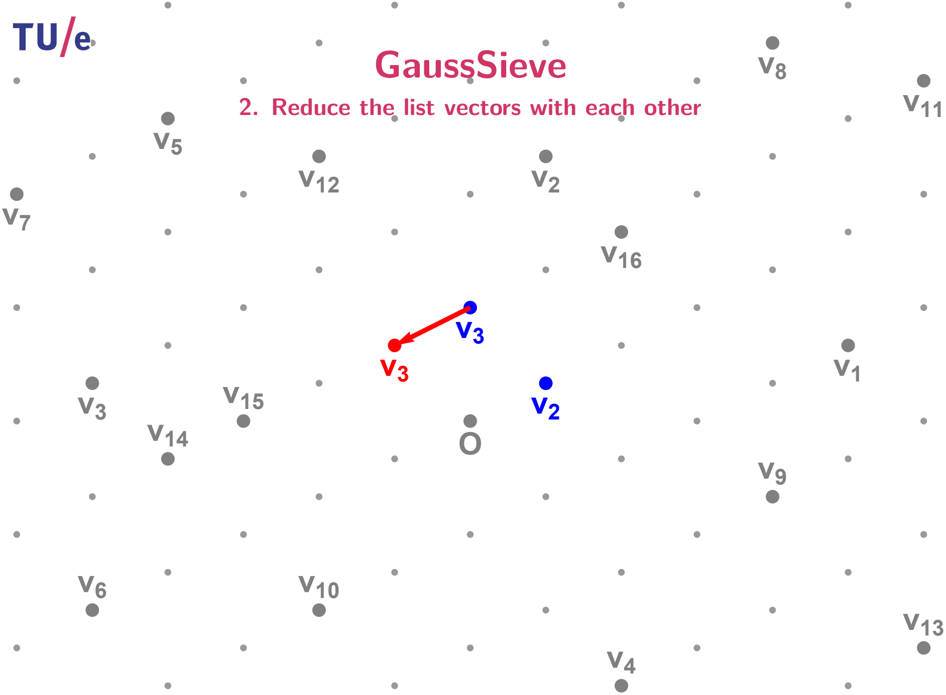

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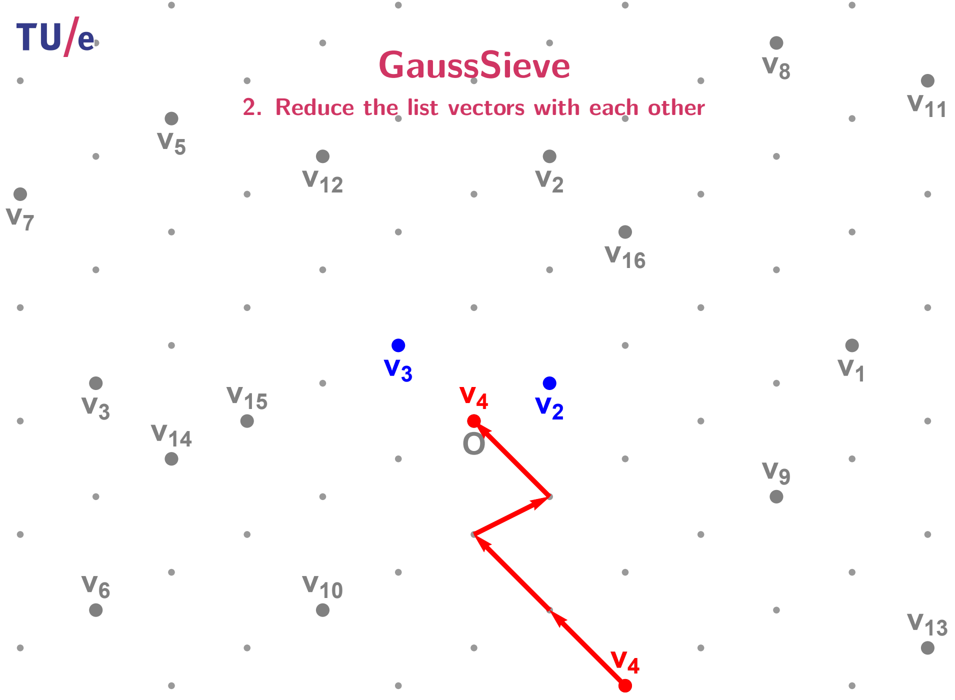

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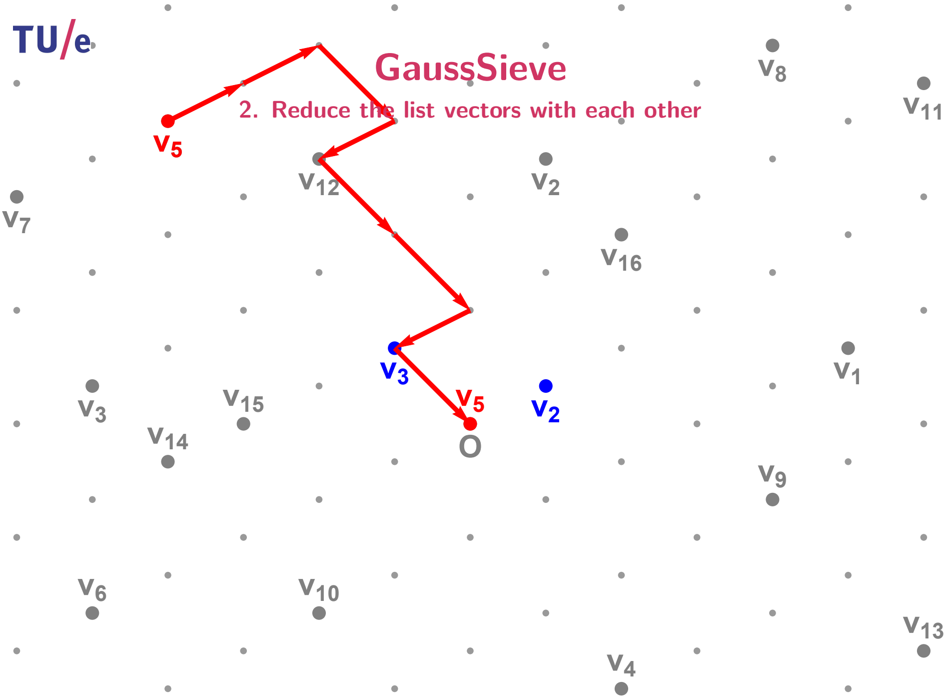

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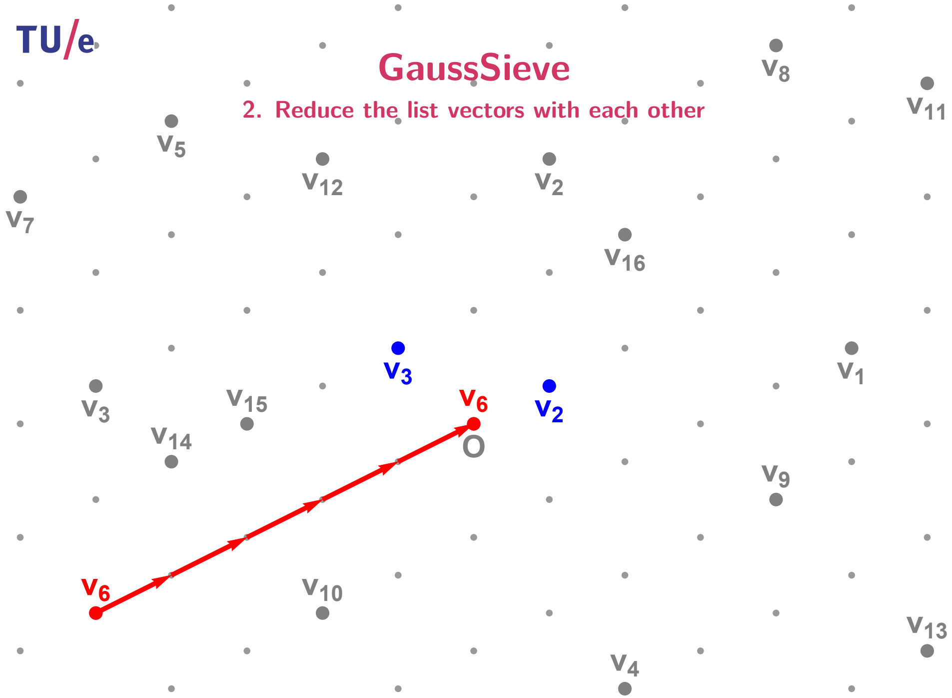

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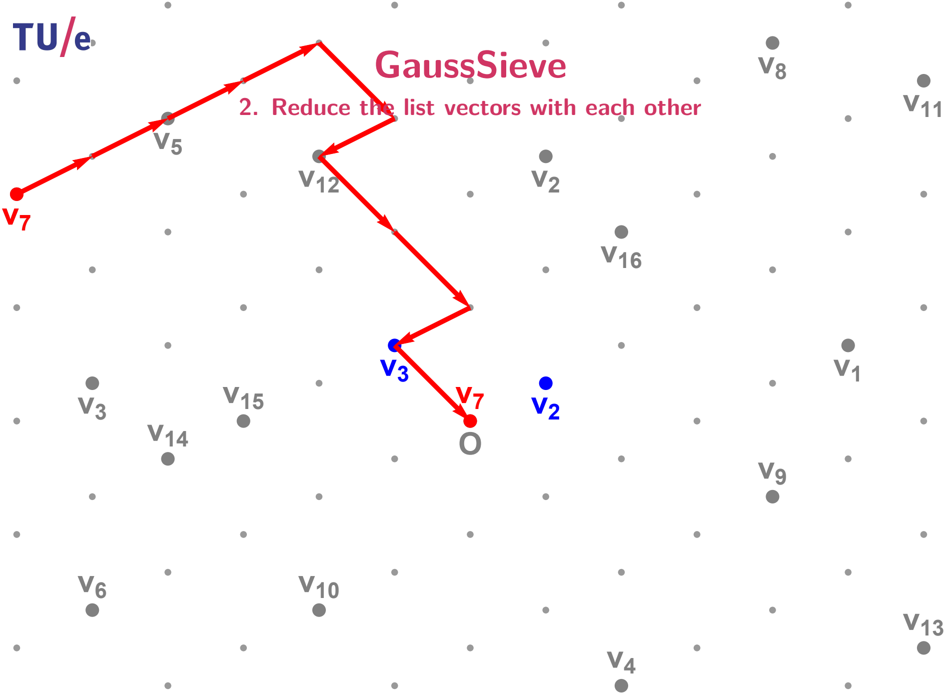

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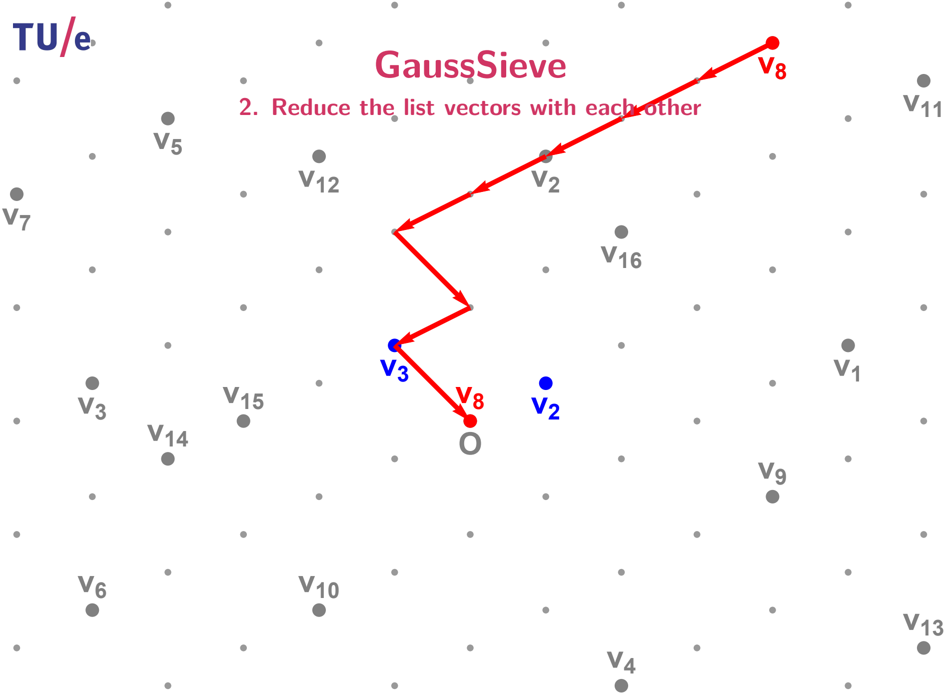

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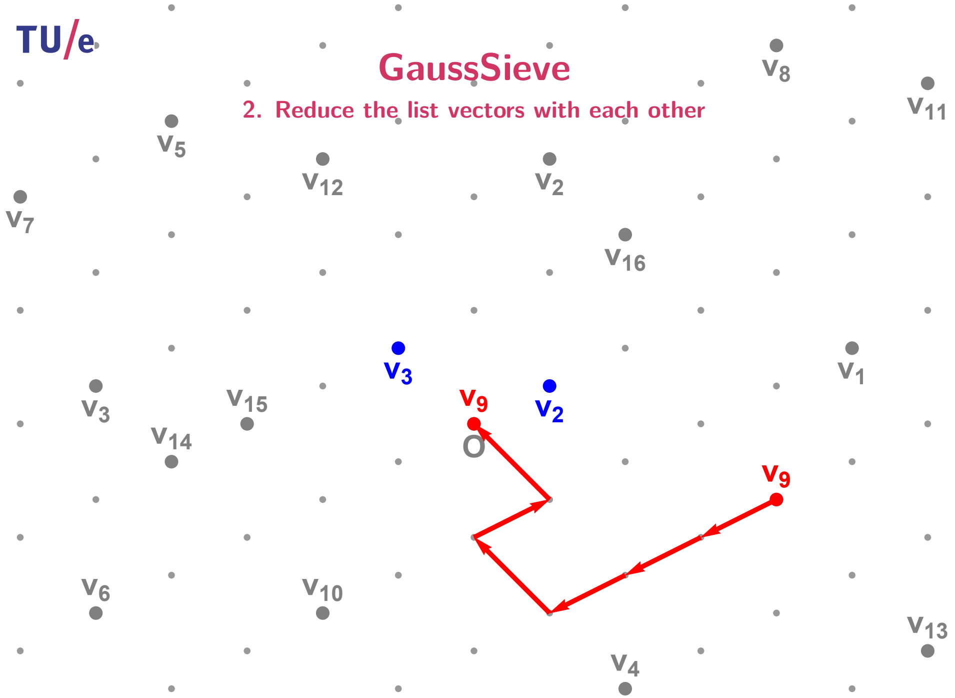

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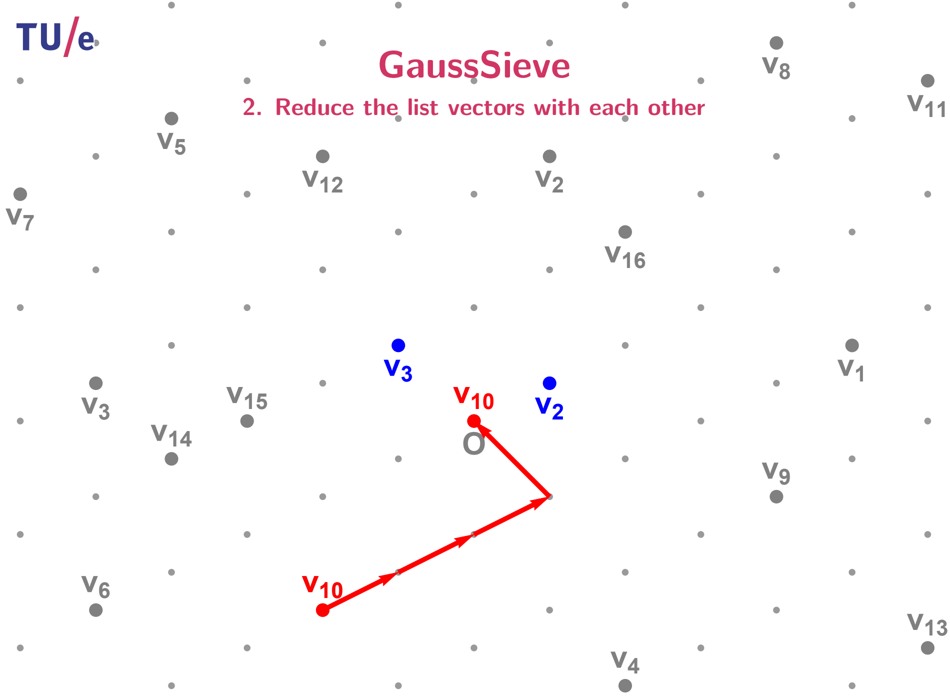

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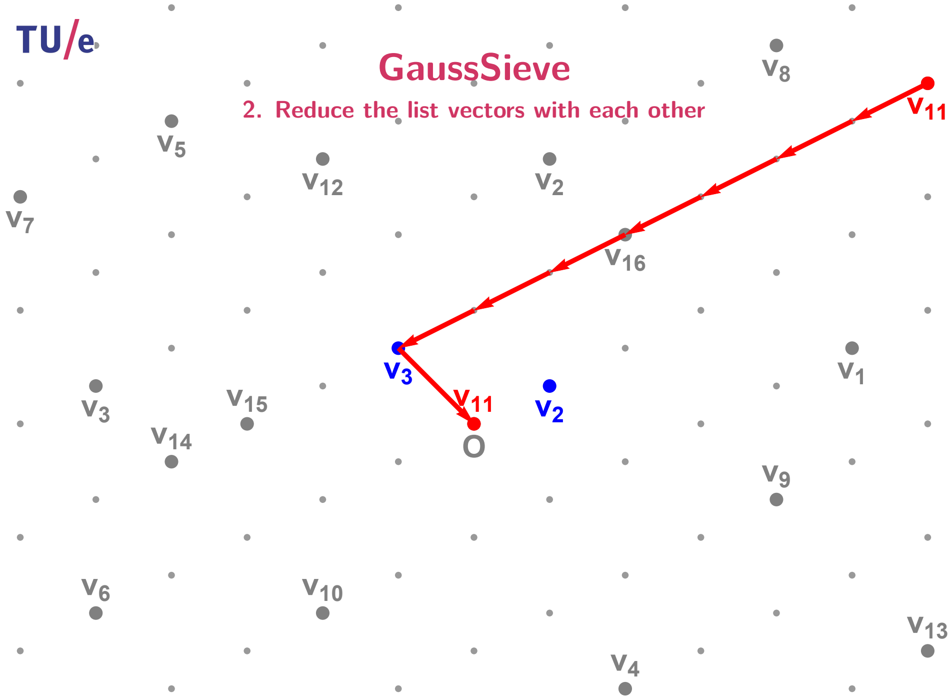

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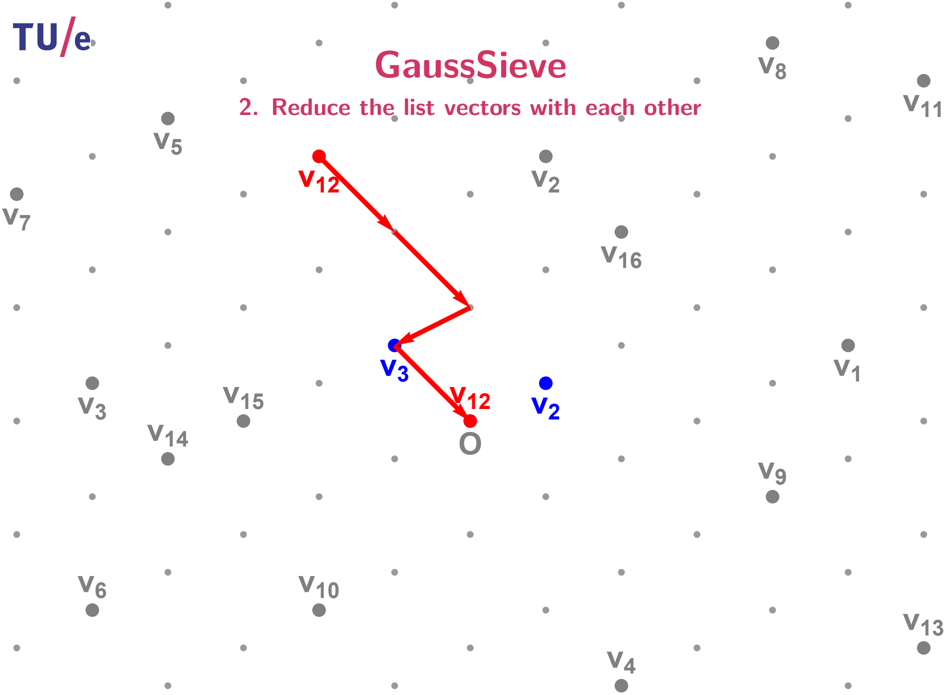

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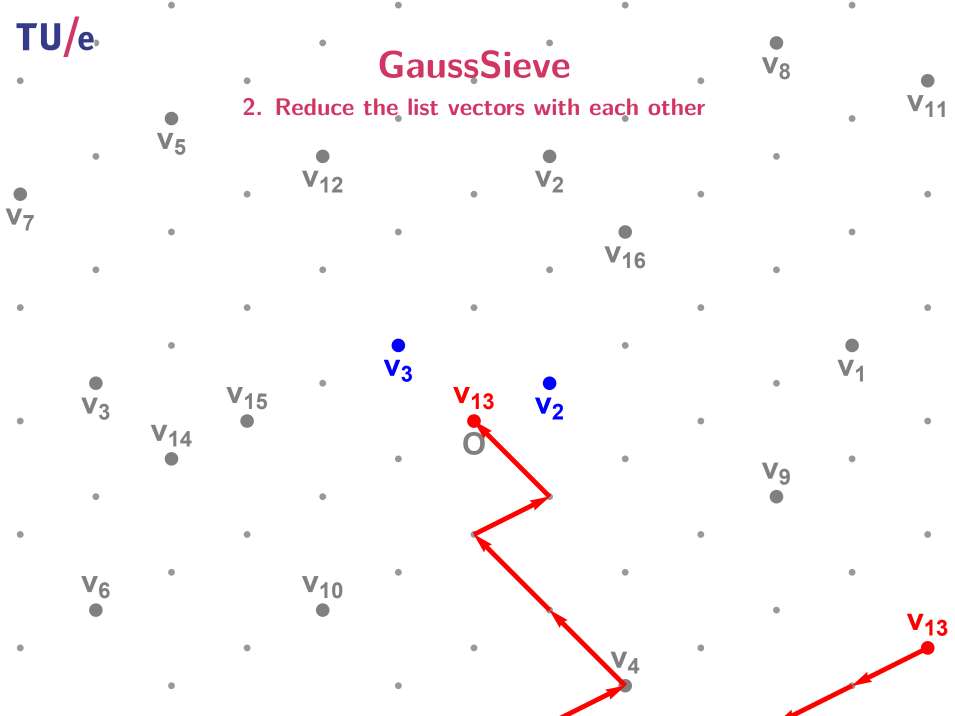

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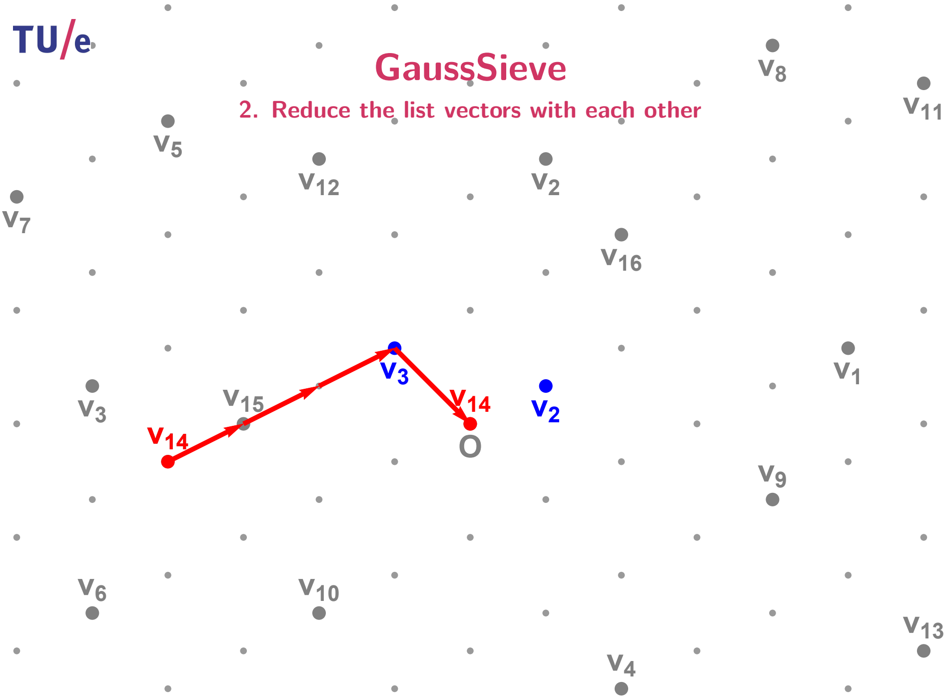

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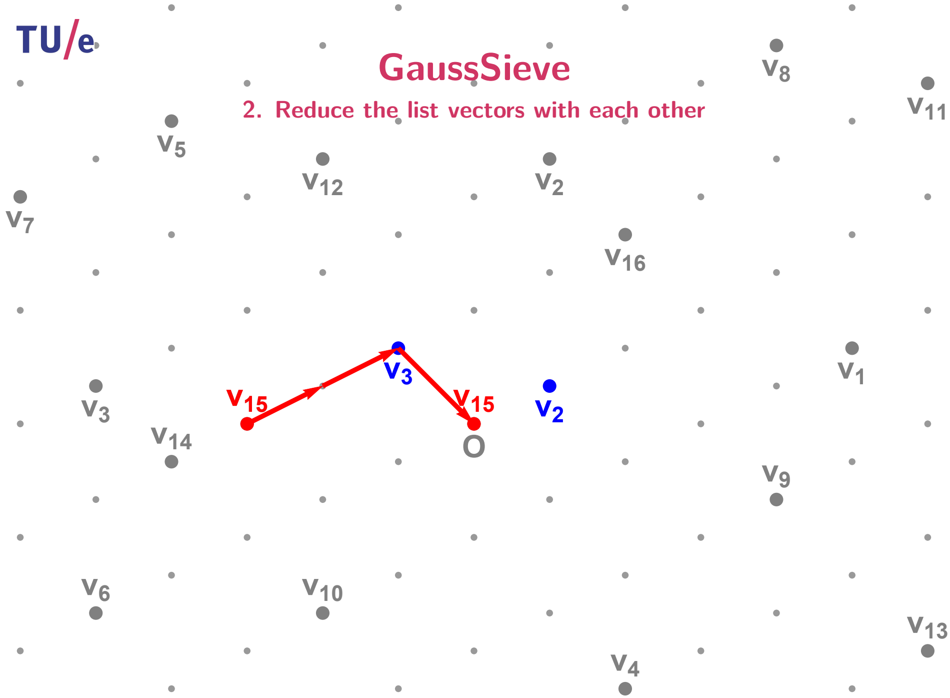

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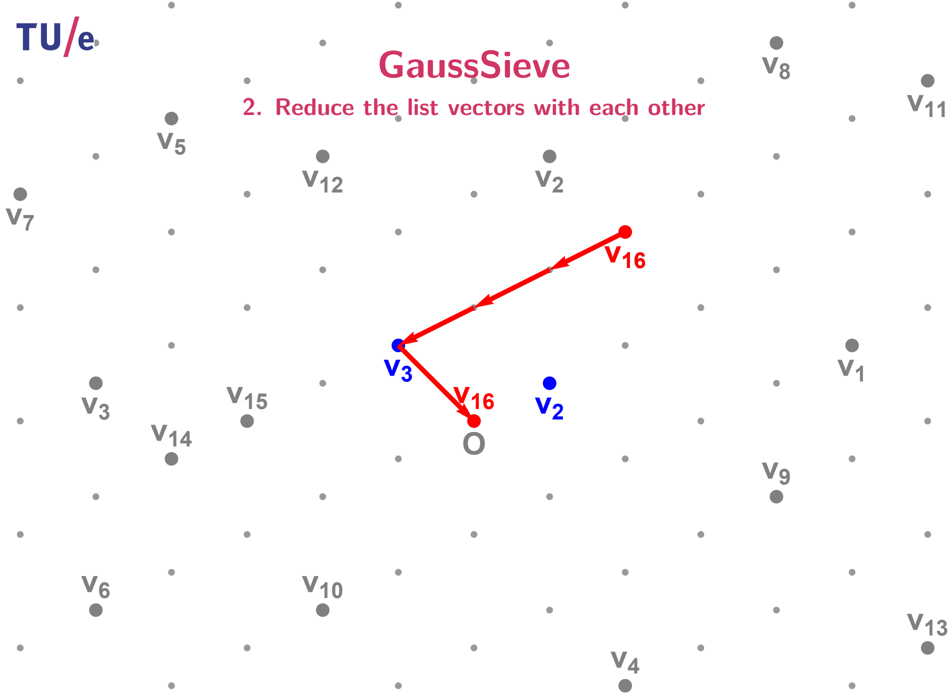

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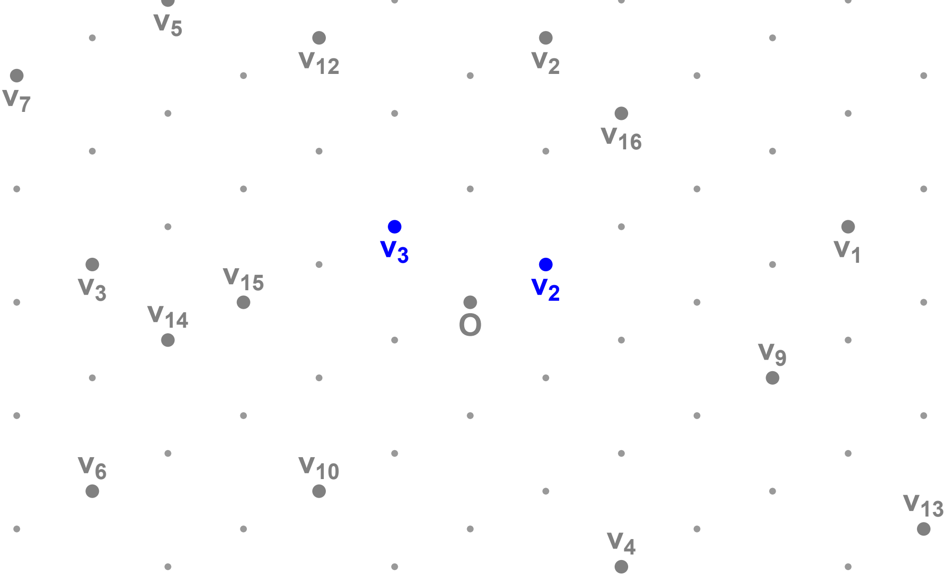

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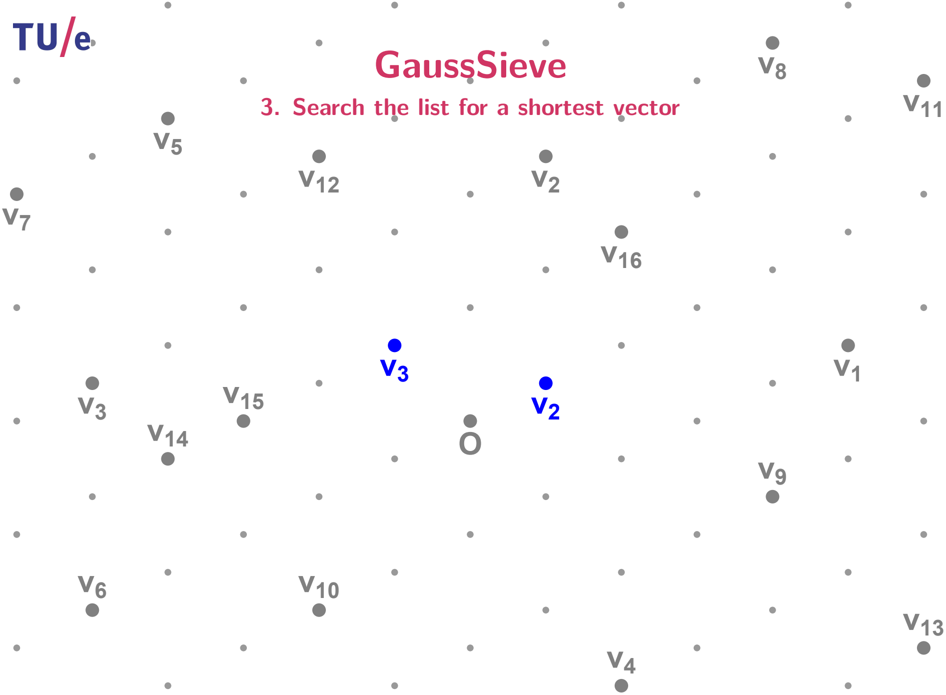

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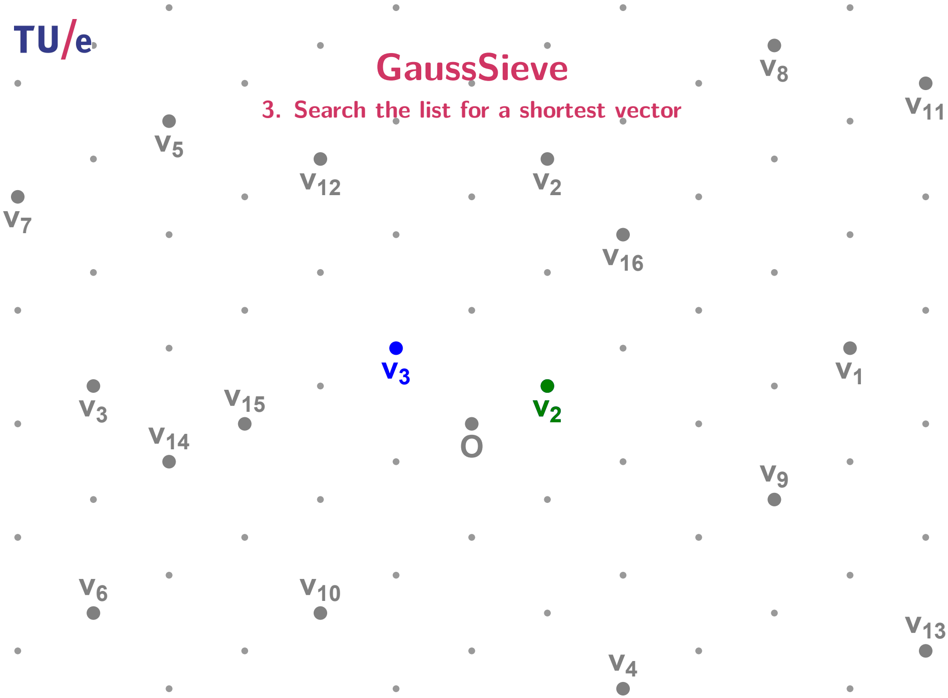
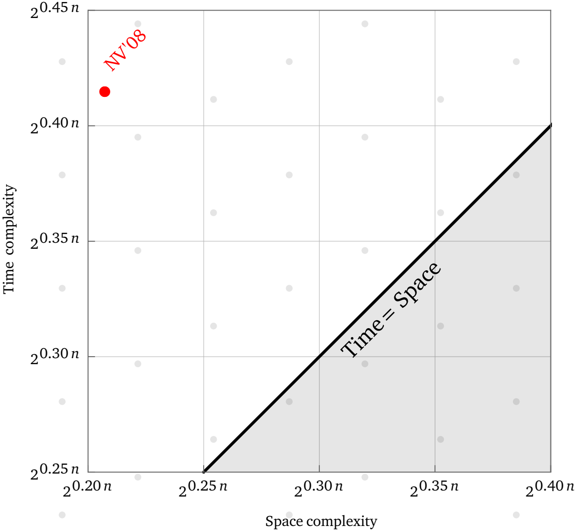


GaussSieve

Space/time trade-off



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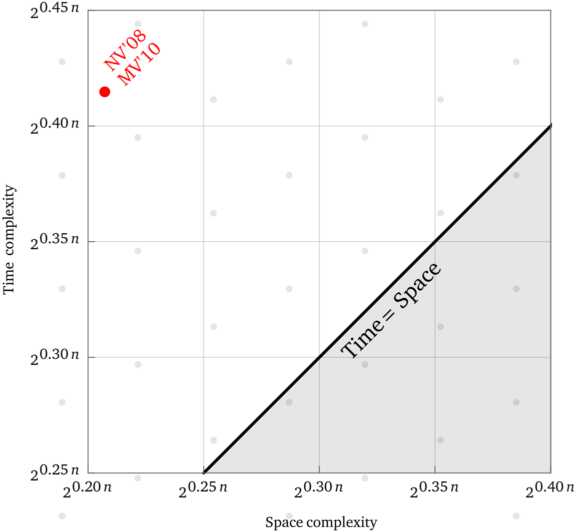
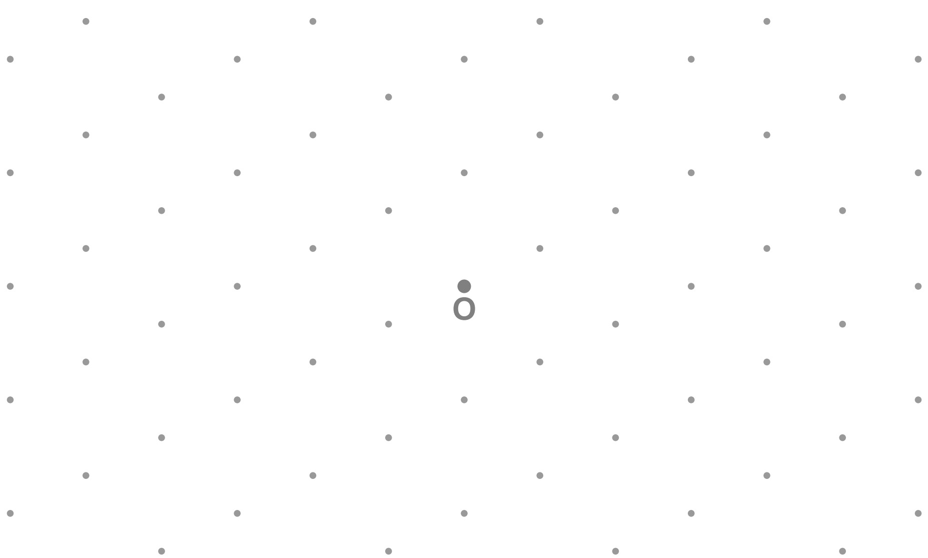


Two-level sieve

1. Sample a list L of random lattice vectors



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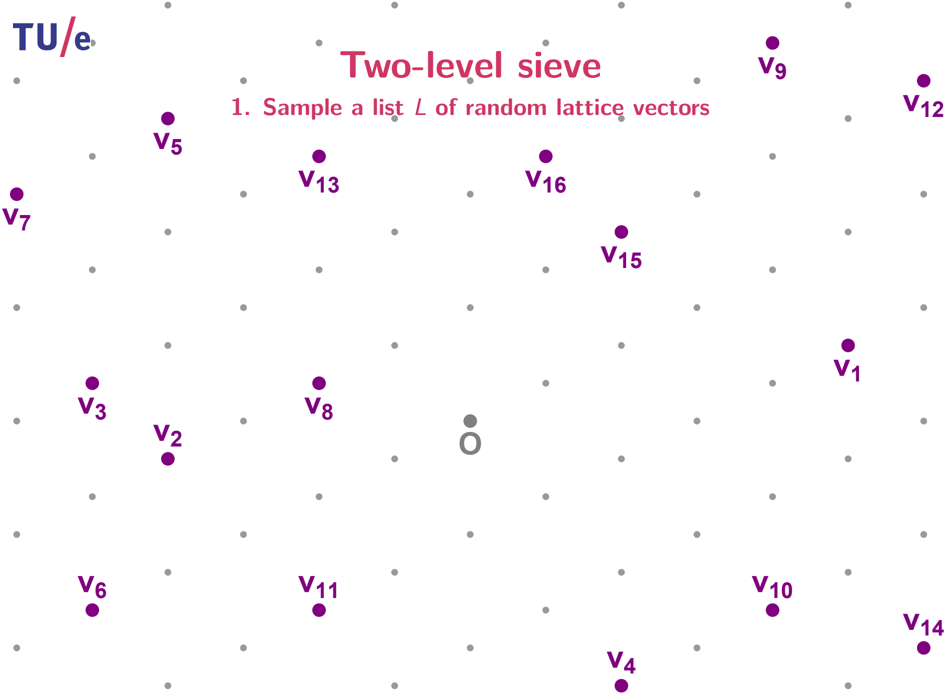
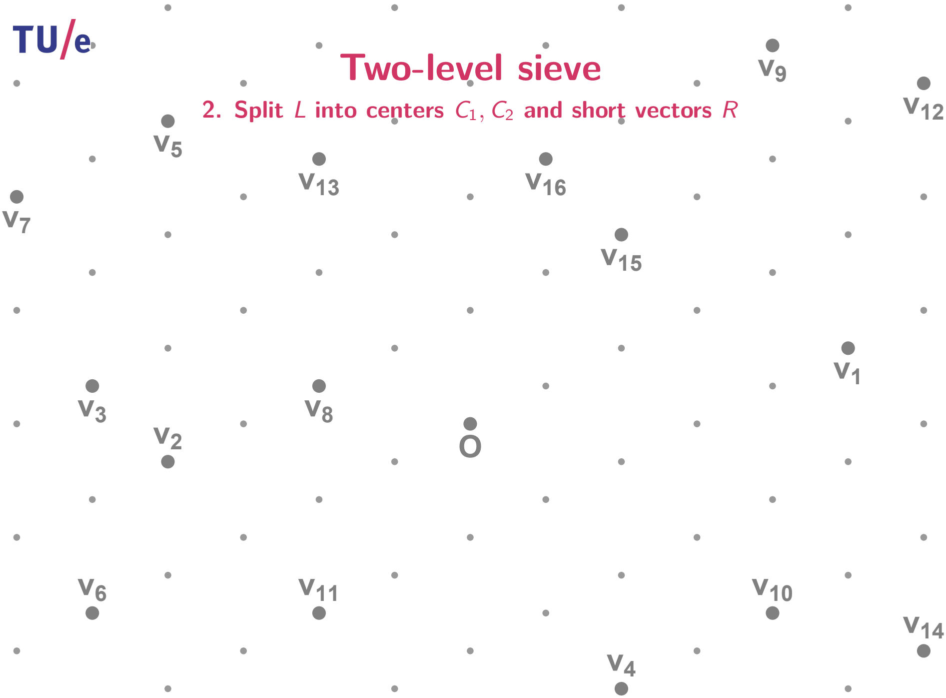


Two-level sieve

2. Split L into centers C_1, C_2 and short vectors R



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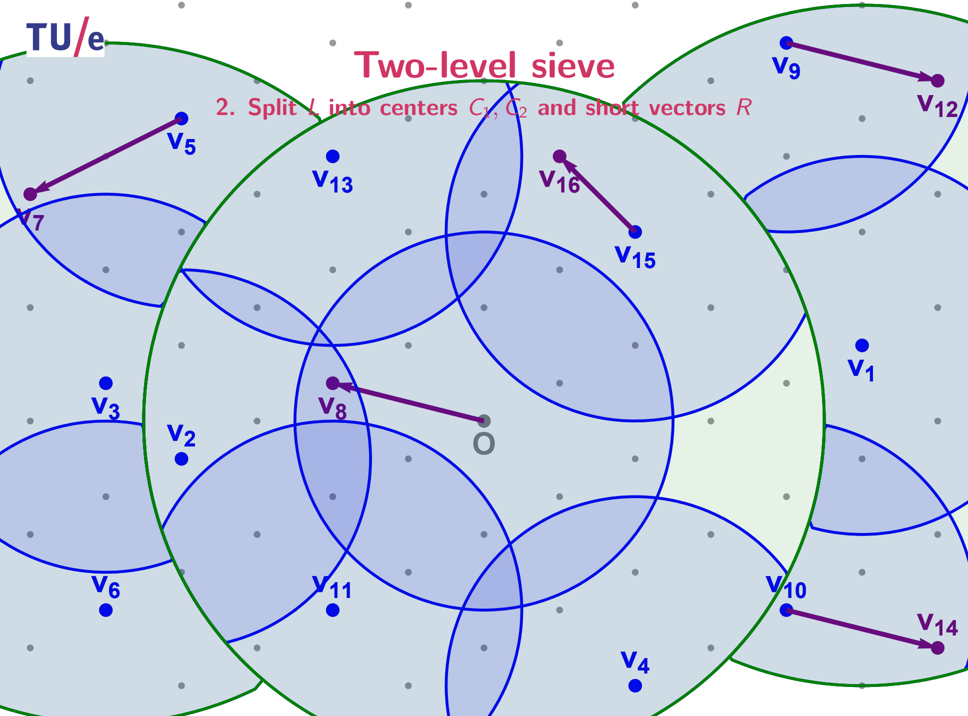
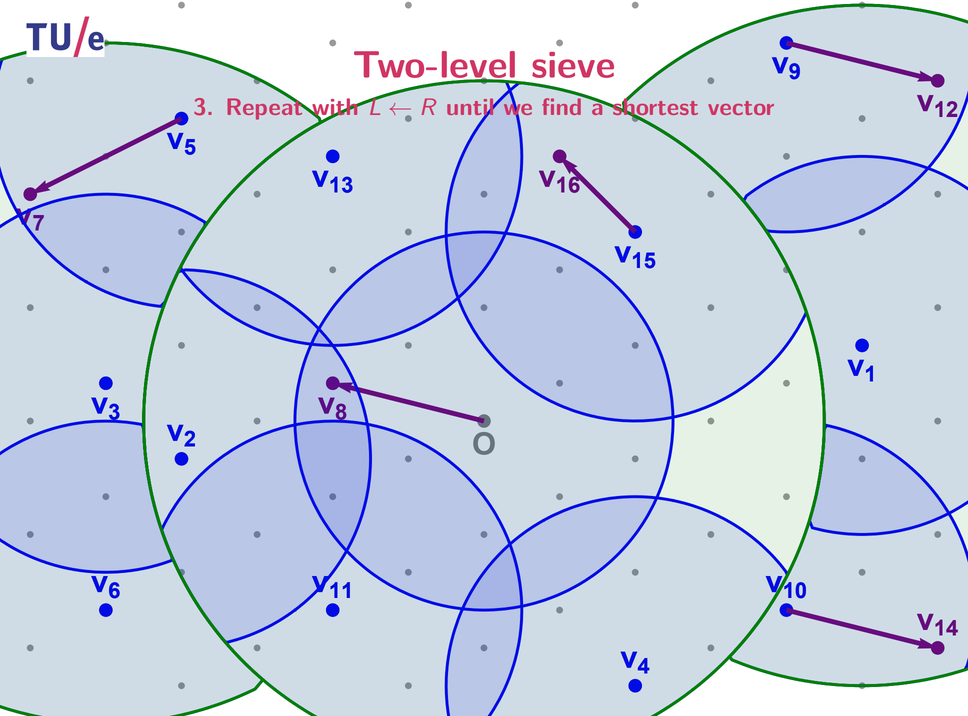


TU/e

Two-level sieve

3. Repeat with $L \leftarrow R$ until we find a shortest vector



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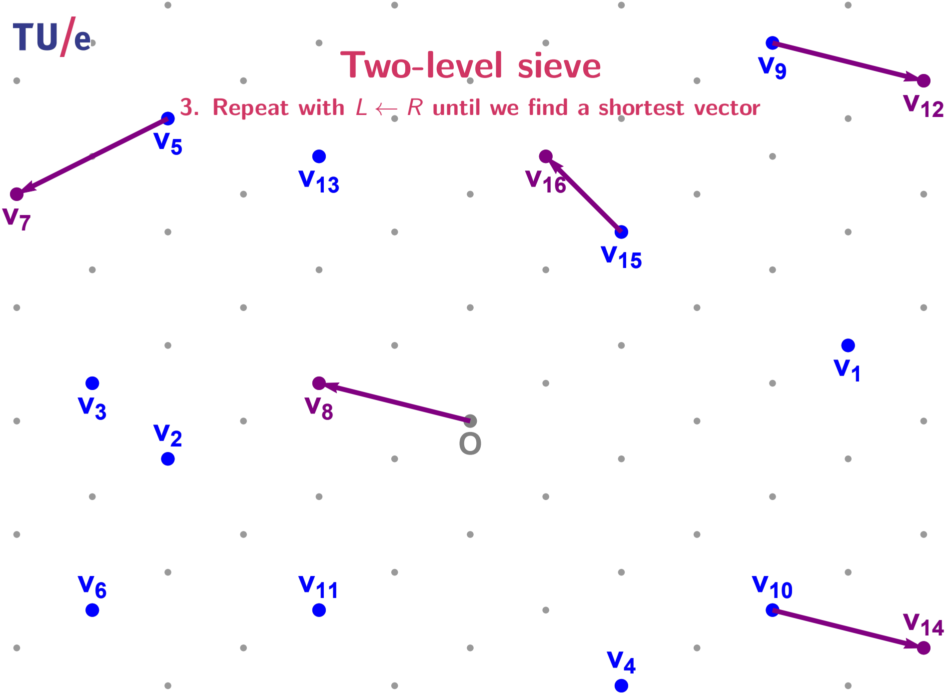

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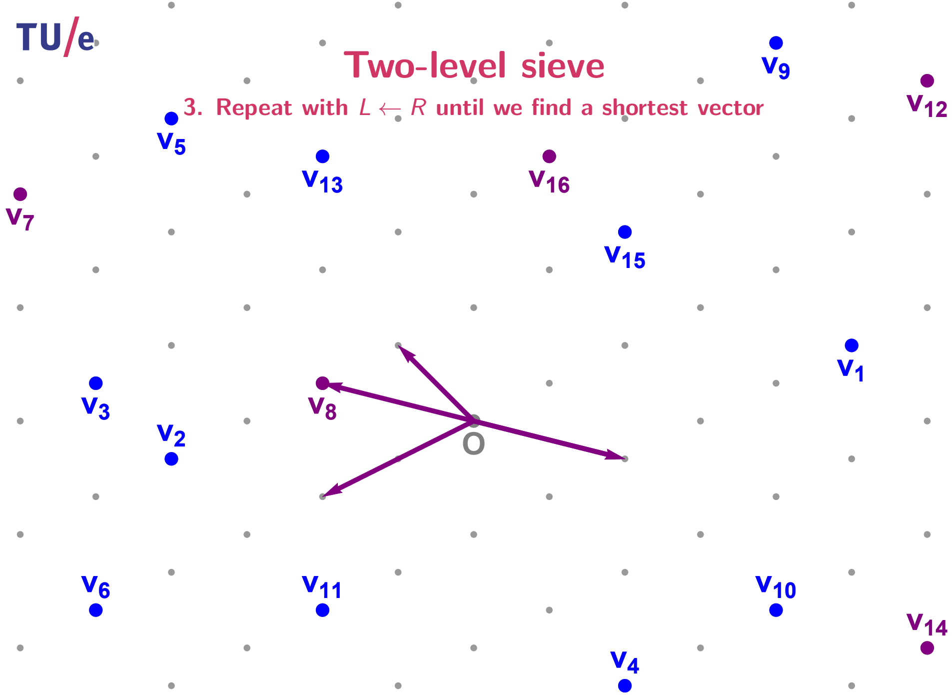

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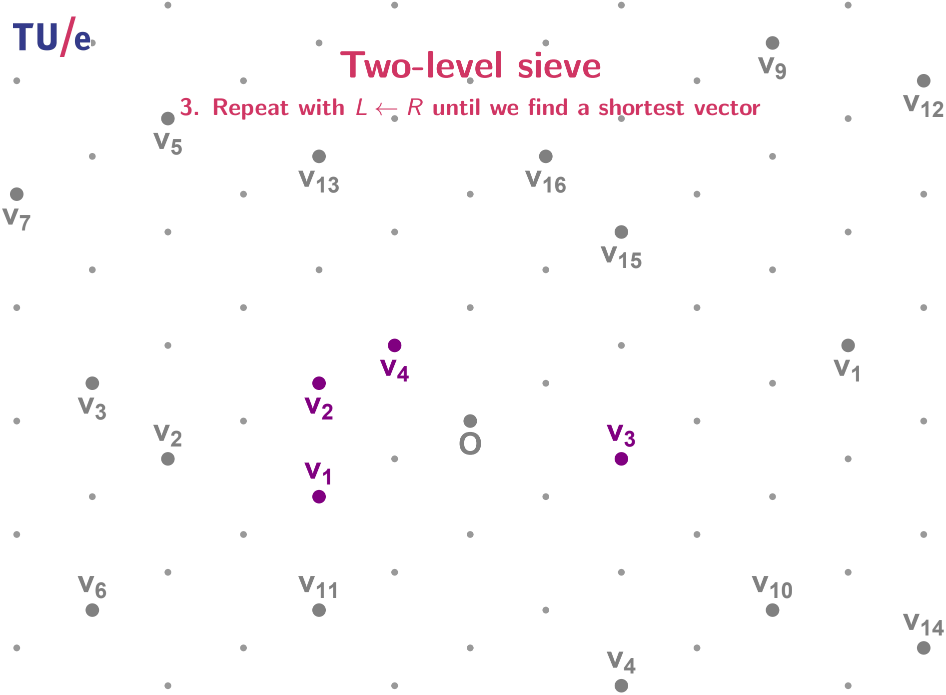

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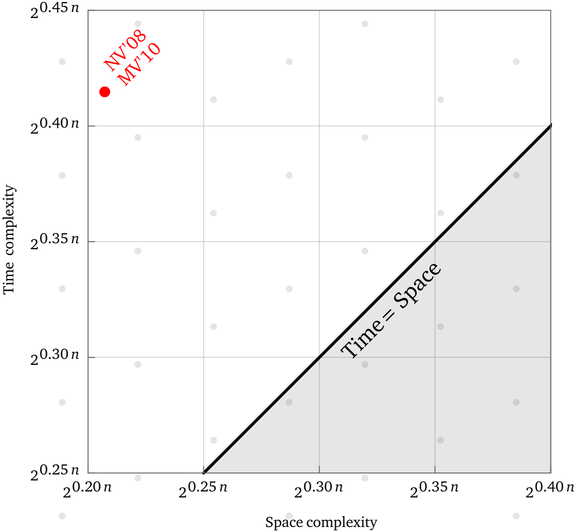

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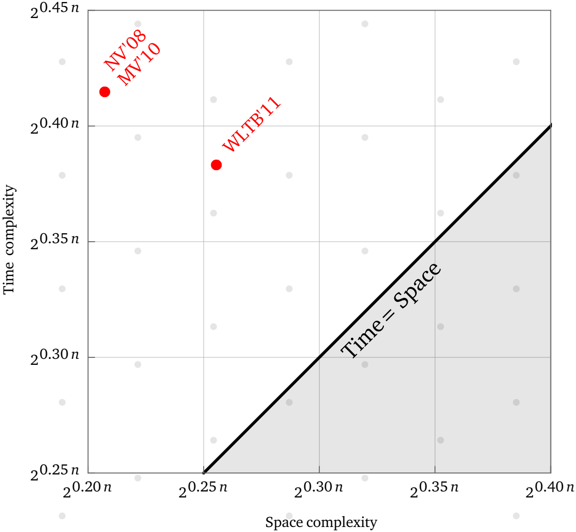

Three-level sieve

Overview

Heuristic (Nguyen and Vidick, J. Math. Crypt. '08)

The one-level sieve runs in time $2^{0.4150n}$ and space $2^{0.2075n}$.

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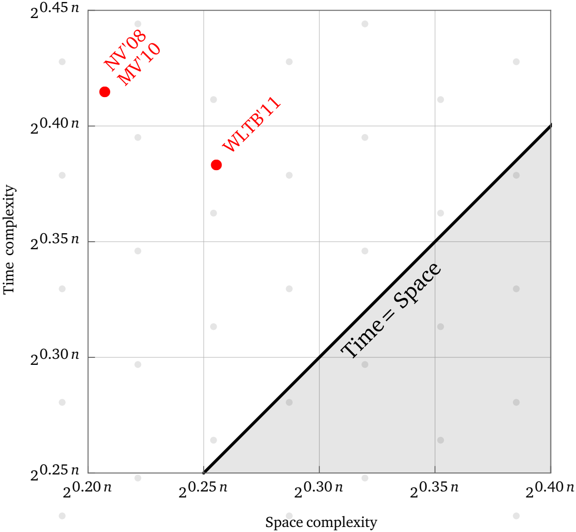
Conjecture

The four-level sieve runs in time $2^{0.3774n}$ and space $2^{0.2925n}$, and higher-level sieves are not faster than this.

Three-level sieve

Space/time trade-off



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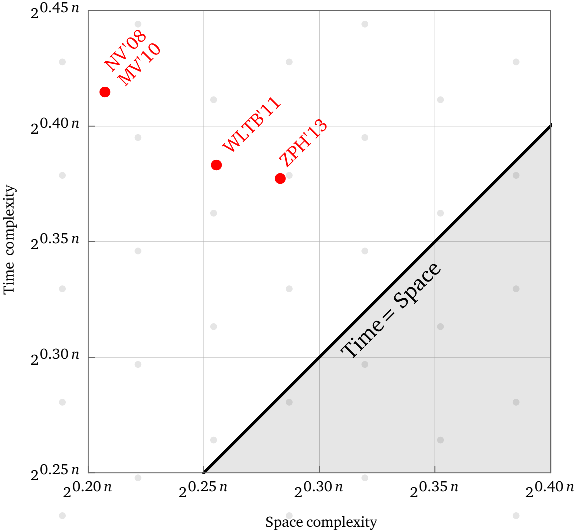
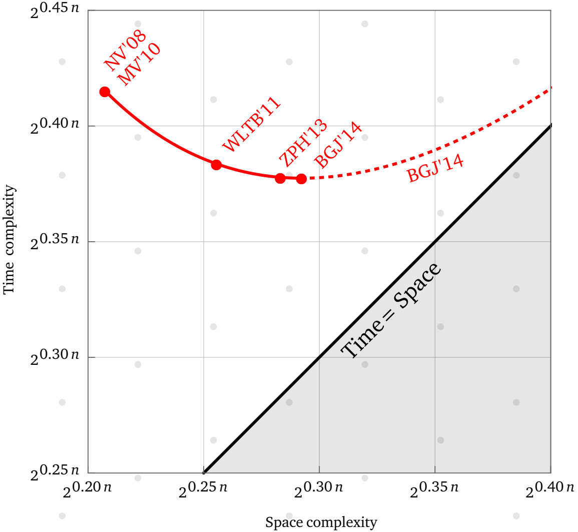


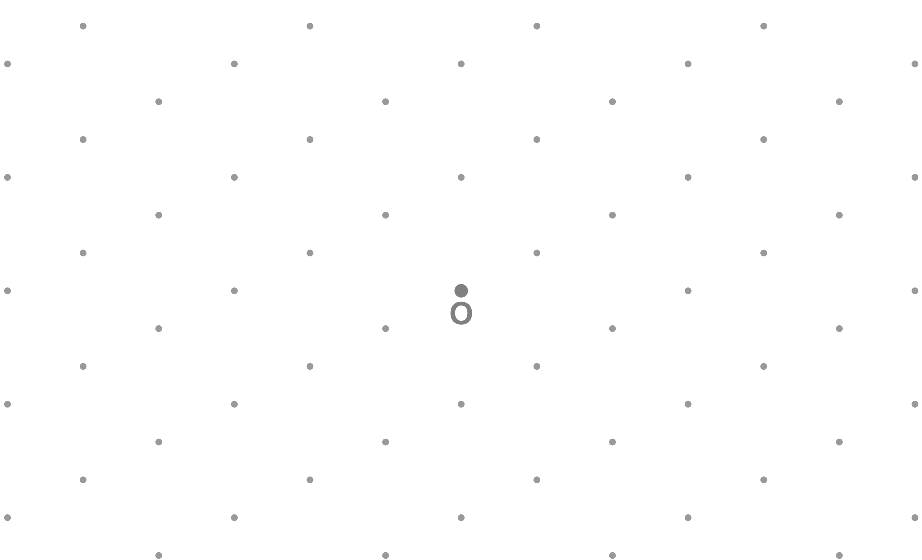
Decomposition approach

Space/time trade-off



HashSieve (GS)

1. Generate random lattice vectors



TU/e

HashSieve (GS)

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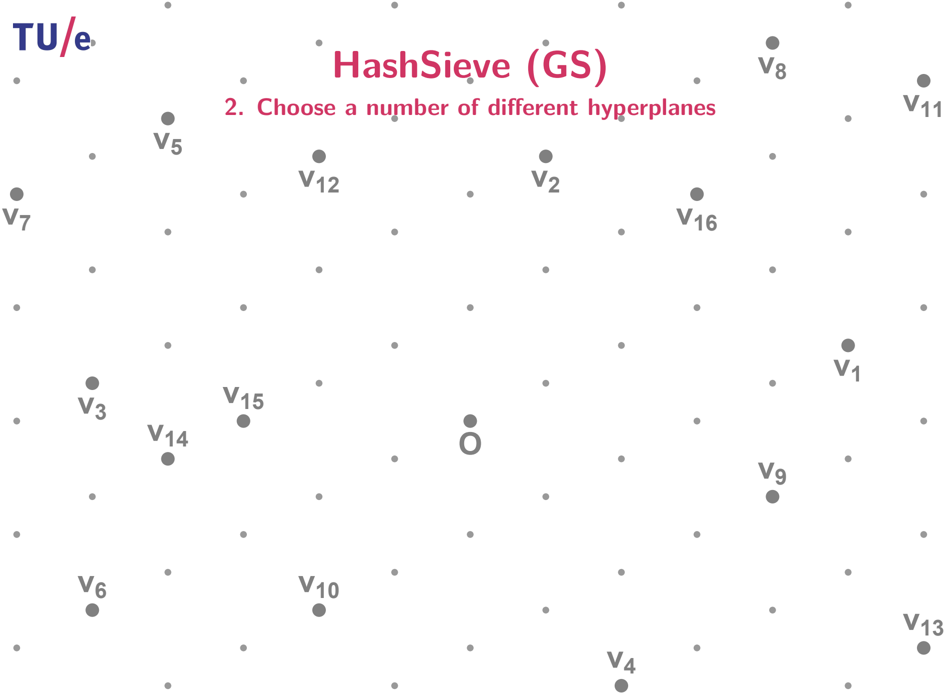
HashSieve (GS)

2. Choose a number of different hyperplanes



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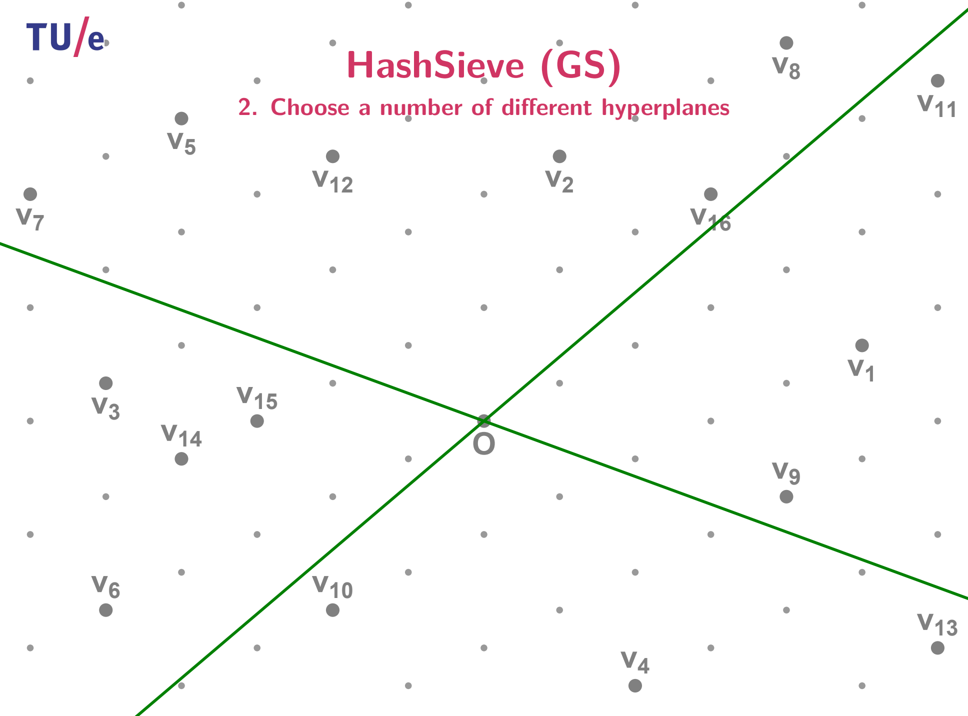
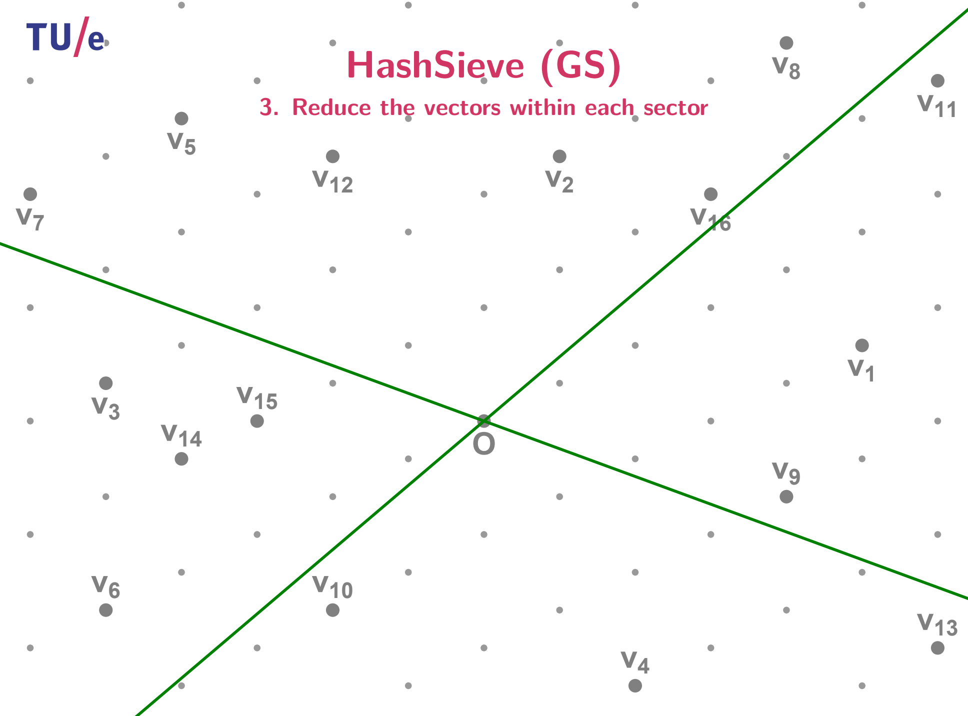


HashSieve (GS)

3. Reduce the vectors within each sector



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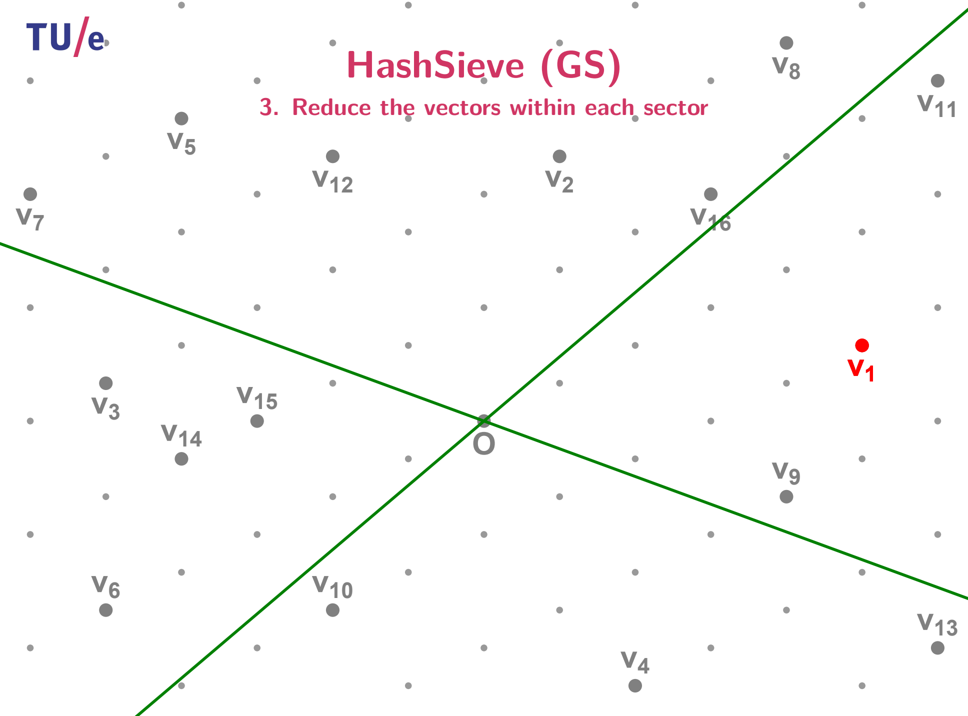

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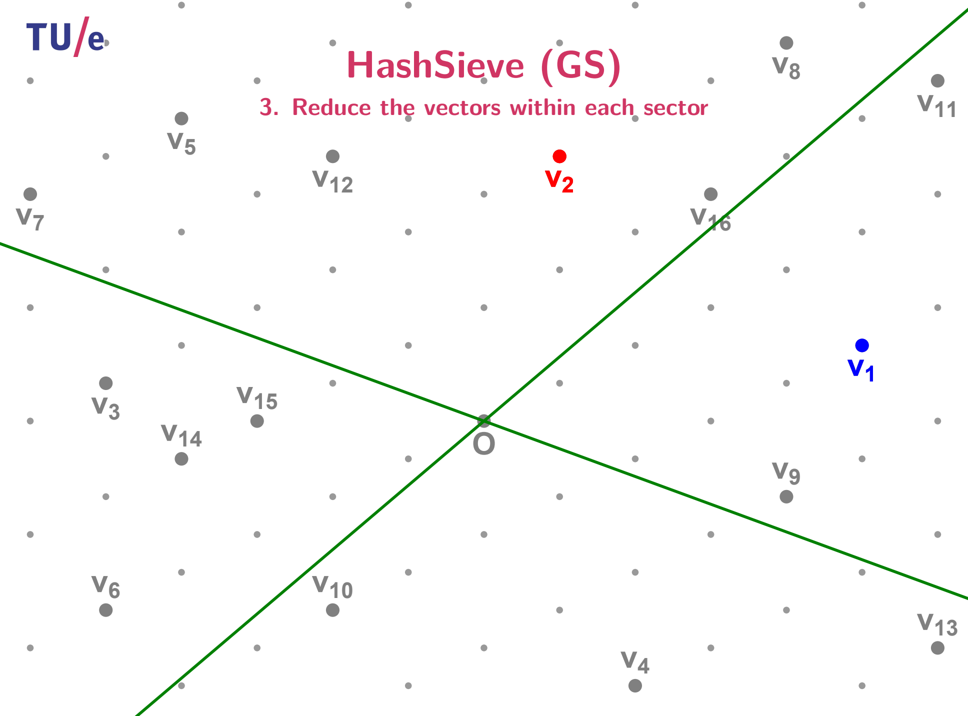

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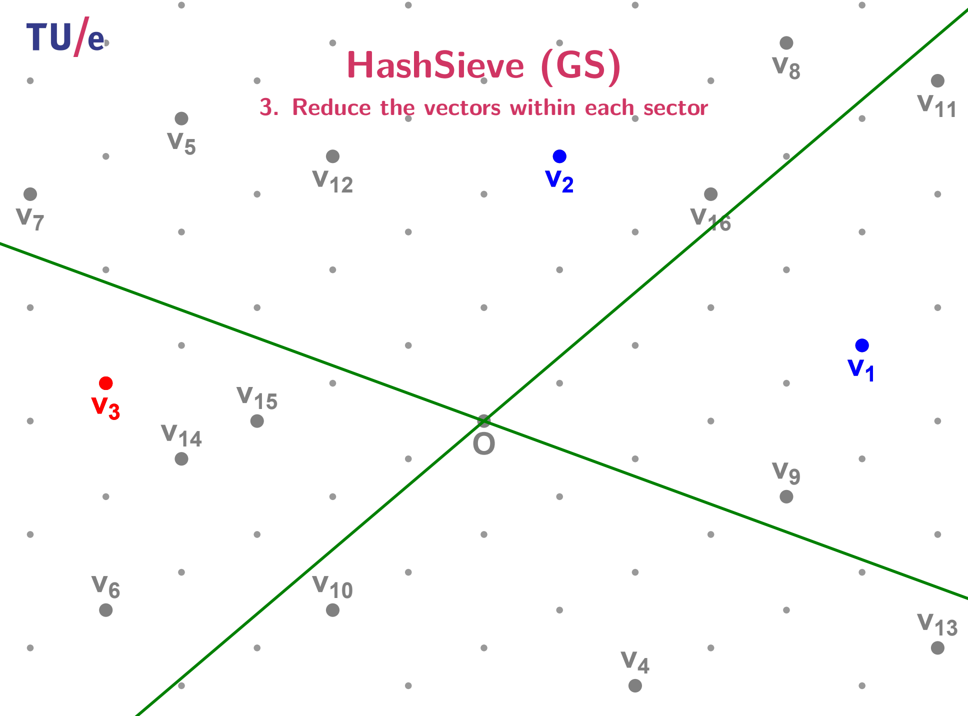

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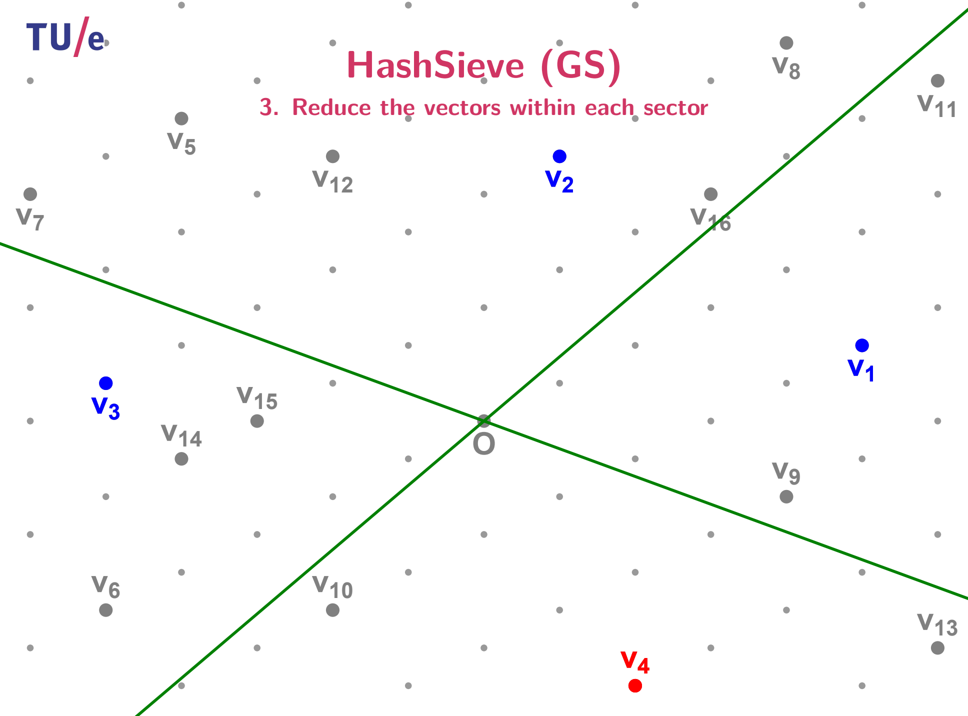

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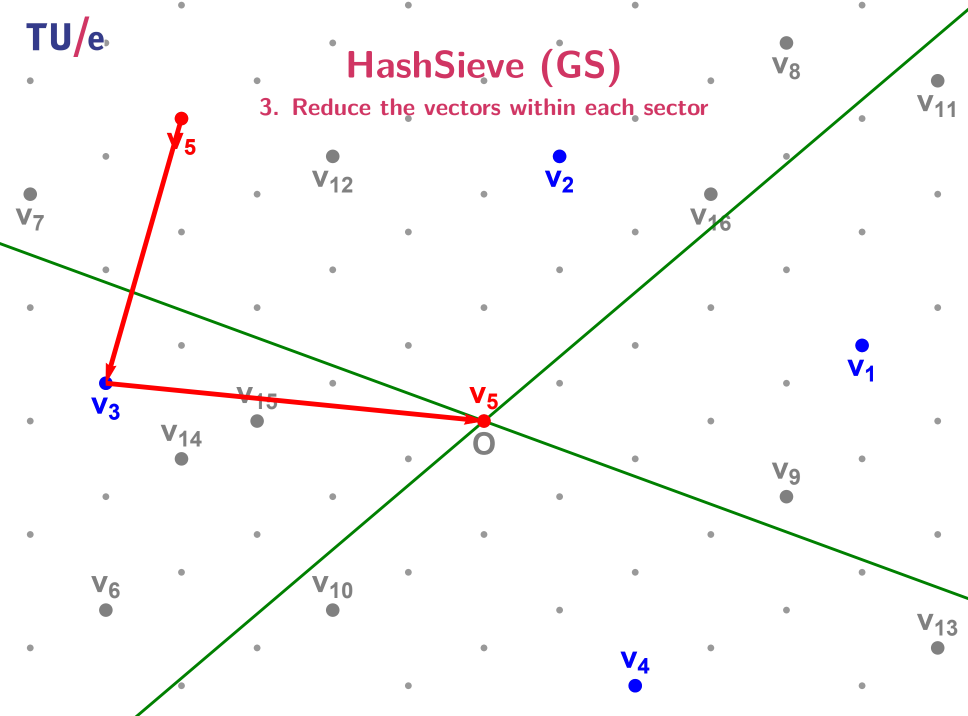

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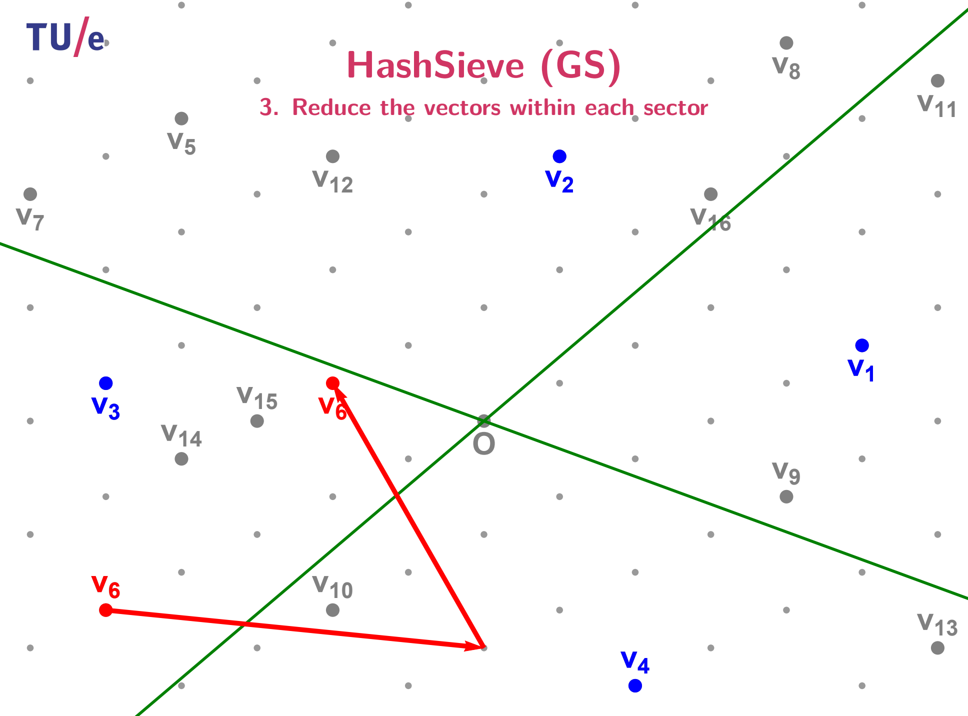

TU/e

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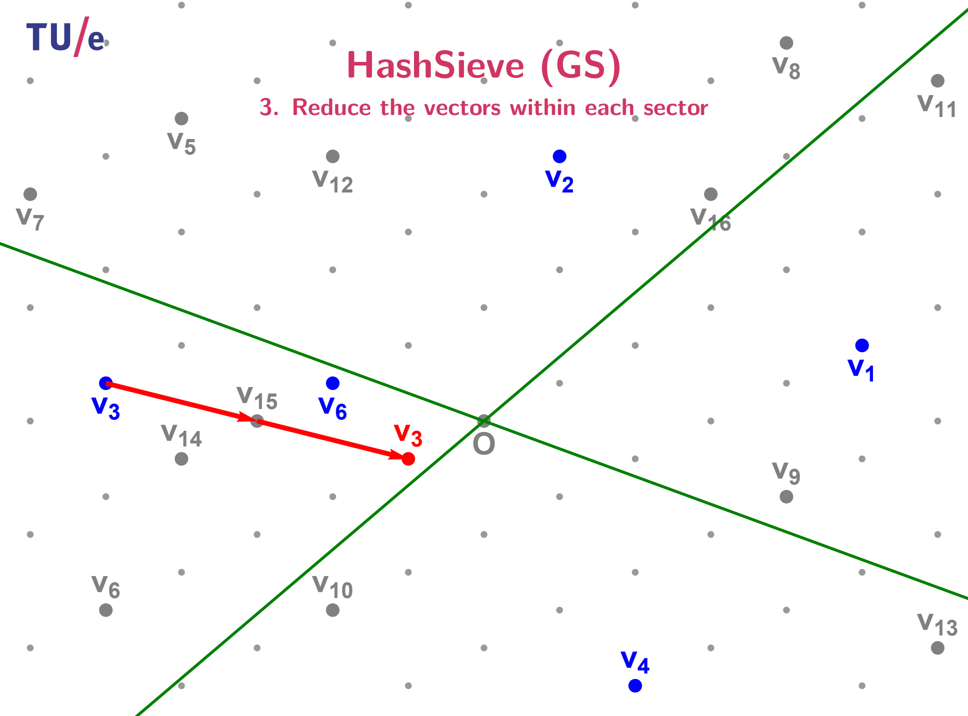

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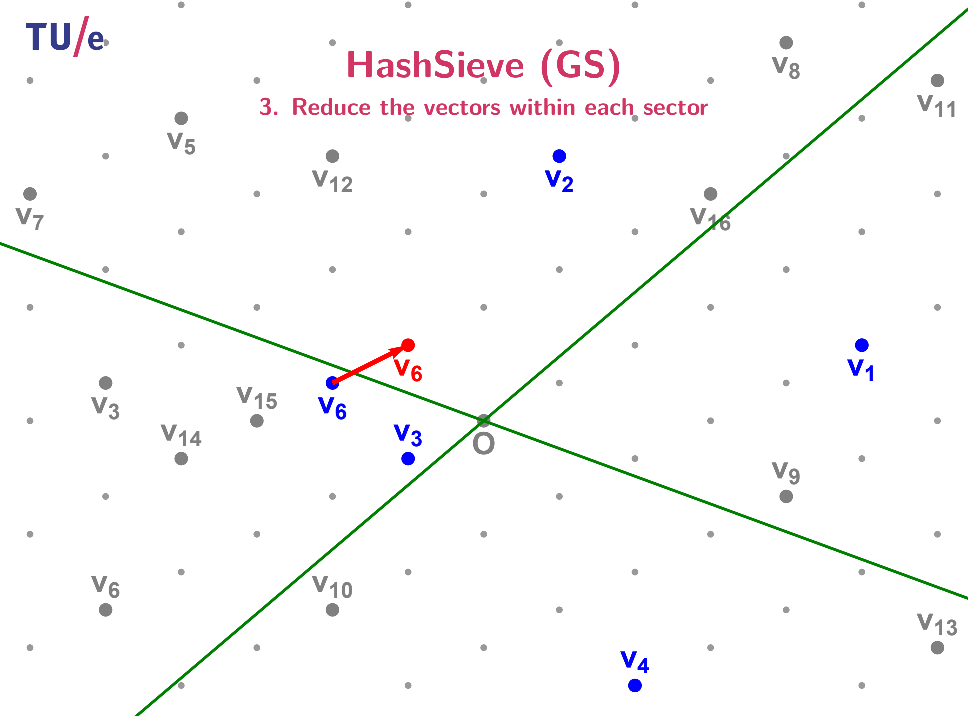

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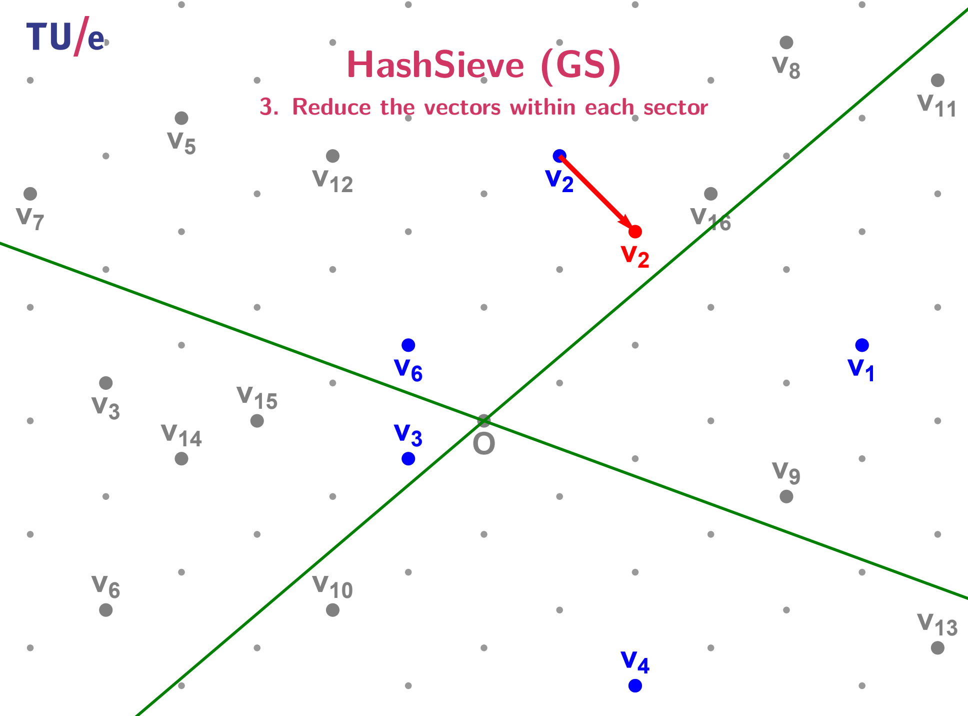

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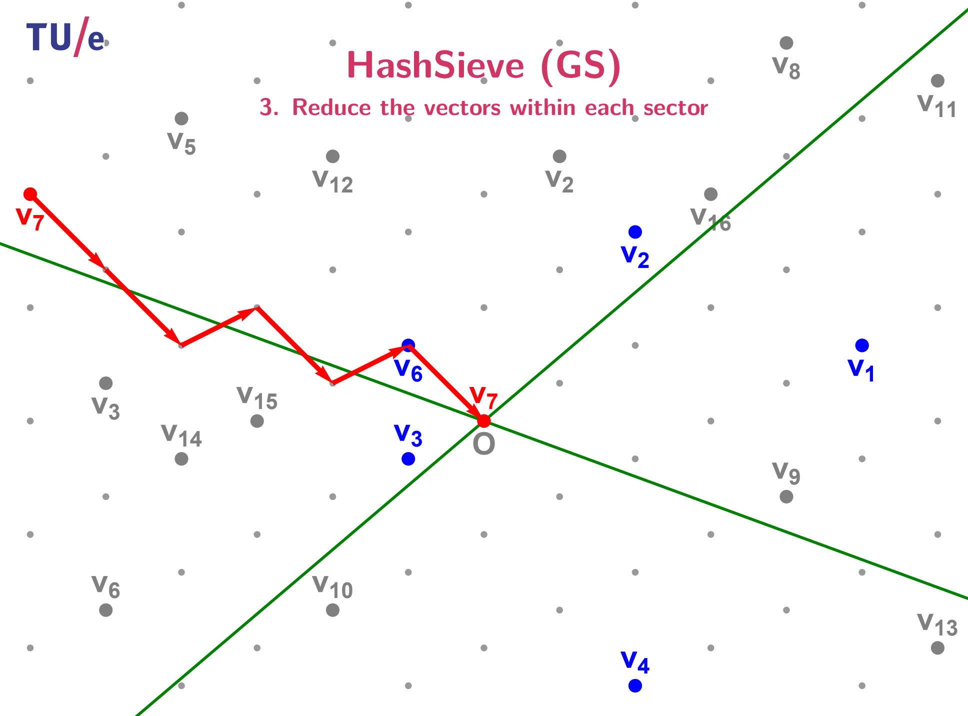

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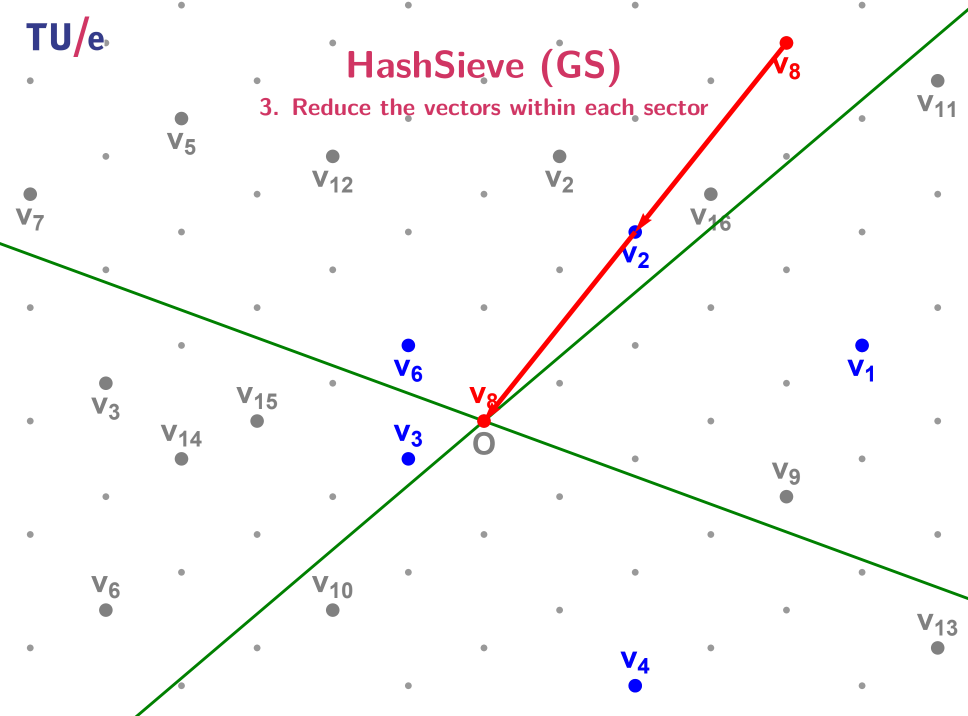

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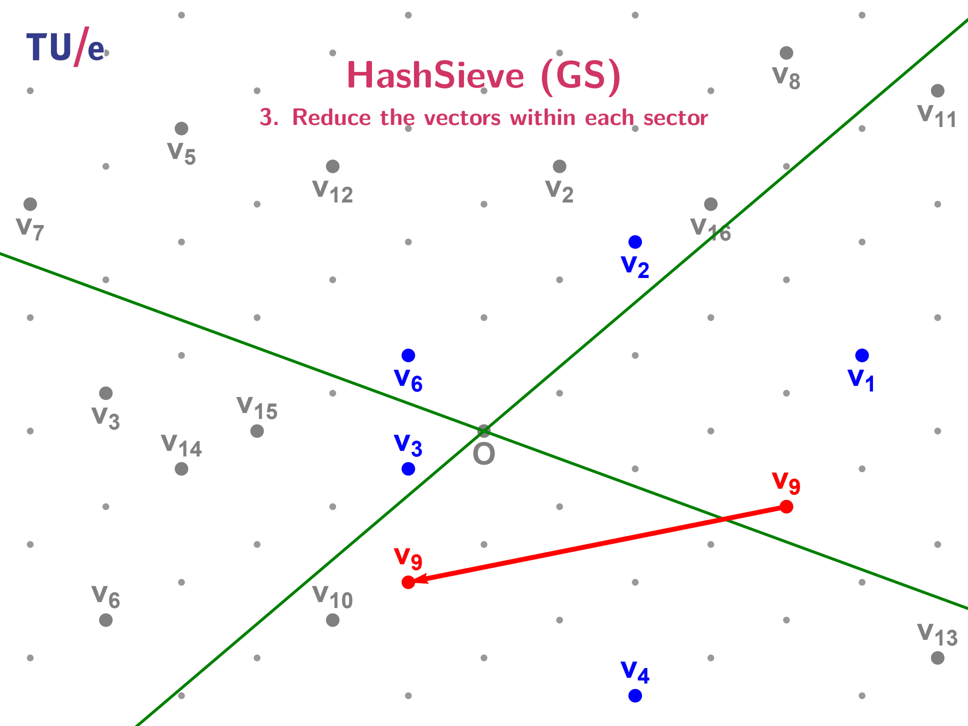

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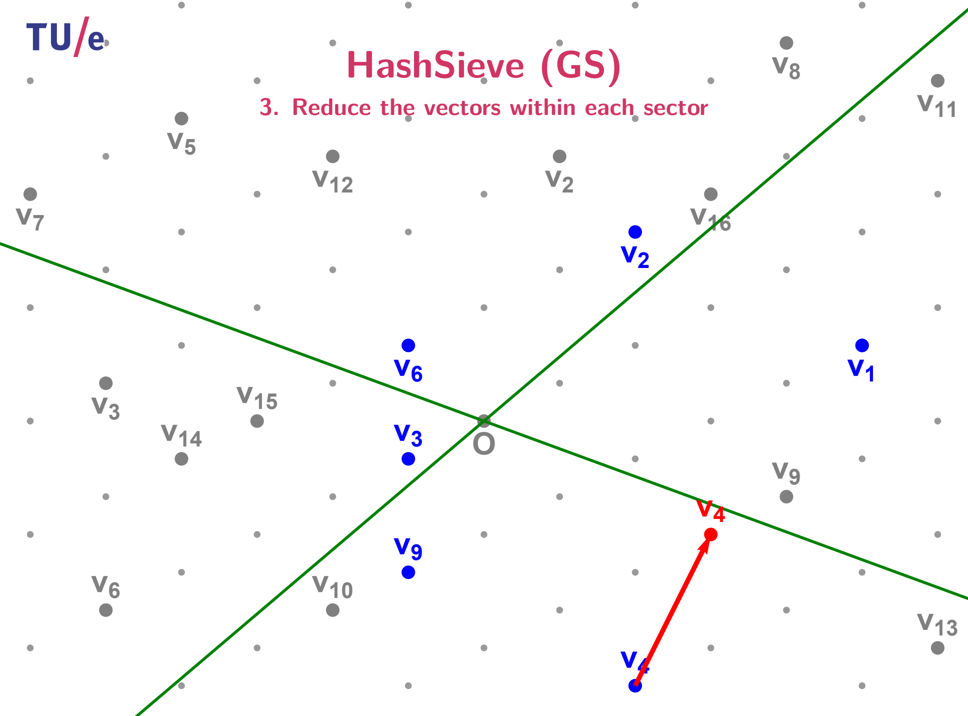

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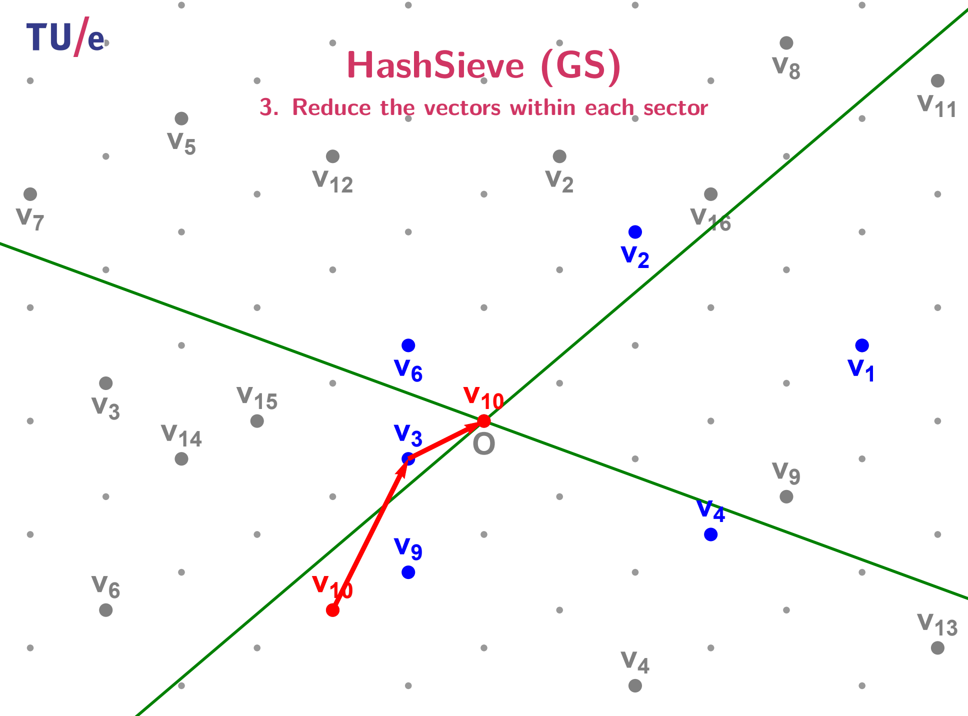

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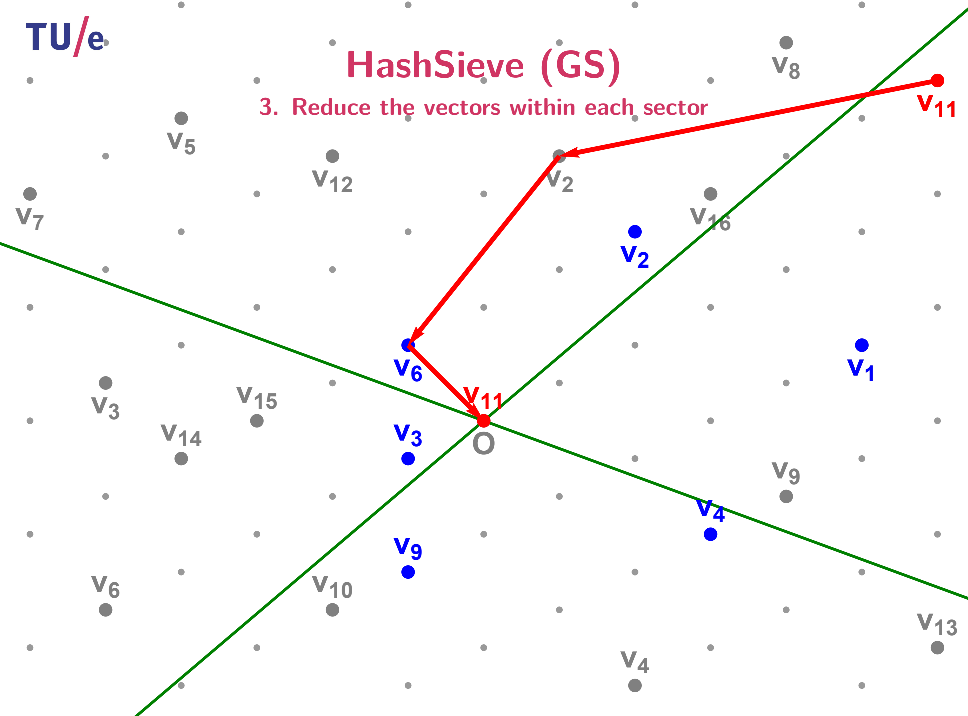

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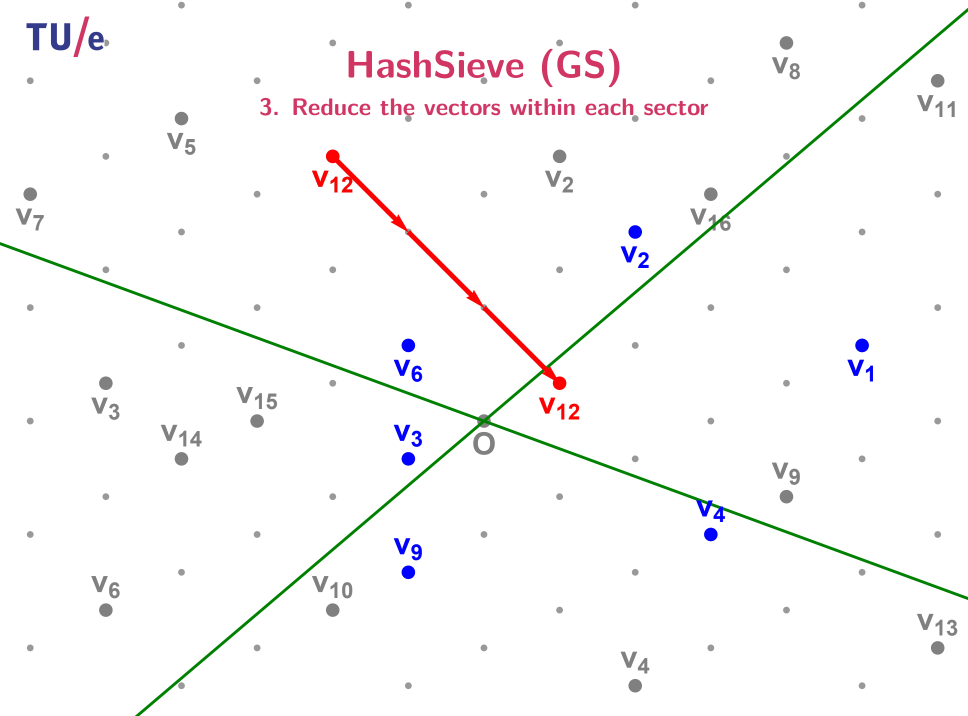

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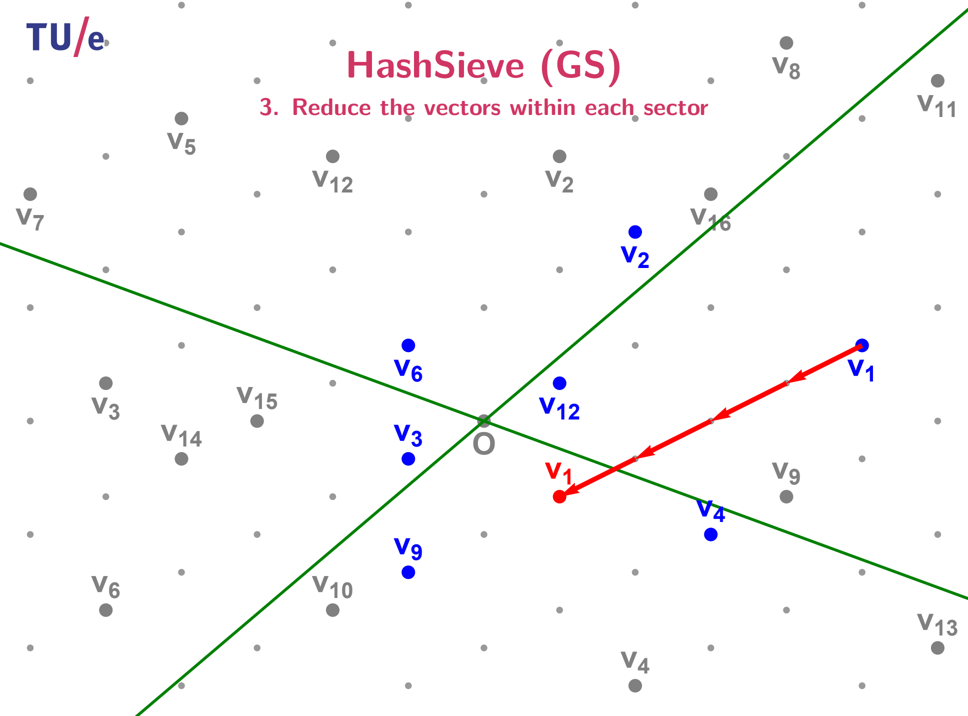

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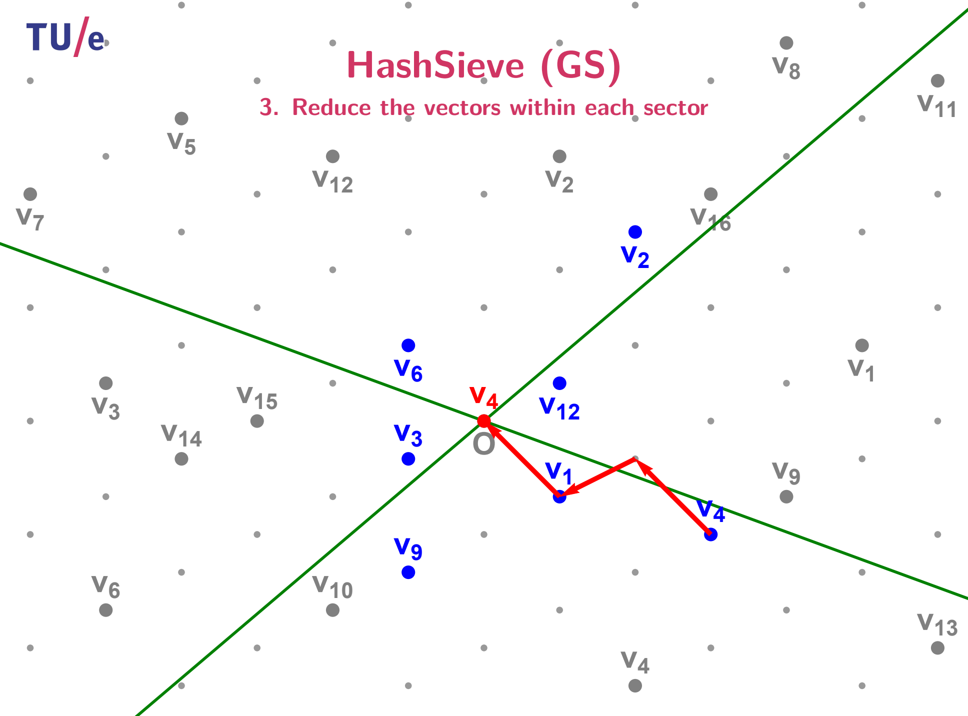

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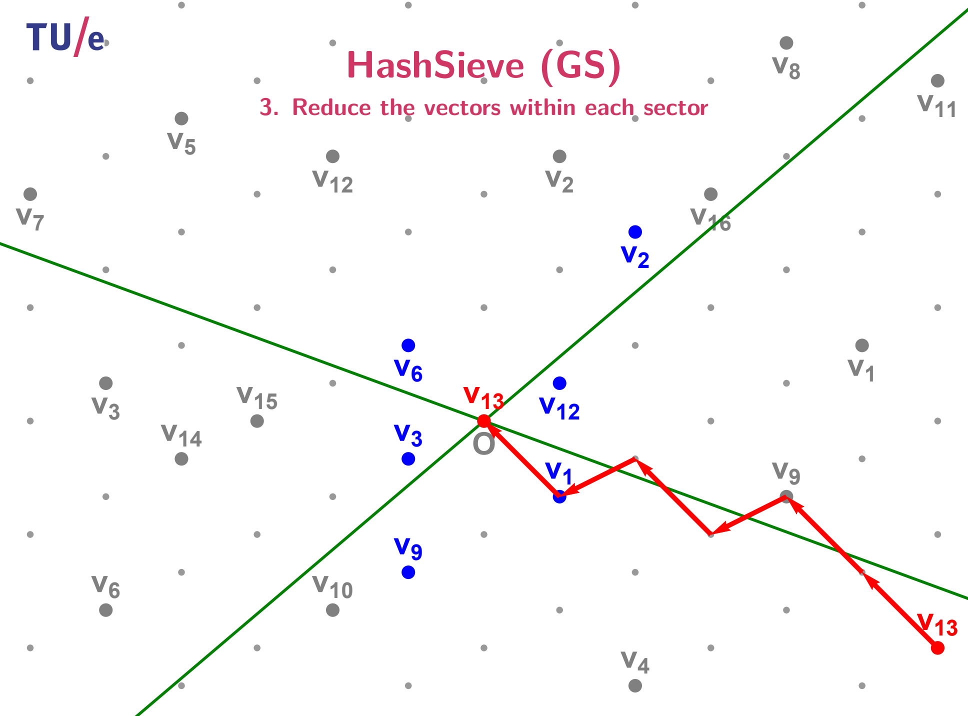

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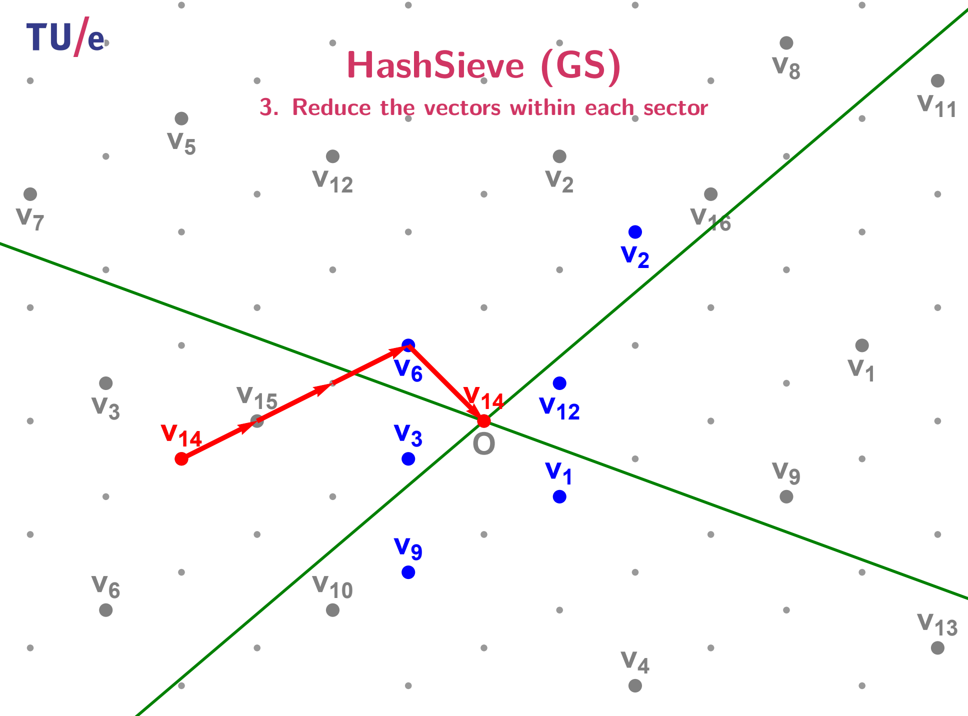

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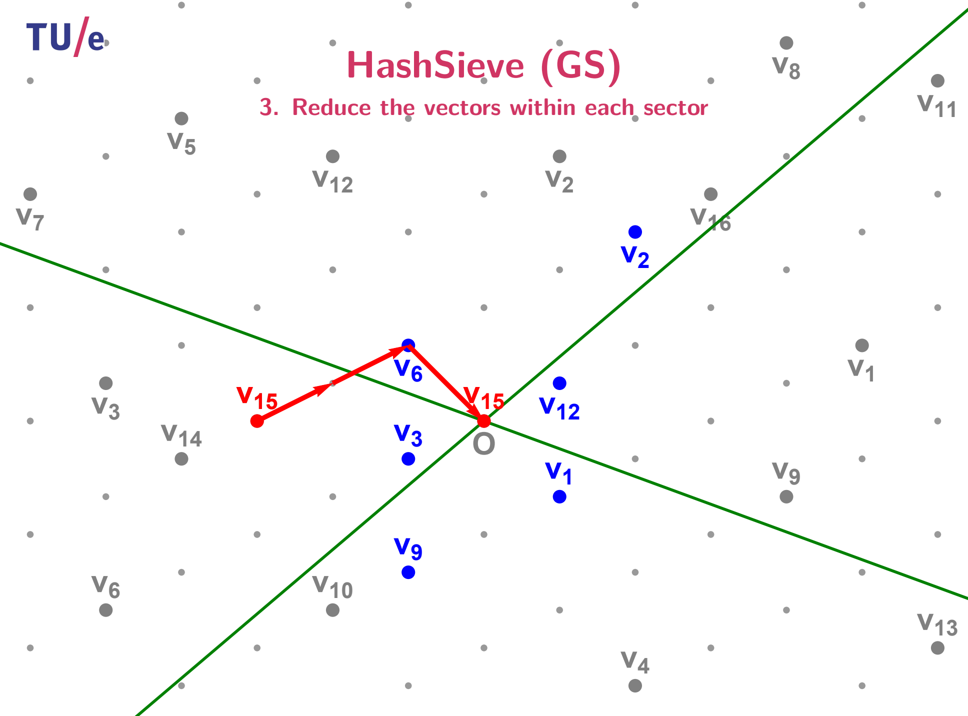

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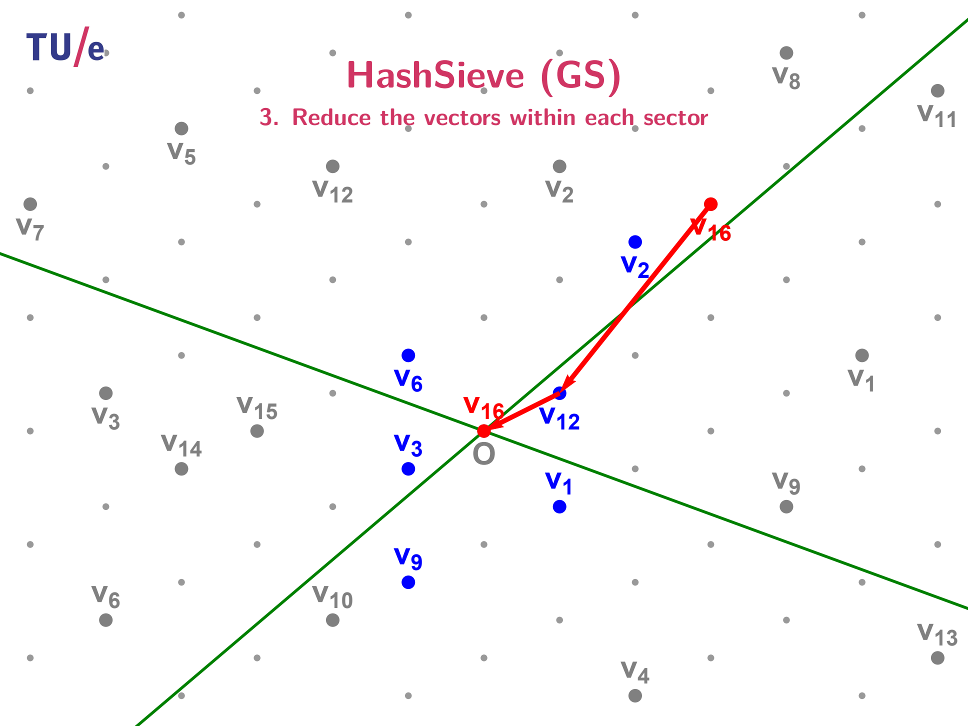

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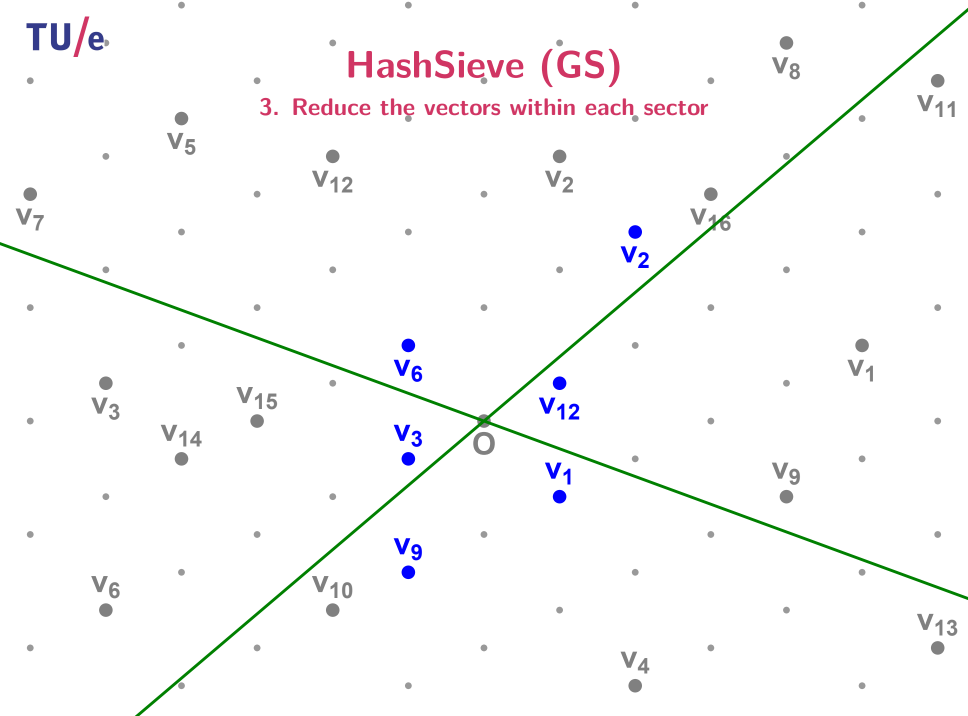
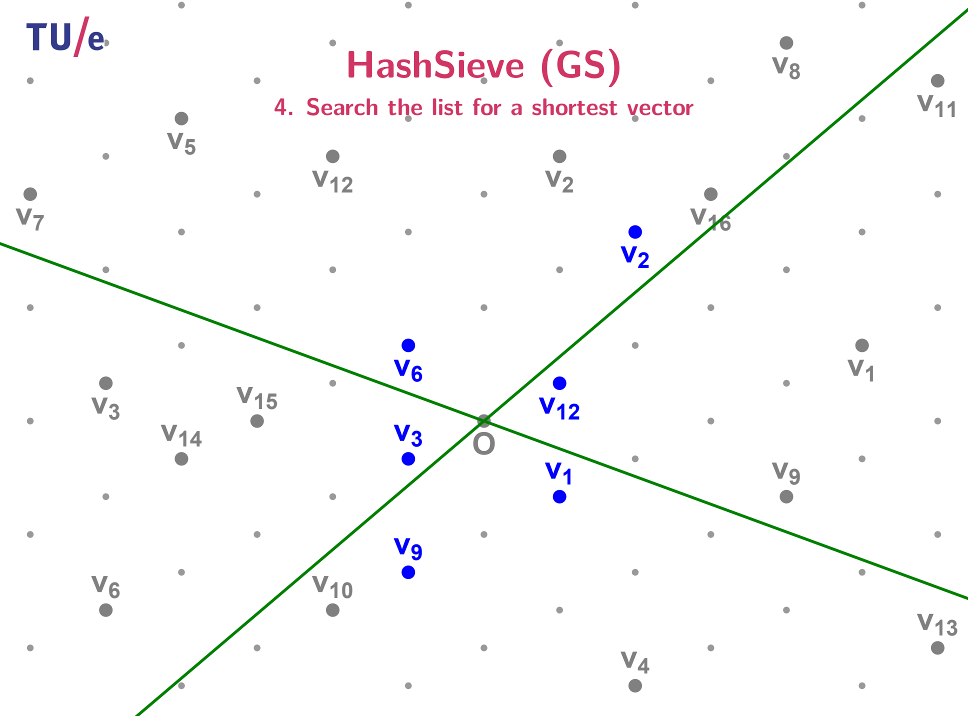


TU/e

HashSieve (GS)

4. Search the list for a shortest vector



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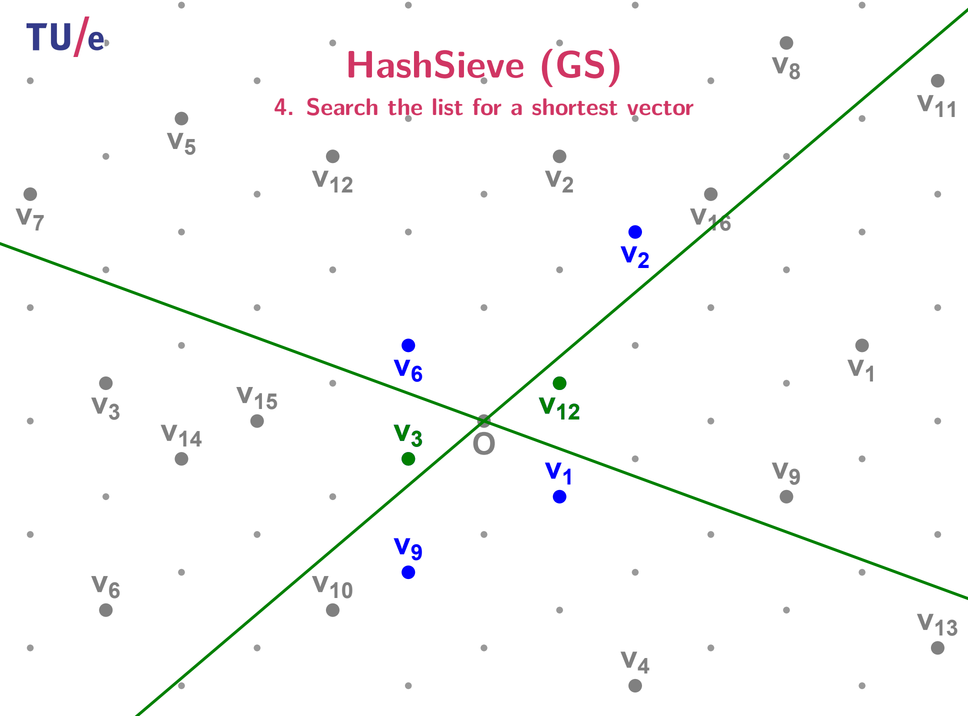

HashSieve (GS)

Overview

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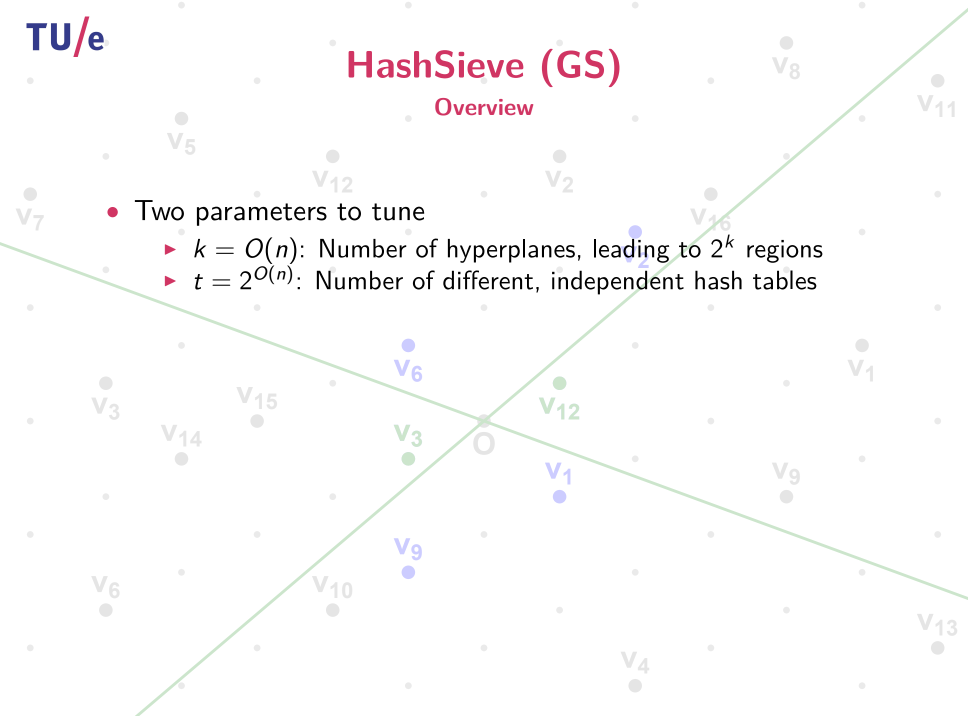
Overview

- Two parameters to tune
  - $k = O(n)$: Number of hyperplanes, leading to 2^k regions
  - $t = 2^{O(n)}$: Number of different, independent hash tables



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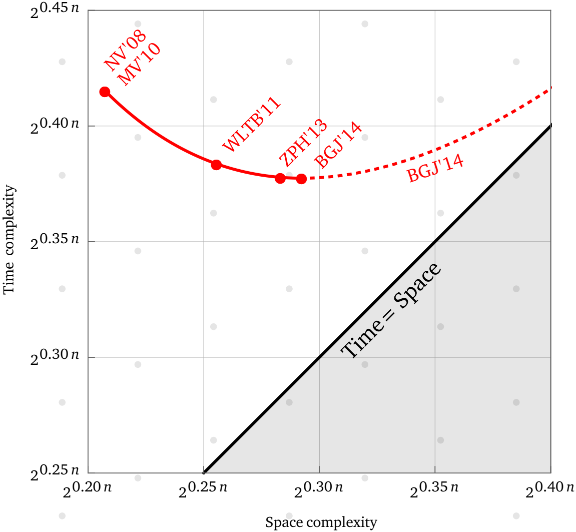
Heuristic

The HashSieve (GS) runs in time and space $2^{0.34n+o(n)}$.

HashSieve (GS)

Space/time trade-off



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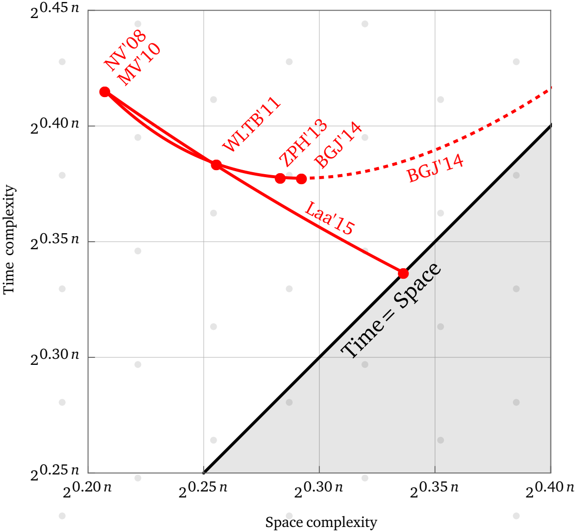

HashSieve (GS)

Experimental results

Dimension	**90**	**92**	**94**	**96**	**98**	**100**
Hyperplanes (k)	20	20	21	21	22	22
Hash tables (t)	3126	3738	4470	465	533	637
Probing?	N	N	N	Y	Y	Y
BKZ-β preproc.	34	34	34	34	40	40
Used vectors ($\cdot 10^6$)	2.43	3.00	4.50	5.57	7.05	10.05
Time (hours)	0.86	1.72	3.74	6.52	10.03	18.19
Memory (GB)	310	380	872	95	113	256

HashSieve (GS)

SVP challenge

22	118	2782	0	Kenji Kashiwabara and Masaharu Fukase	vec	Other	2013-08-12	1.00811
23	118	2868	8	Yuanmi Chen and Phong Nguyen	vec	ENUM	2013-02-13	1.04441
24	116	2743	0	Thorsten Kleinjung	vec	Sieving	2014-05-2	1.00492
25	116	2764	0	Kenji Kashiwabara and Masaharu Fukase	vec	Other	2014-03-21	1.01287
26	116	2786	0	Kenji Kashiwabara and Masaharu Fukase	vec	Other	2013-08-3	1.02075

40	108	2508	0	Yuanmi Chen and Phong Nguyen	vec	Other	2010-06-16	0.95162
41	108	2755	0	Yuanmi Chen and Phong Nguyen	vec	Other	2010-05-30	1.04519
42	107	2626	0	Artur Mariano	vec	Sieving	2015-05-22	1.00056
43	107	2713	0	Artur Mariano	vec	Sieving	2015-05-22	1.03379
44	107	2716	0	Artur Mariano	vec	Sieving	2015-05-22	1.03490
45	107	2719	0	Artur Mariano	vec	Sieving	2015-05-22	1.03609
46	107	2720	0	Artur Mariano	vec	Sieving	2015-05-22	1.03657
47	107	2721	0	Artur Mariano	vec	Sieving	2015-05-22	1.03688
48	107	2722	0	Artur Mariano	vec	Sieving	2015-05-22	1.03721
49	107	2724	8	Po-Chun Kuo, Michael Schneider	vec	ENUM,BKZ	2011-03-12	1.03655

HashSieve (NV)

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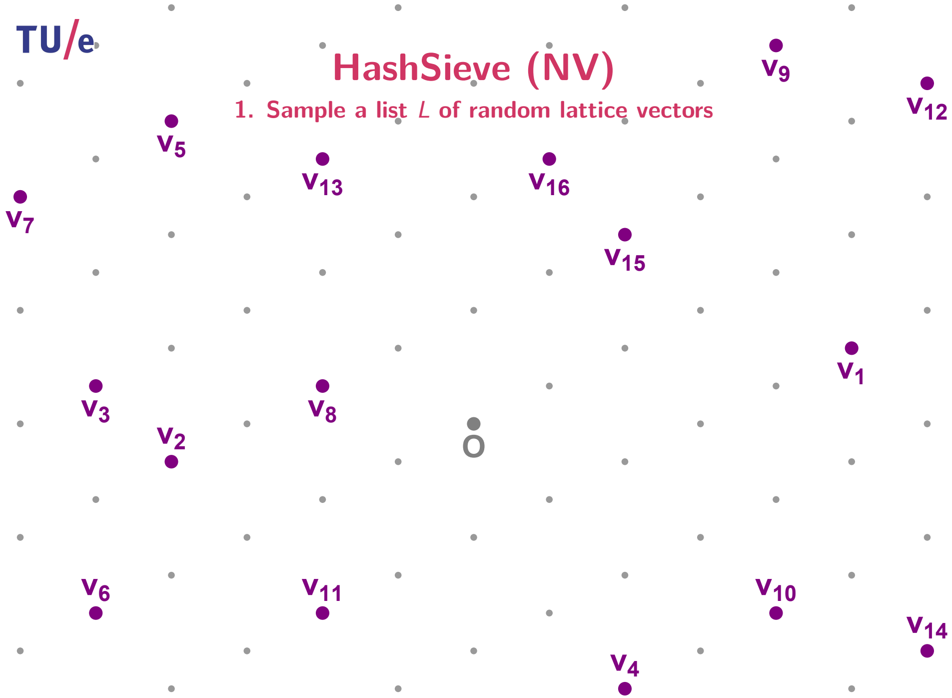

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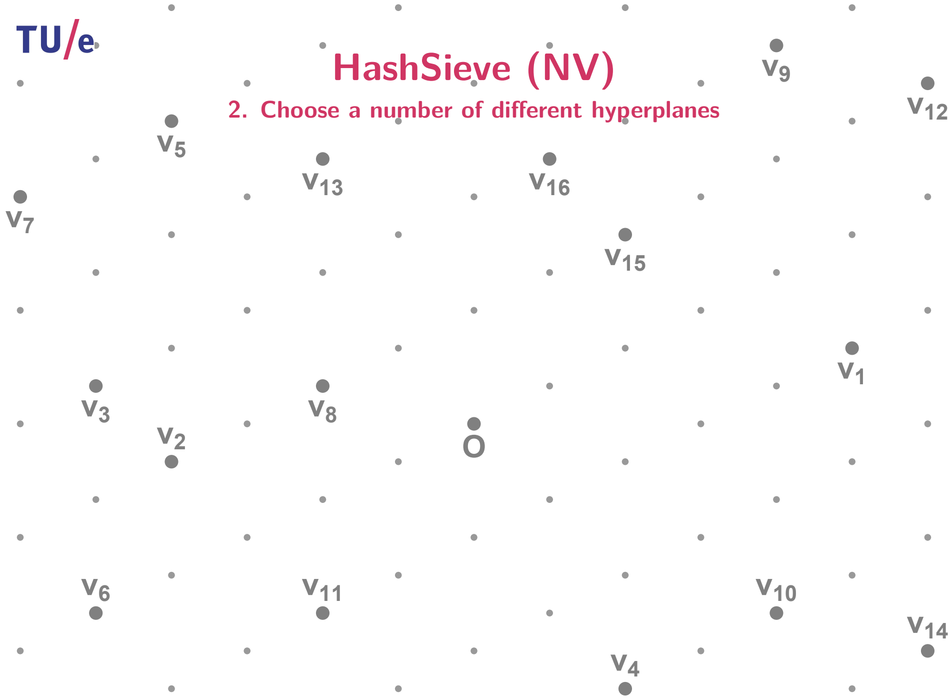
2. Choose a number of different hyperplanes



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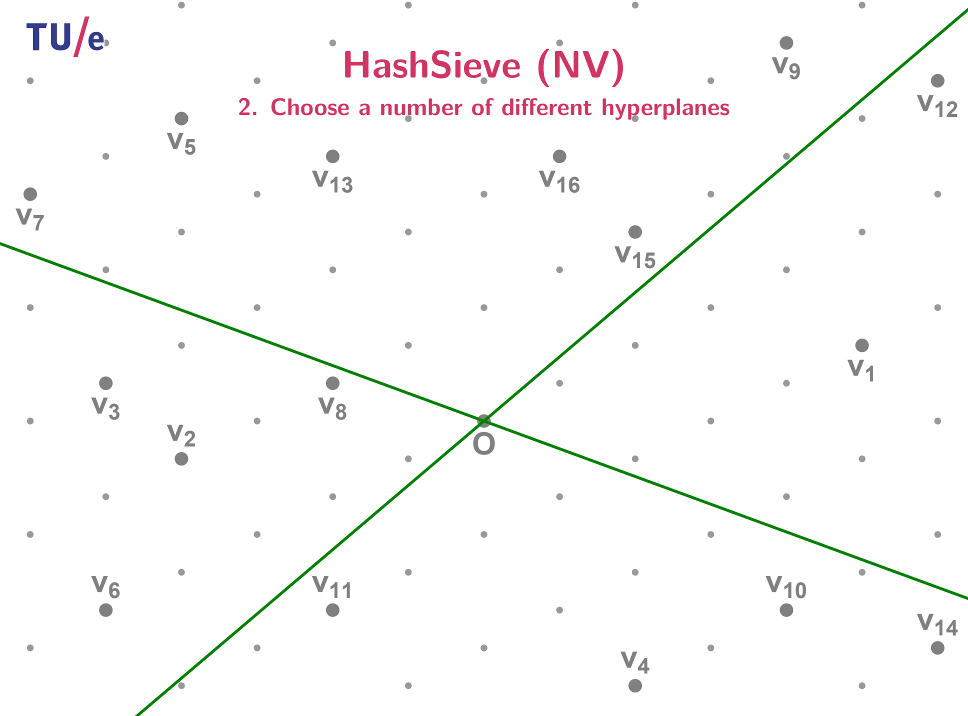
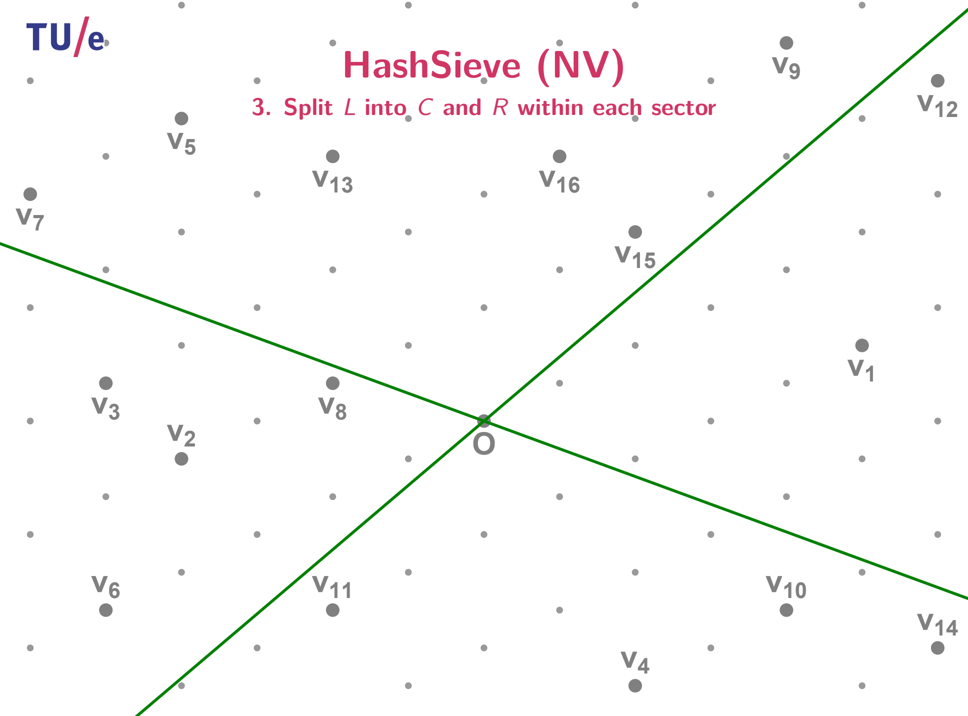


HashSieve (NV)

3. Split L into C and R within each sector



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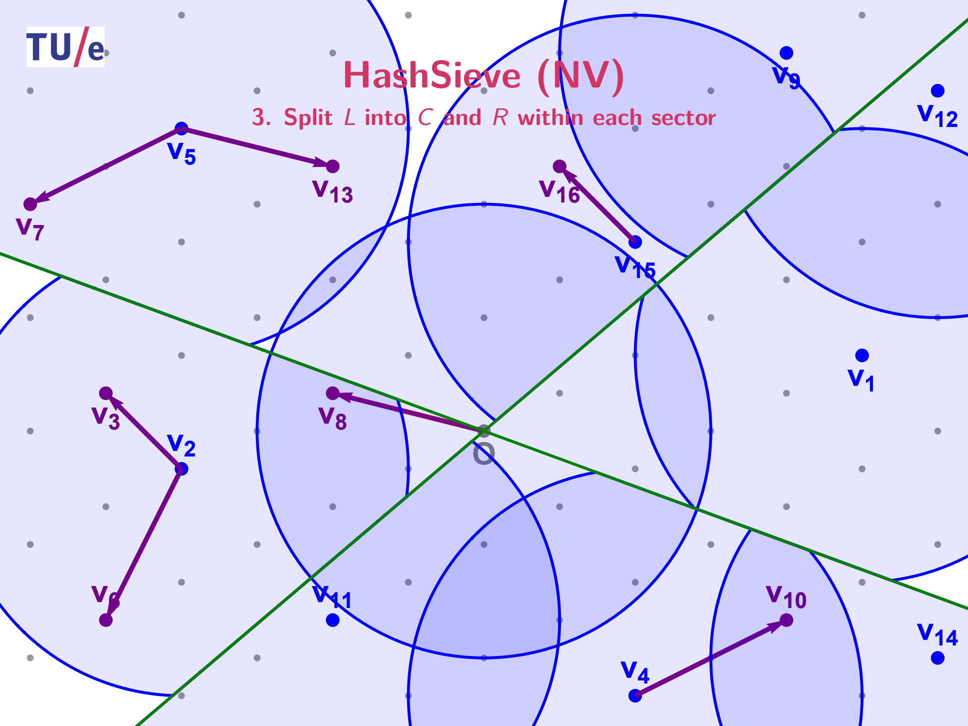

TU/e

HashSieve (NV)

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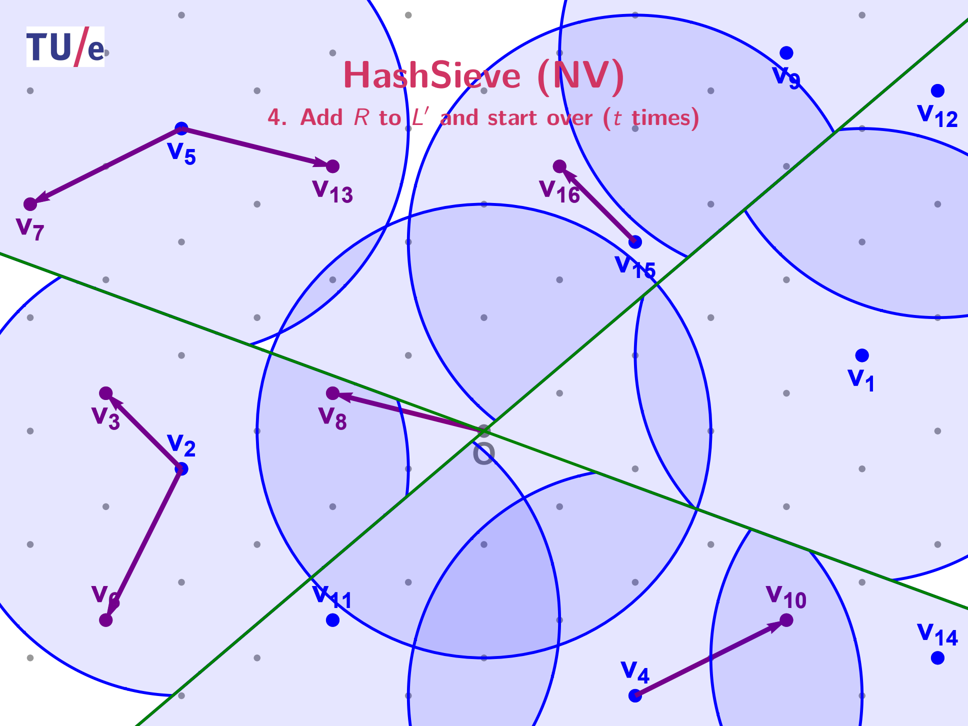

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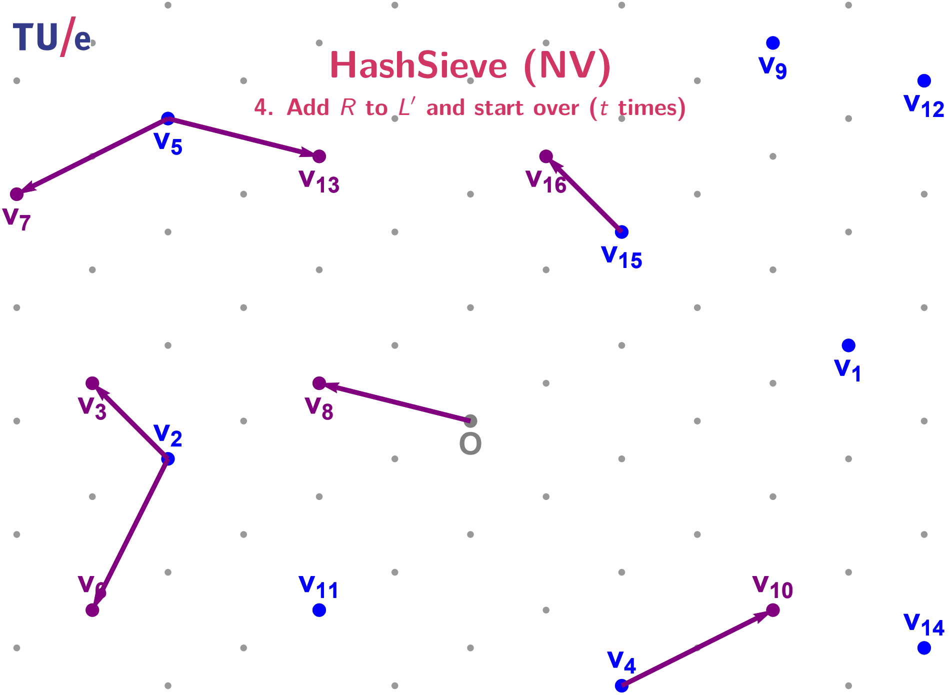

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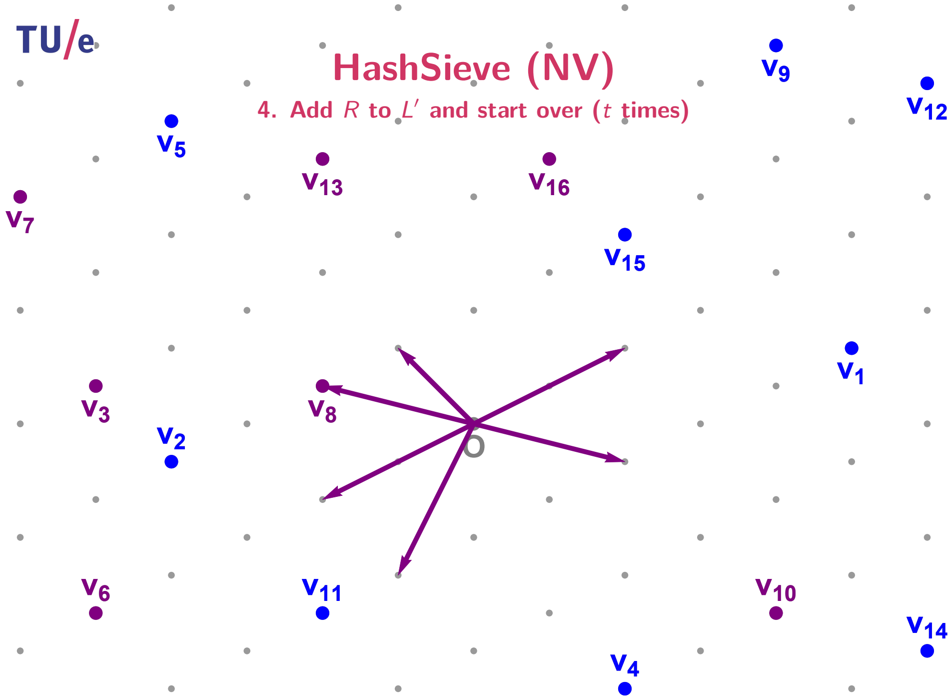

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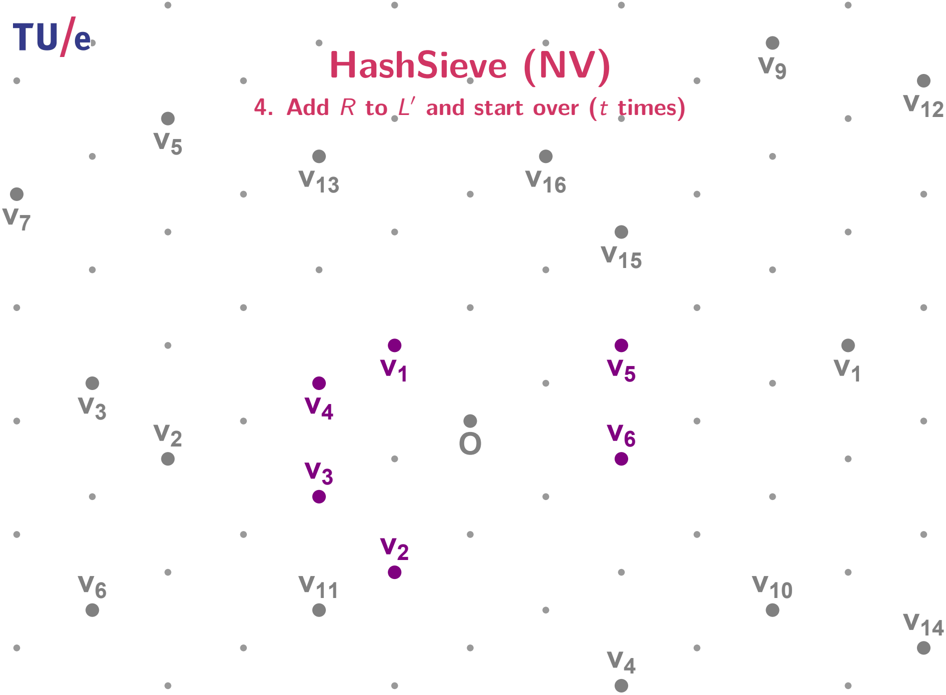

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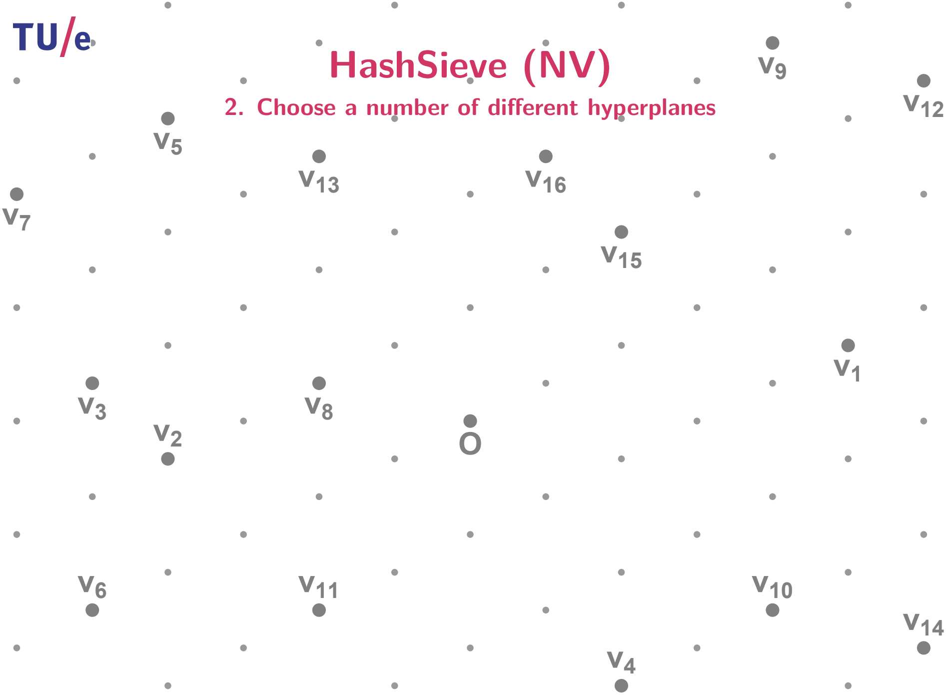

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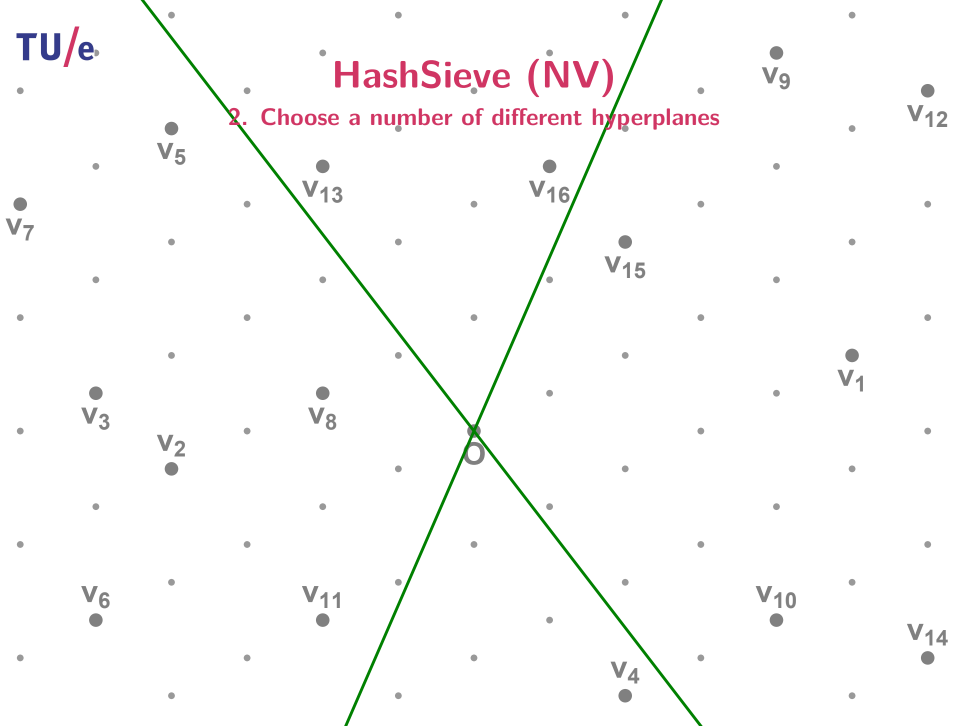

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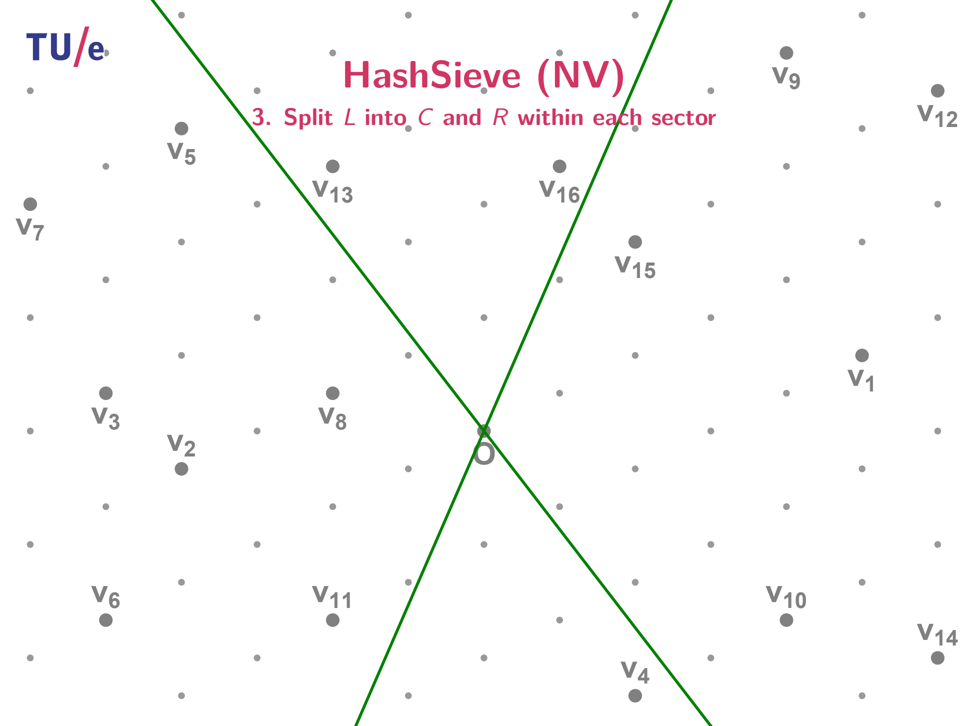

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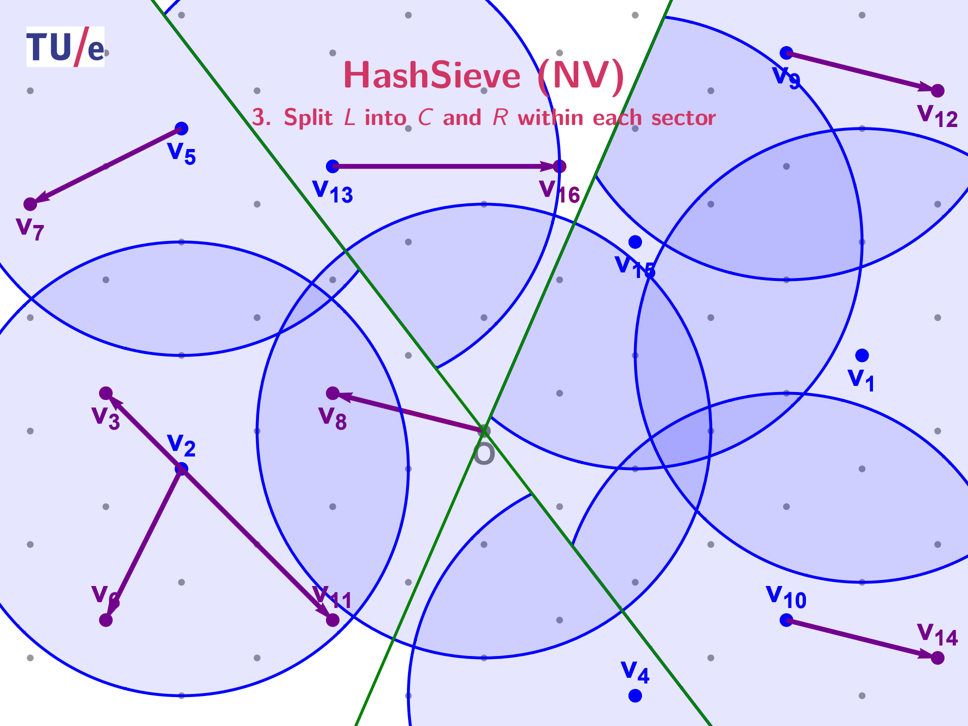

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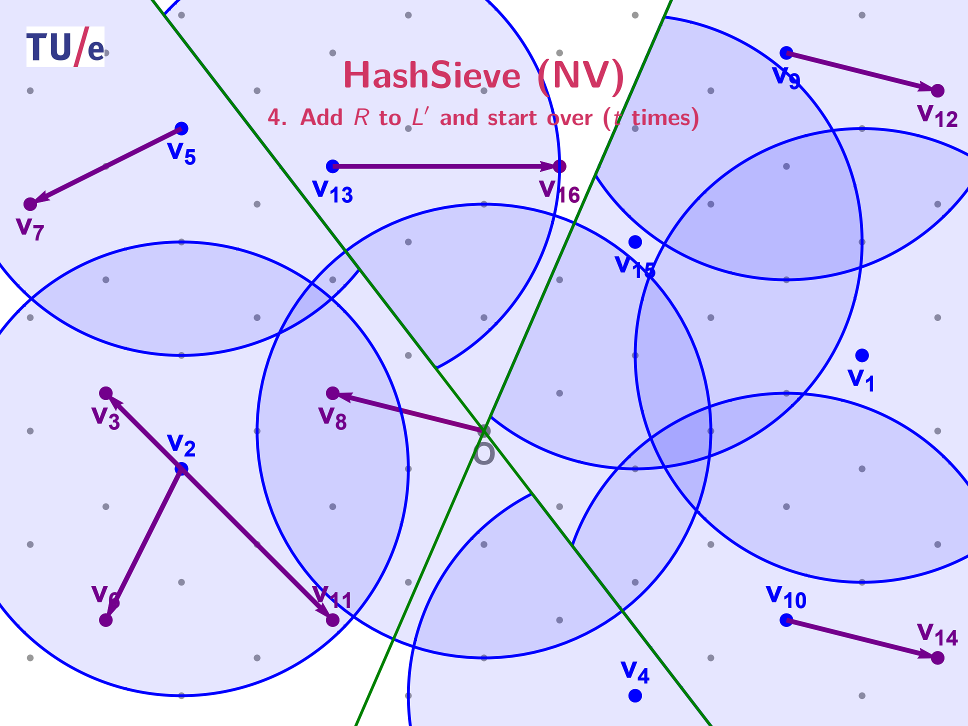

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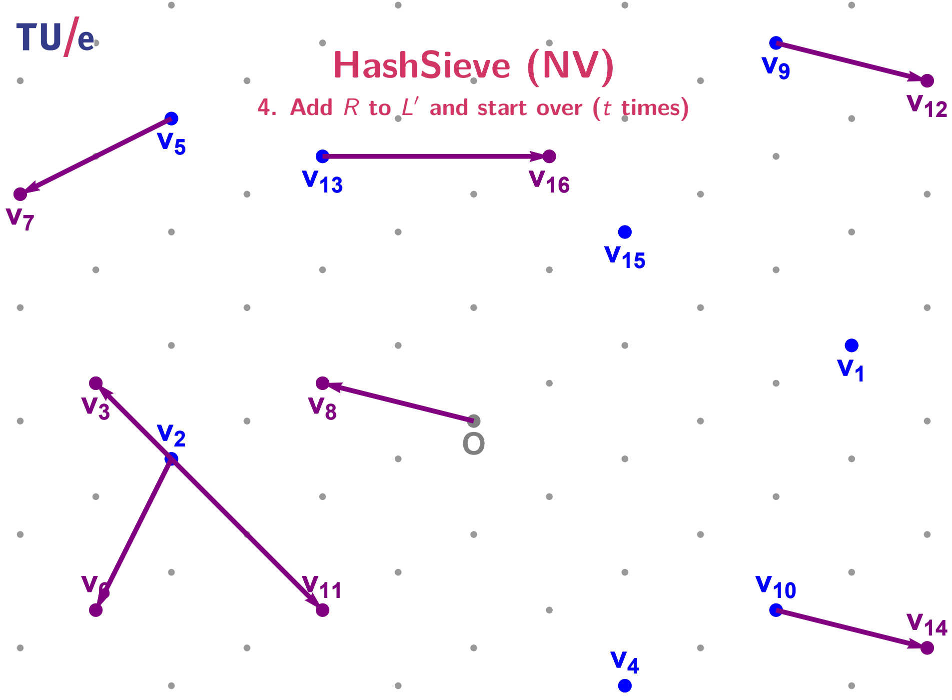

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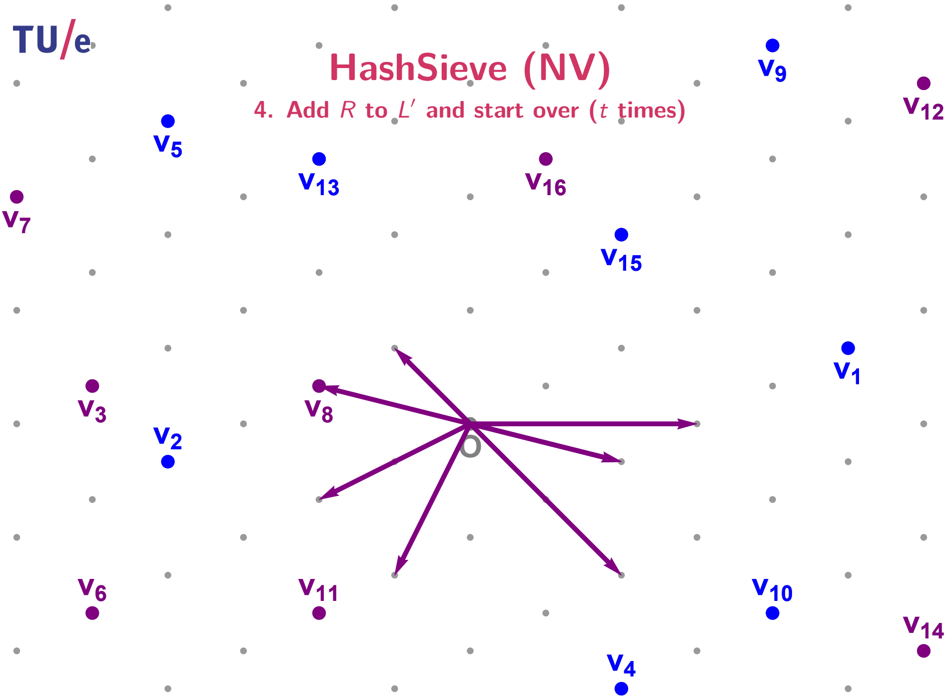

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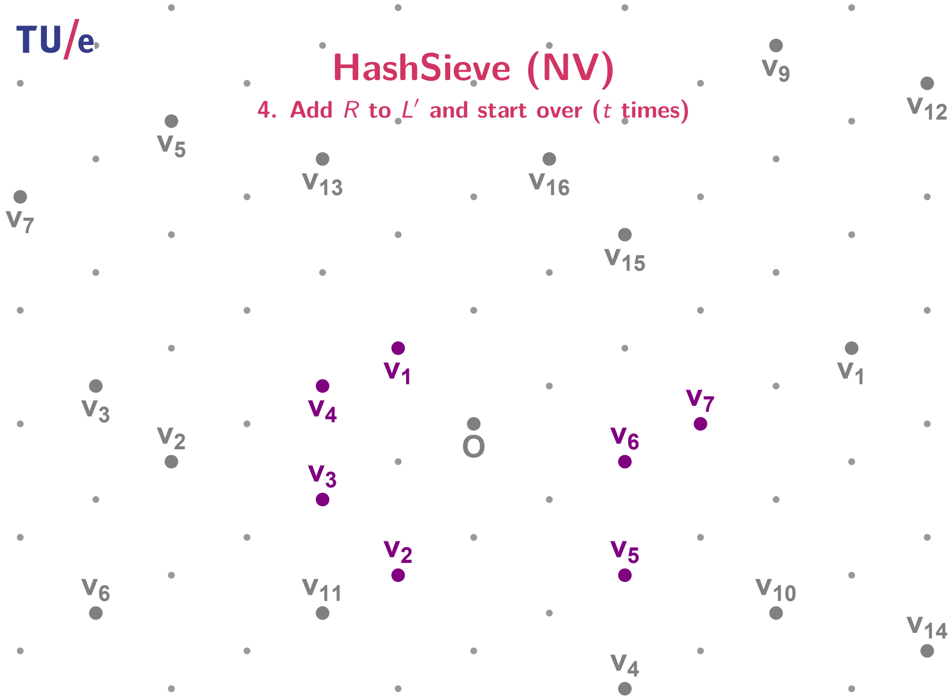

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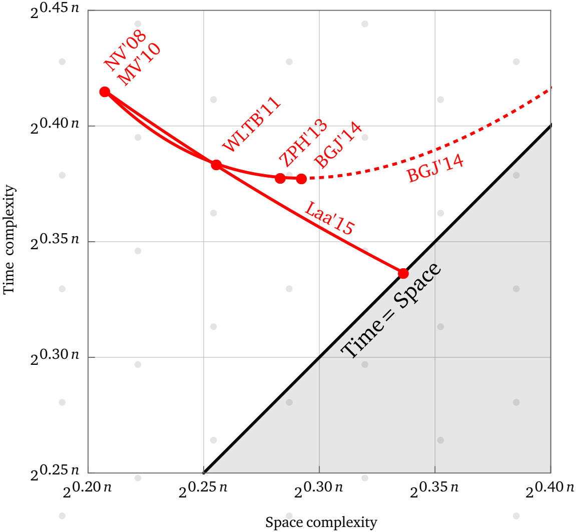
Heuristic

The HashSieve (NV) runs in time $2^{0.34n+o(n)}$ and space $2^{0.21n+o(n)}$.

HashSieve (NV)

Space/time trade-off



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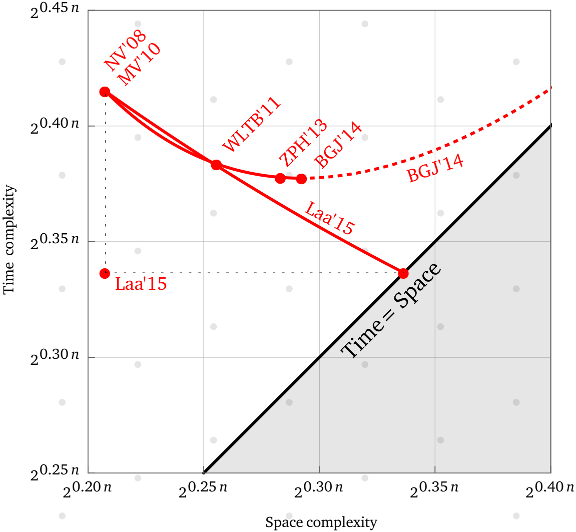
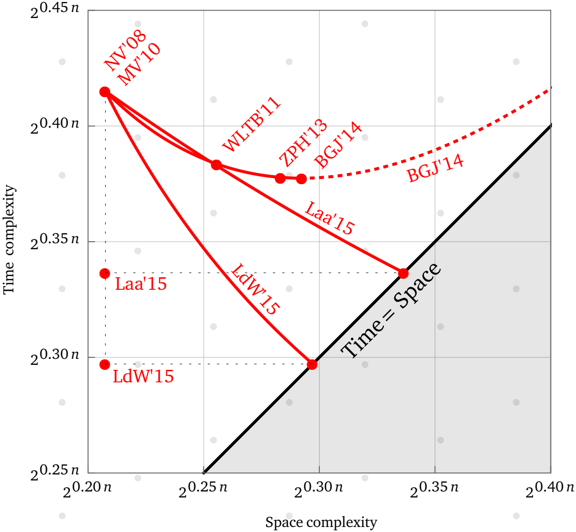


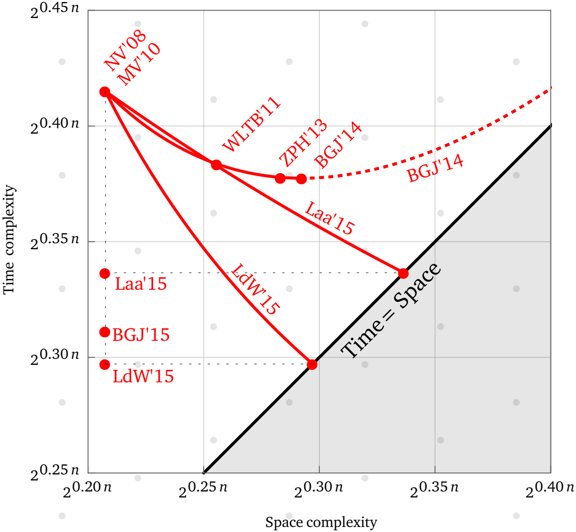
SphereSieve

Space/time trade-off



Another NNS sieve

Space/time trade-off



CrossPolytopeSieve

Main ideas

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CrossPolytopeSieve

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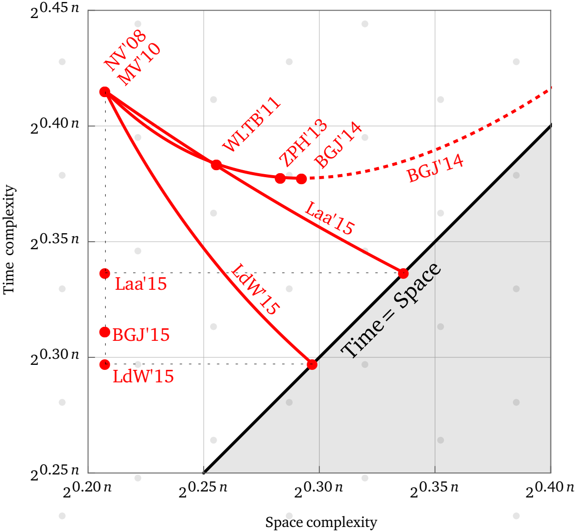
- HashSieve: divide space into "equal" parts with hyperplanes
- Observation: orthogonal hyperplanes better than random ones
  - Corresponds to "hypercube hashing"
- Observation: hypercube not "most symmetric" object ($k \ll n$)
- Idea: divide space into regions using regular polytopes
  - Hypercube $\implies$ HashSieve with orthogonal hyperplanes
  - Cross polytope $\implies$ CrossPolytopeSieve
  - Simplex $\implies$ similar to CrossPolytopeSieve

CrossPolytopeSieve

Space/time trade-off



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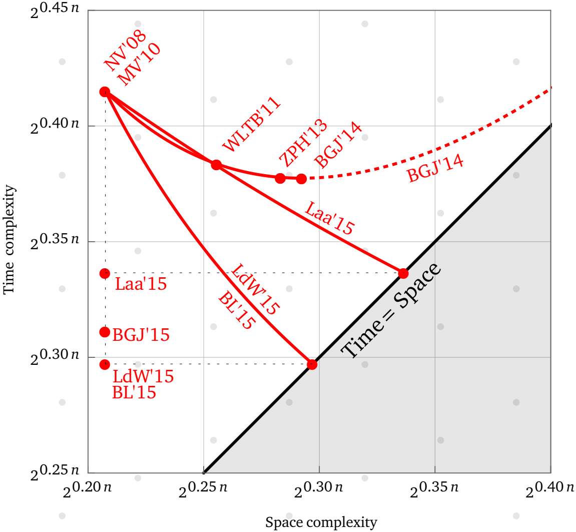
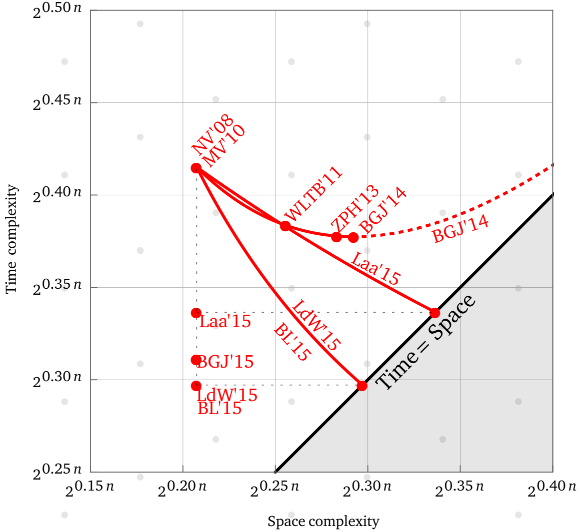


Ongoing work

Space/time trade-off



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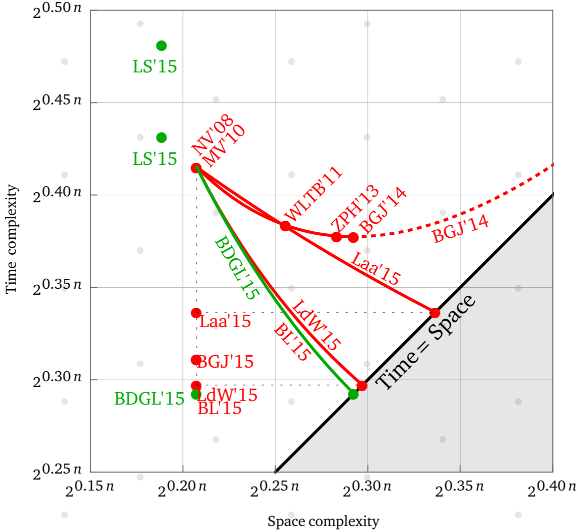

Questions