Sieving for shortest vectors in lattices using locality-sensitive hashing

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ENS Lyon, Lyon, France
(June 10, 2015)

Lattices

What is a lattice?

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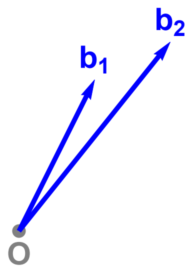

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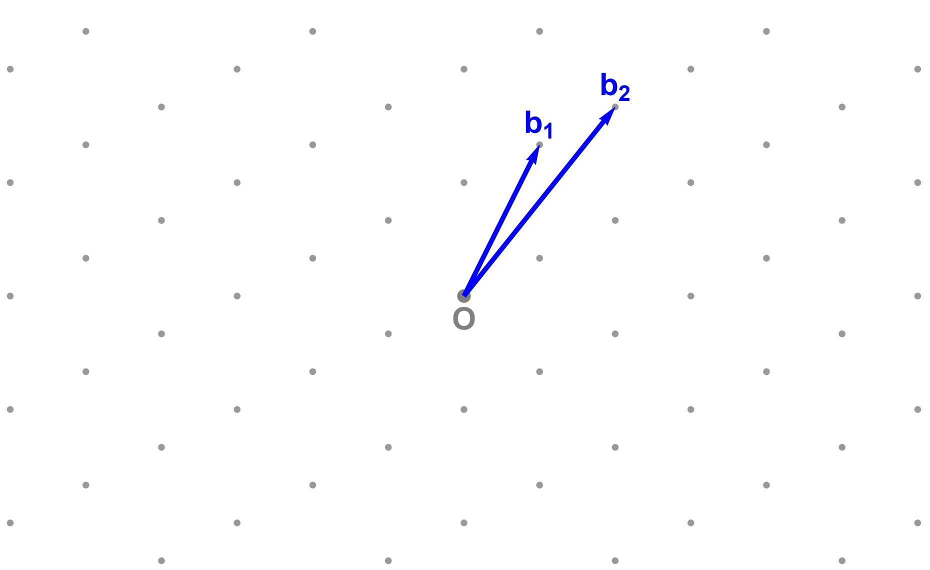
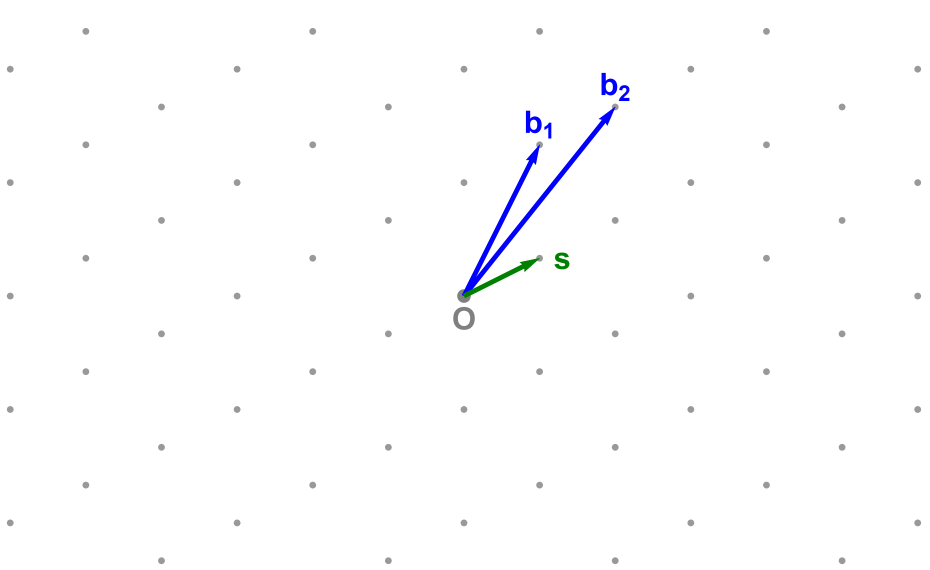


Lattices

Shortest Vector Problem (SVP)



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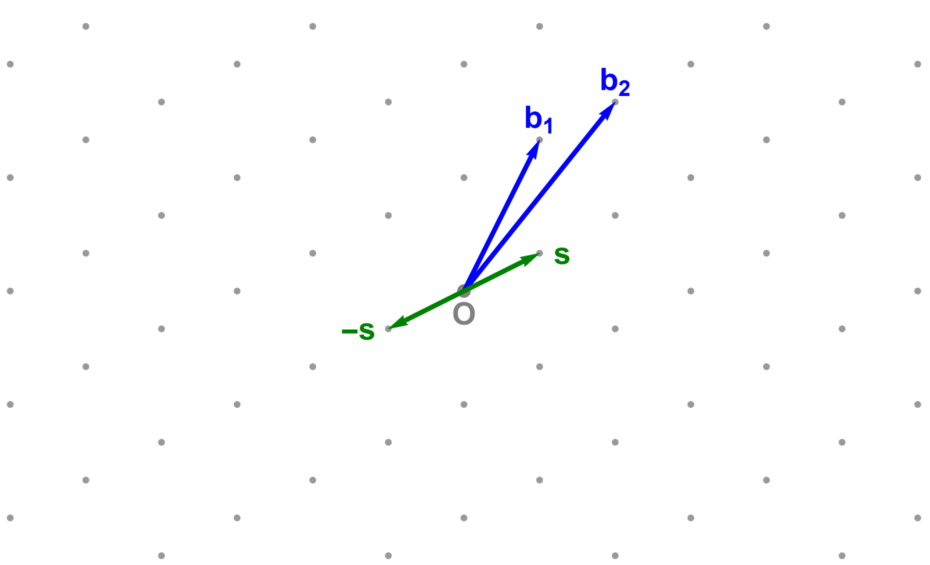

Lattices

SVP algorithms

	Algorithm	$\log_2(\text{Time})$	$\log_2(\text{Space})$
Provable SVP	Enumeration [Poh81, Kan83, . . . , GNR10]	$\Omega(n \log n)$	$O(\log n)$
	AKS-sieve [AKS01, NV08, MV10, HPS11]	$3.398n$	$1.985n$
	ListSieve [MV10, MDB14]	$3.199n$	$1.327n$
	AKS-sieve-birthday [PS09, HPS11]	$2.648n$	$1.324n$
	ListSieve-birthday [PS09]	$2.465n$	$1.233n$
	Voronoi cell algorithm [MV10b]	$2.000n$	$1.000n$
	Discrete Gaussian sampling [ADRS15, ADS15]	$1.000n$	$1.000n$
Heuristic SVP	NV-sieve [NV08]	$0.415n$	$0.208n$
	GaussSieve [MV10, . . . , IKMT14, BNvdP14]	$0.415n$?	$0.208n$
	Two-level sieve [WLTB11]	$0.384n$	$0.256n$
	Three-level sieve [ZPH13]	$0.3778n$	$0.283n$
	Decomposition approach [BGJ14]	$0.3774n$	$0.293n$
	HashSieve [Laa15, MLB15]	$0.337n$	$0.208n$*
	Another NNS sieve [BGJ15]	$0.311n$	$0.208n$
	SphereSieve [LdW15]	$0.298n$	$0.208n$
	CrossPolytopeSieve [BL15]	$0.298n$	$0.208n$*

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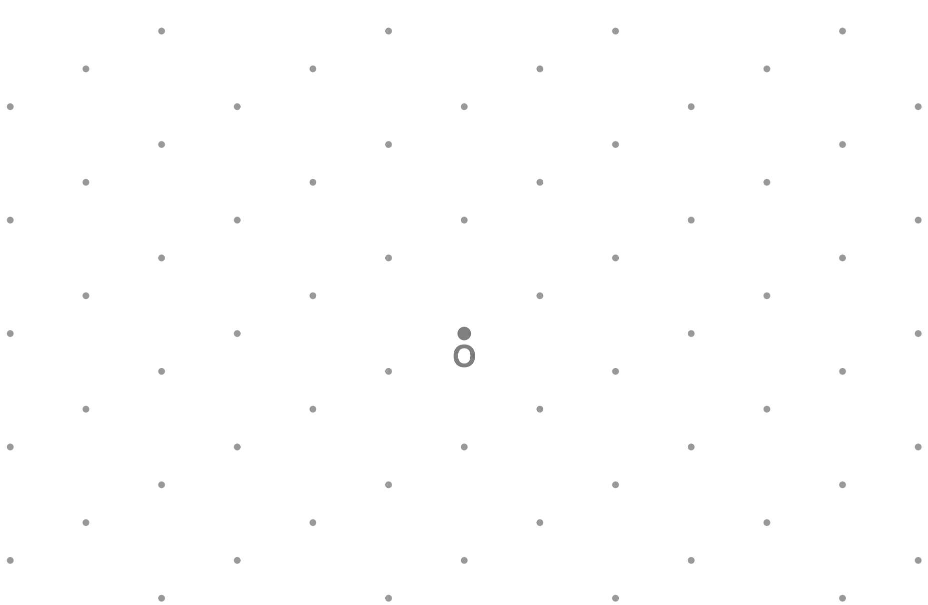
Nguyen-Vidick sieve



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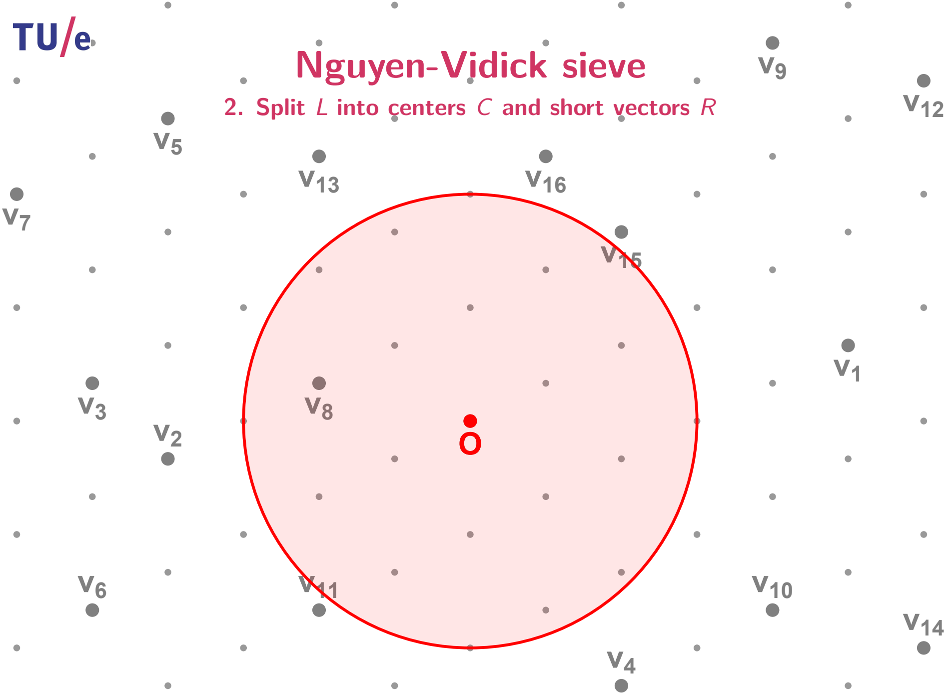

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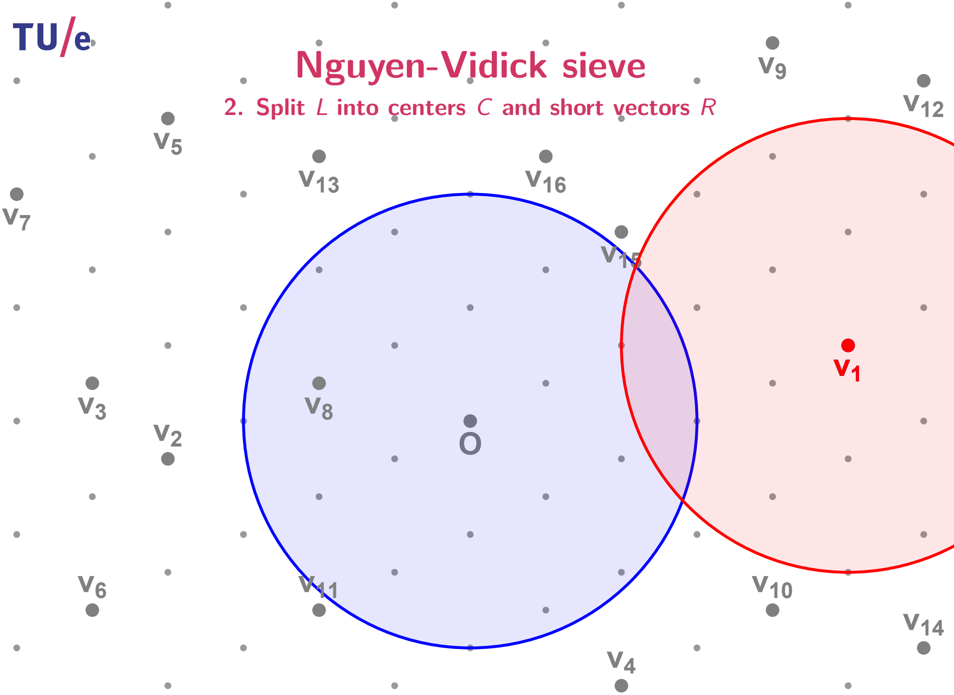

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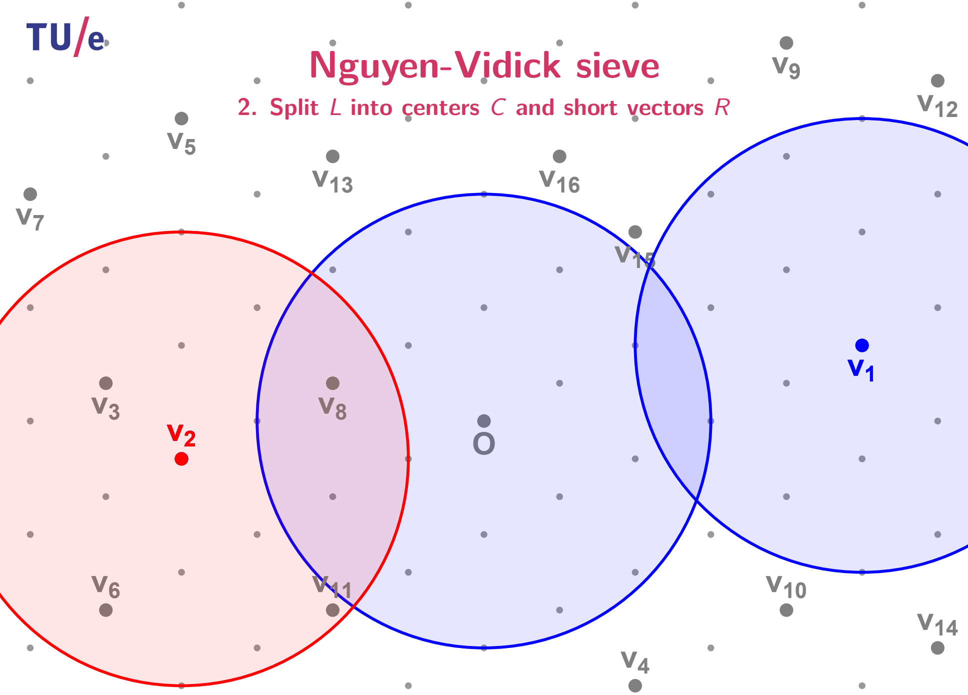

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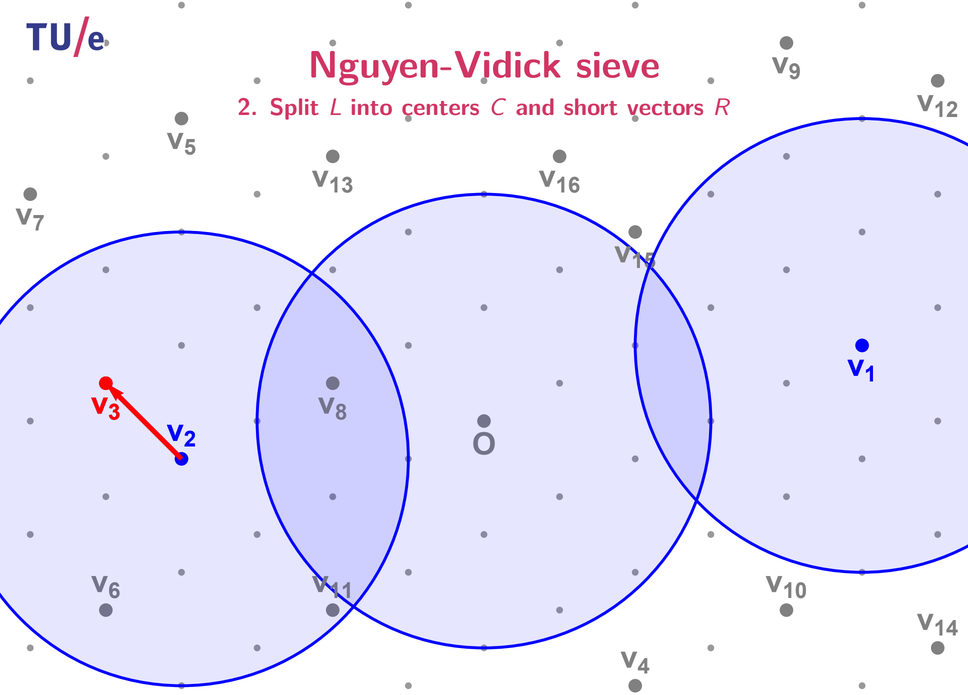

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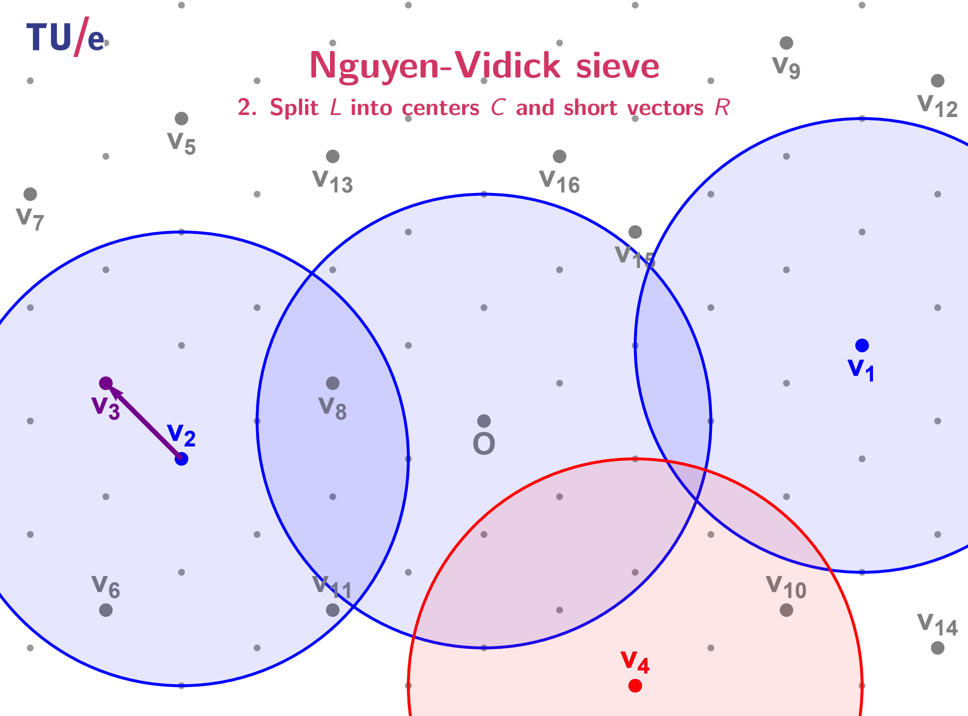

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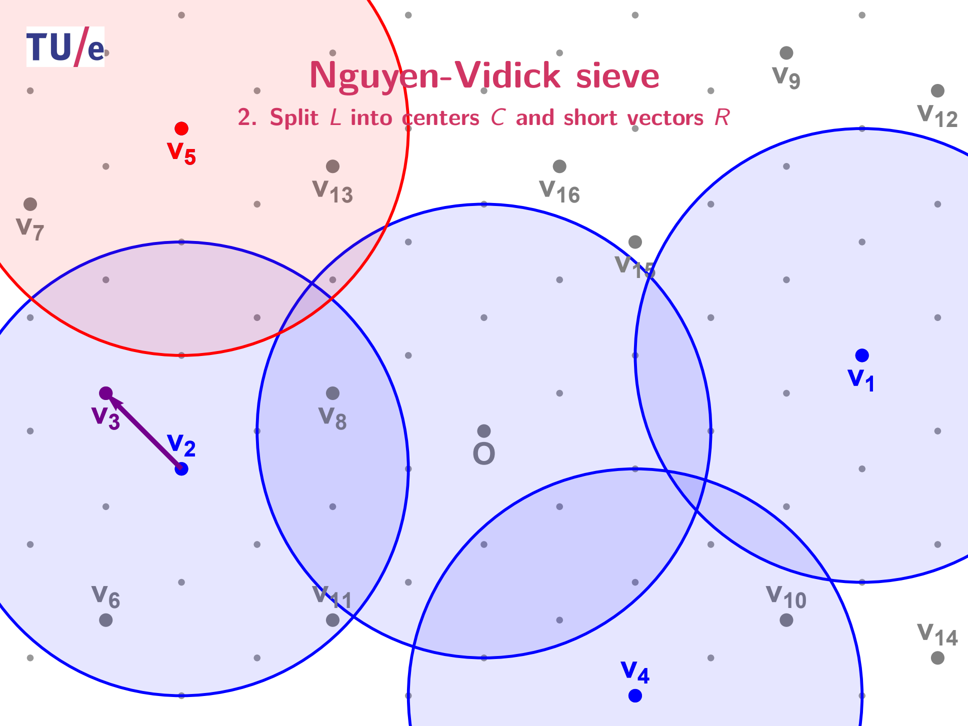

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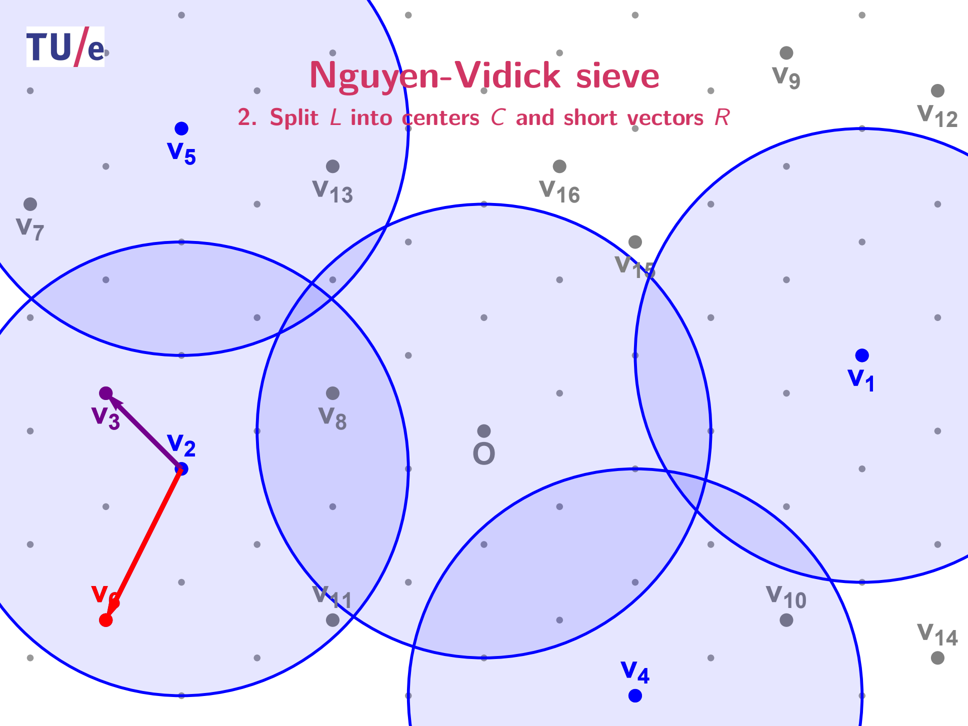

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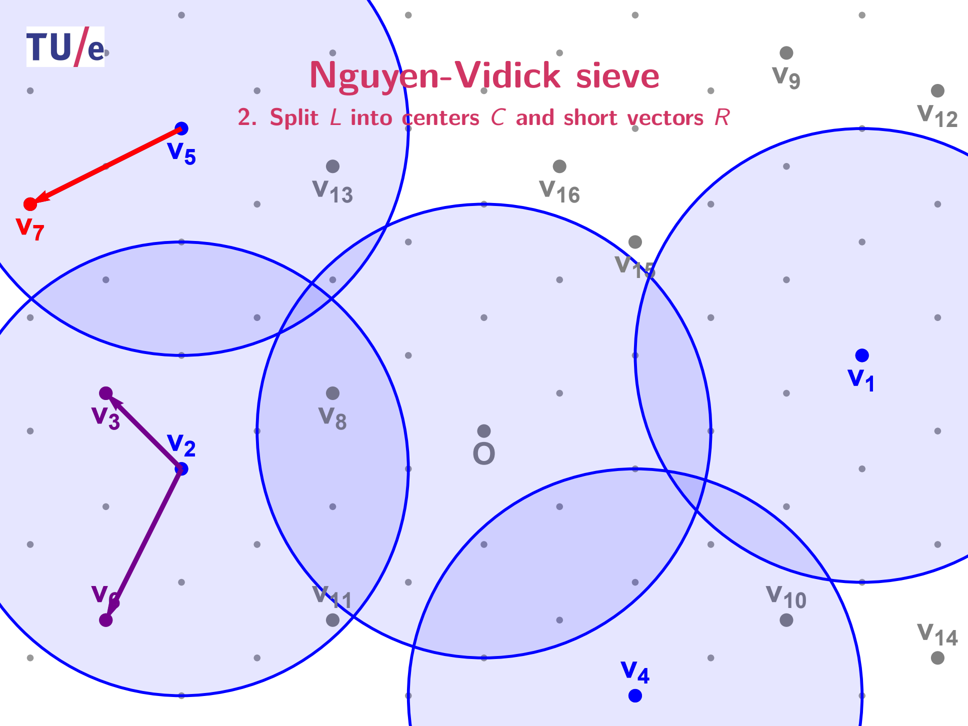

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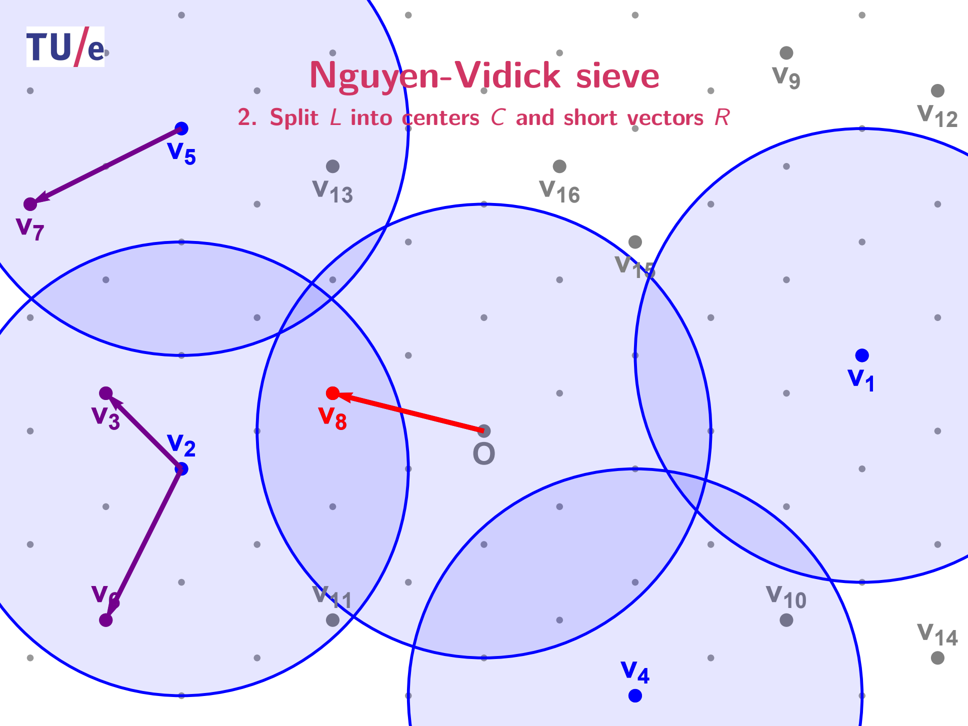

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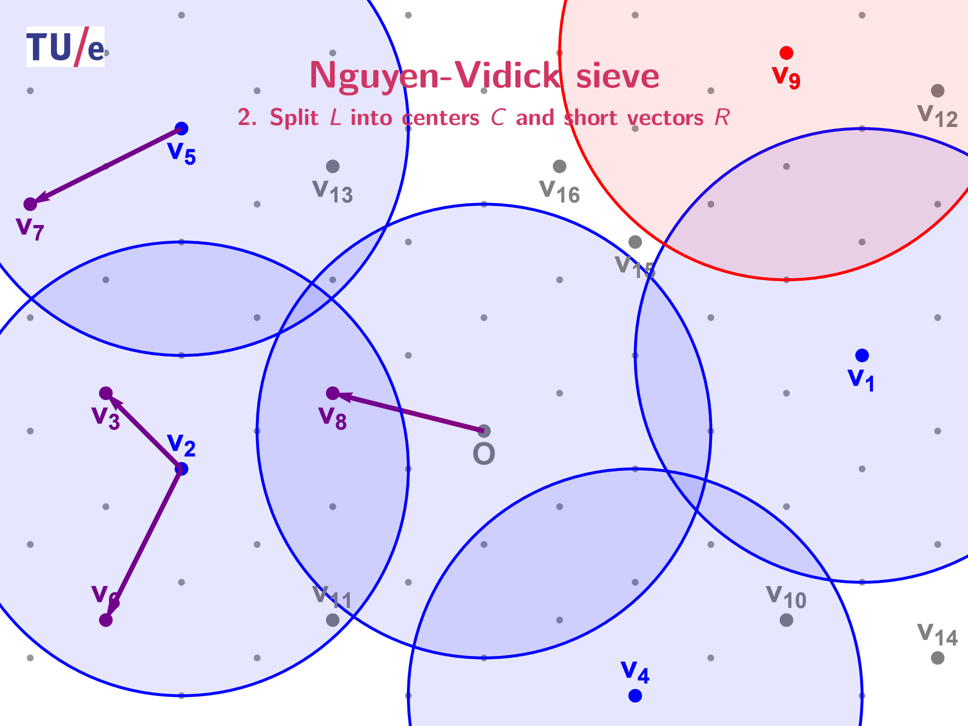

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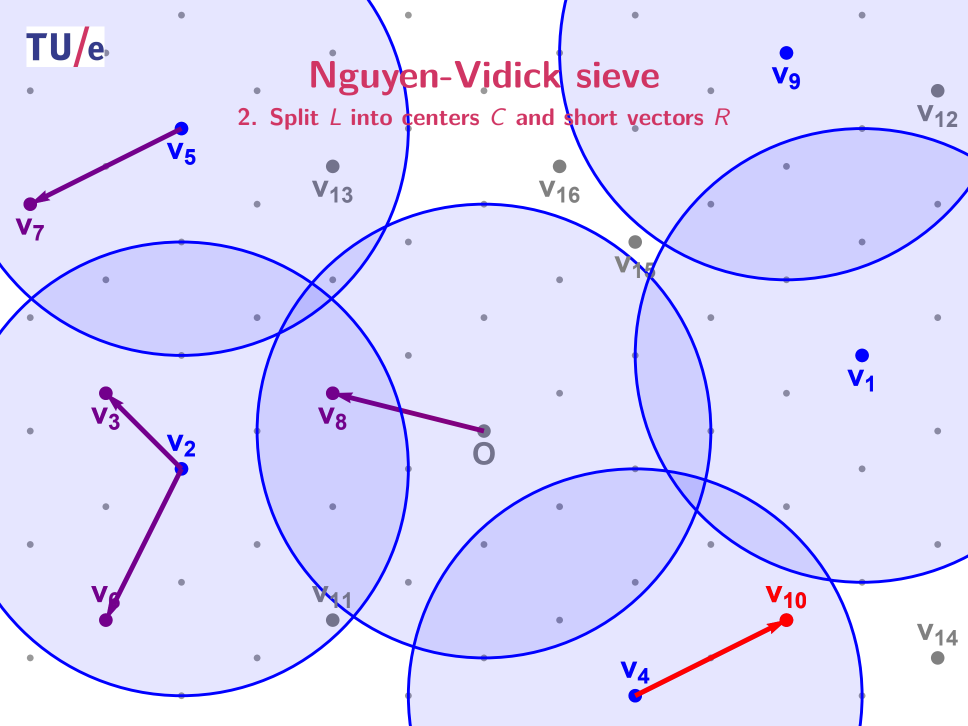

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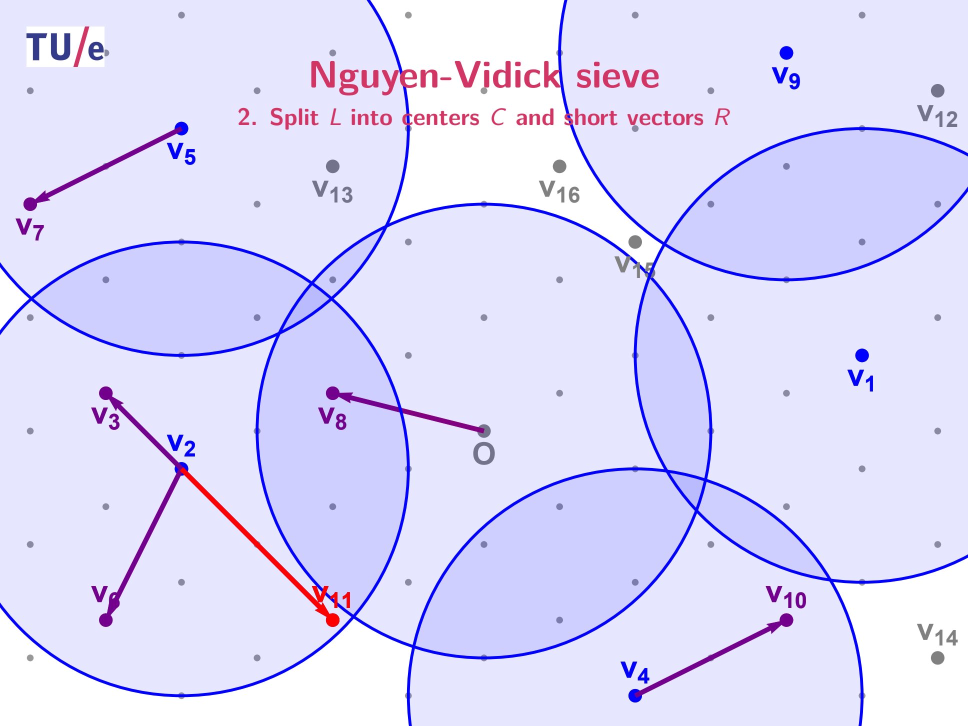

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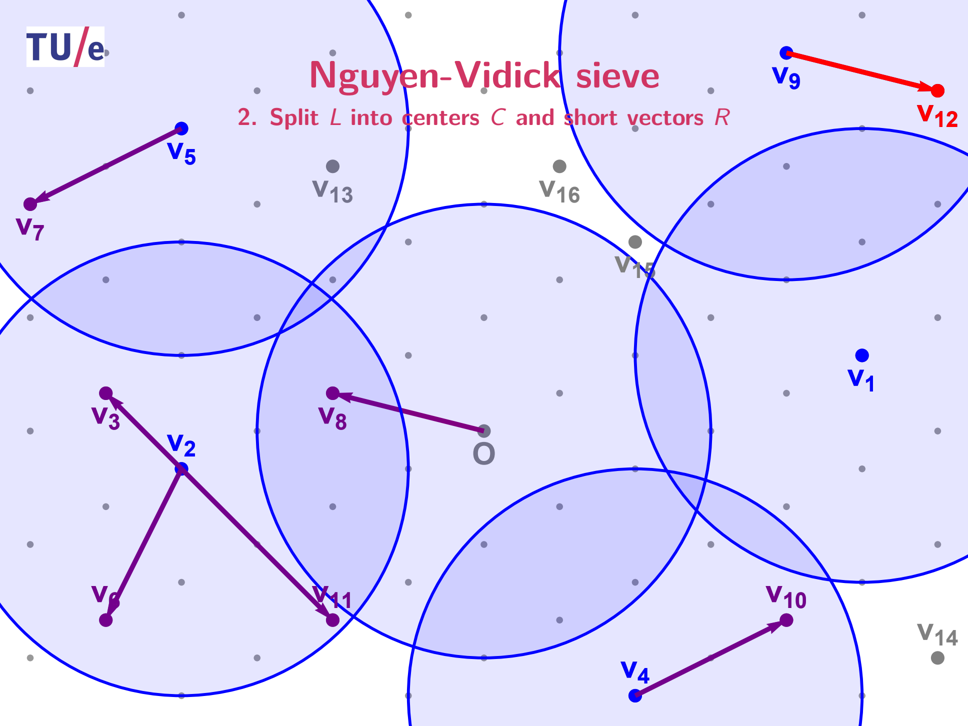

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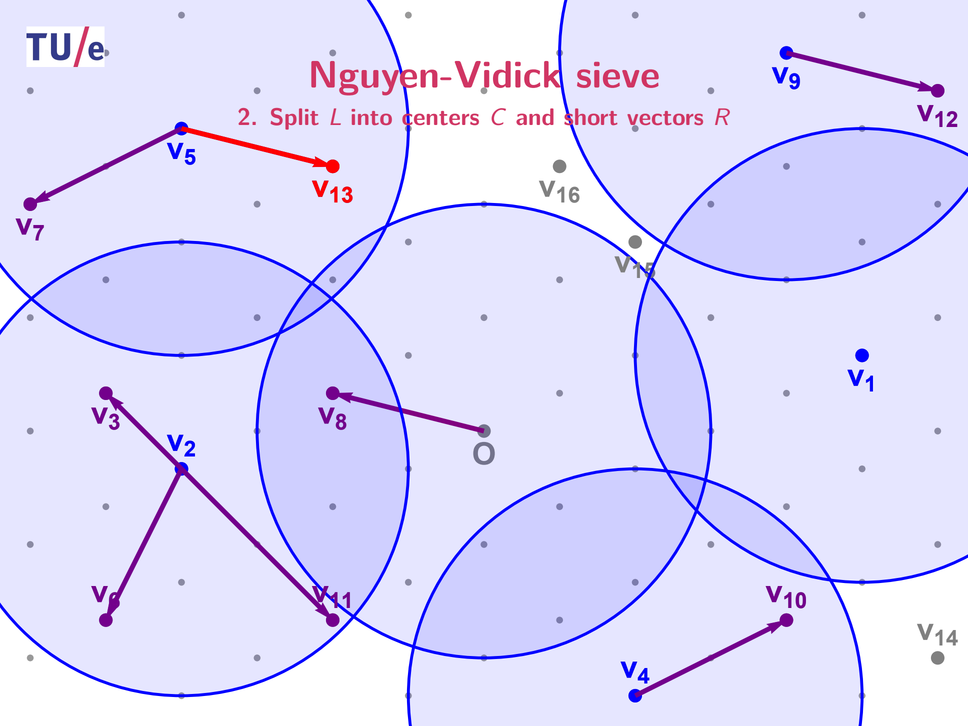

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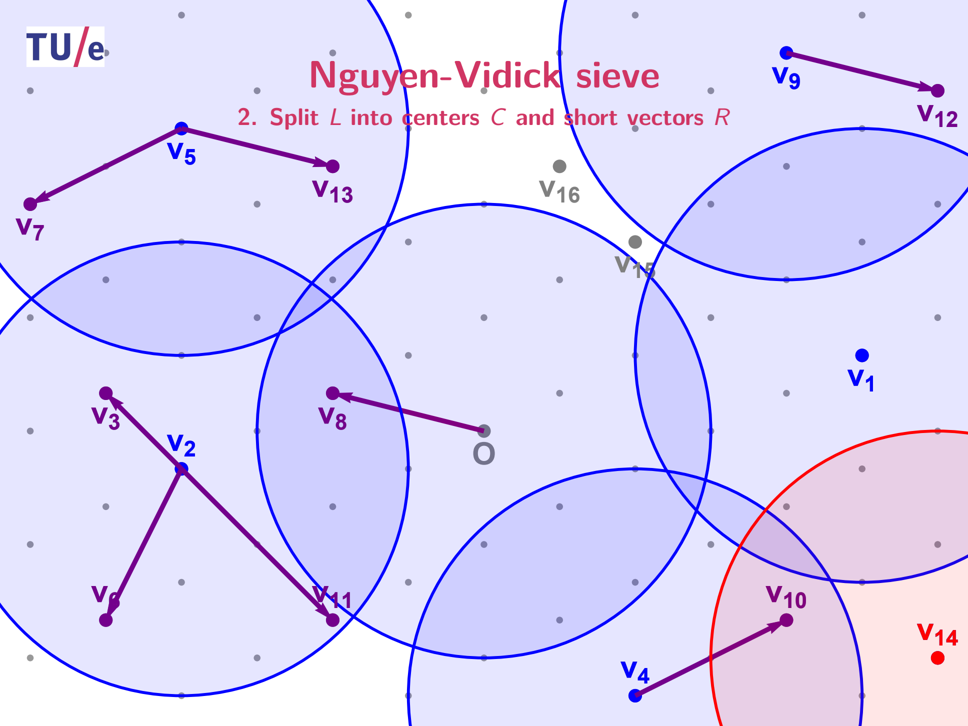

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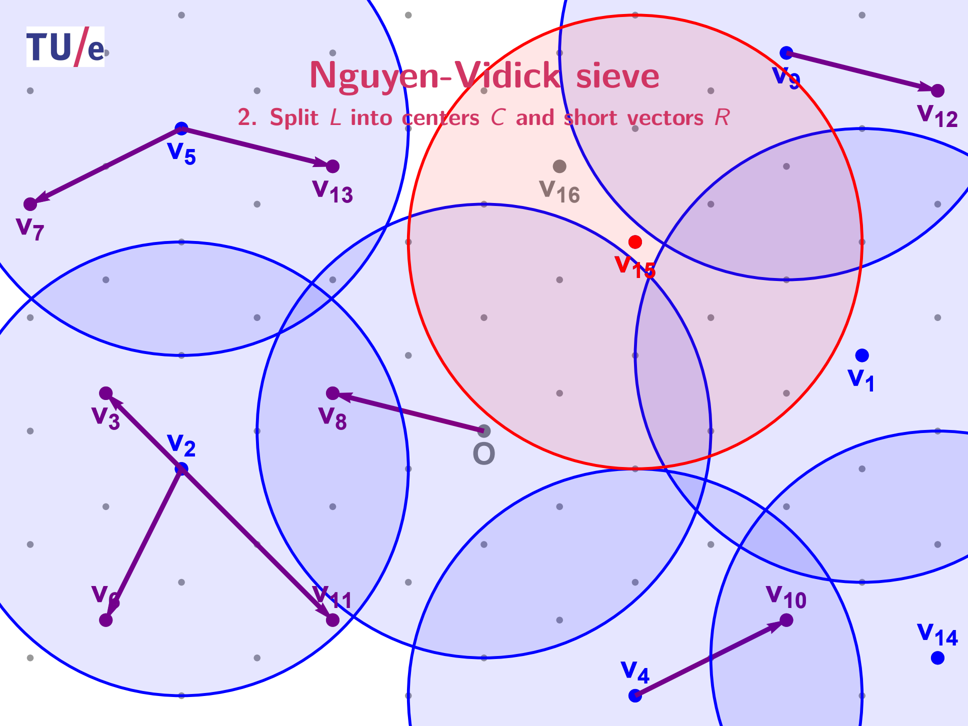

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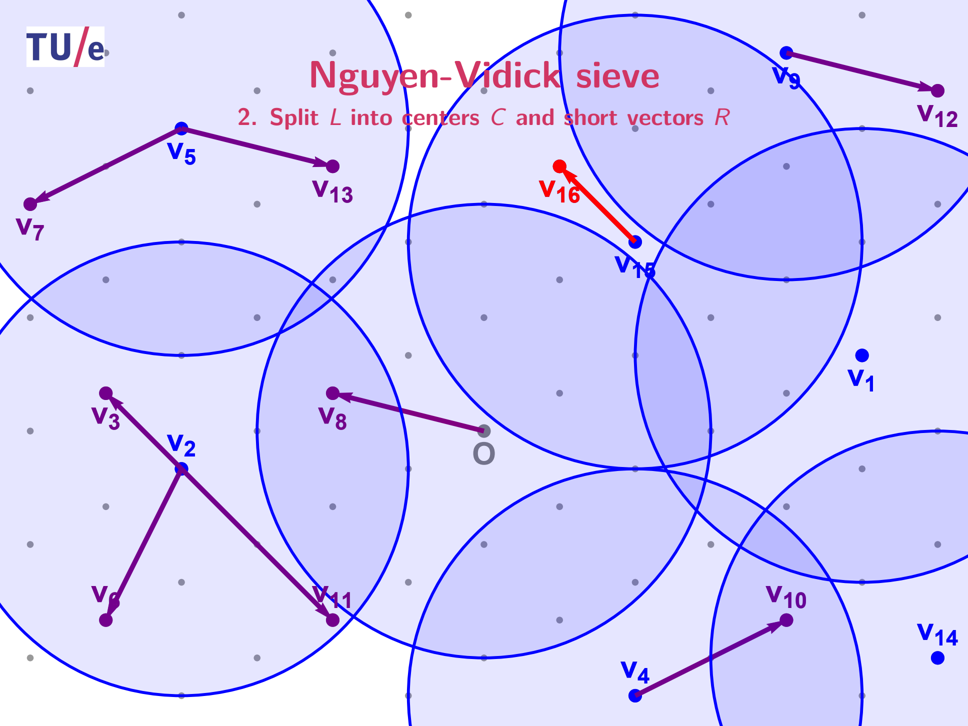

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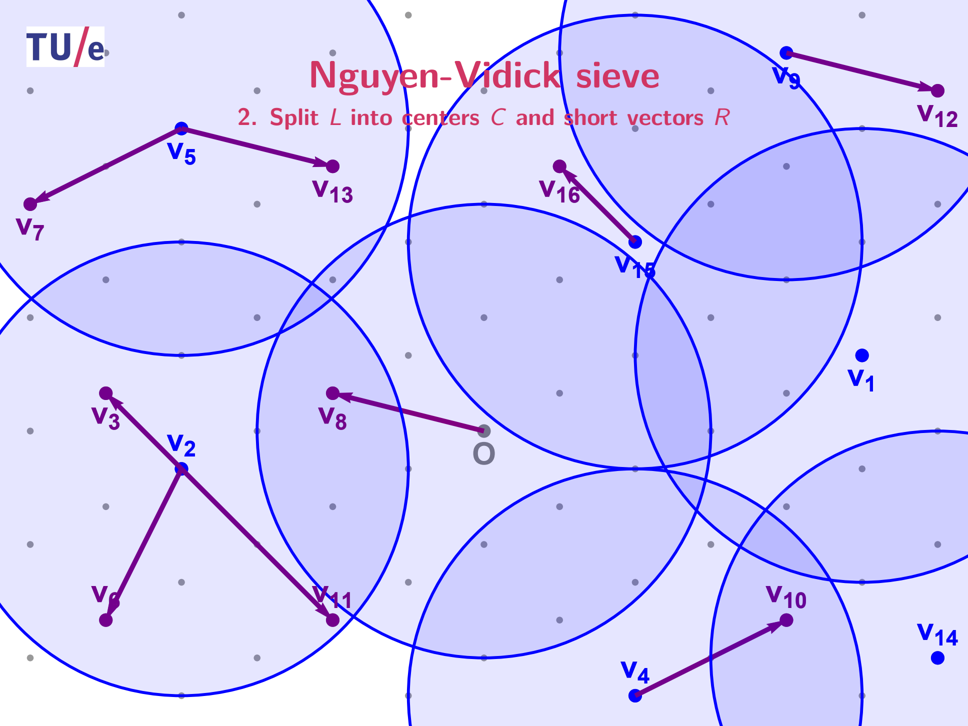

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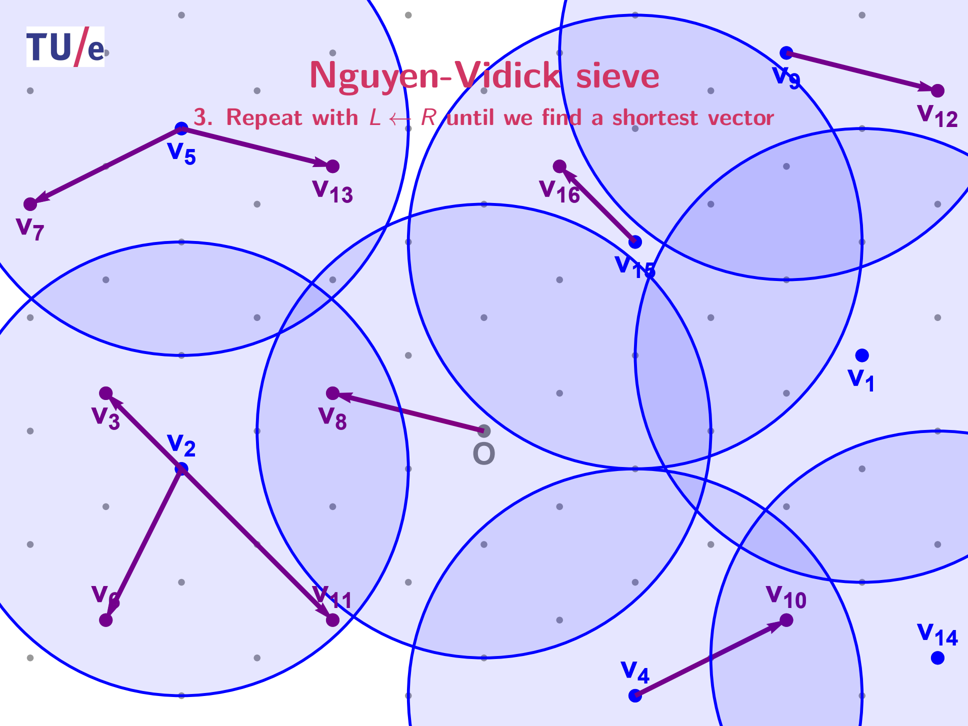

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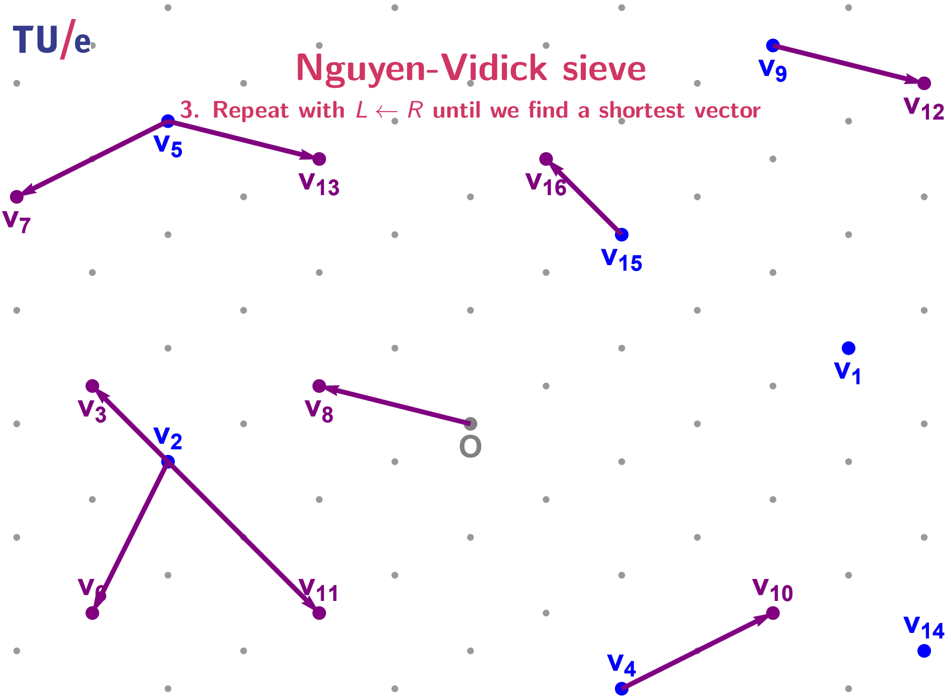

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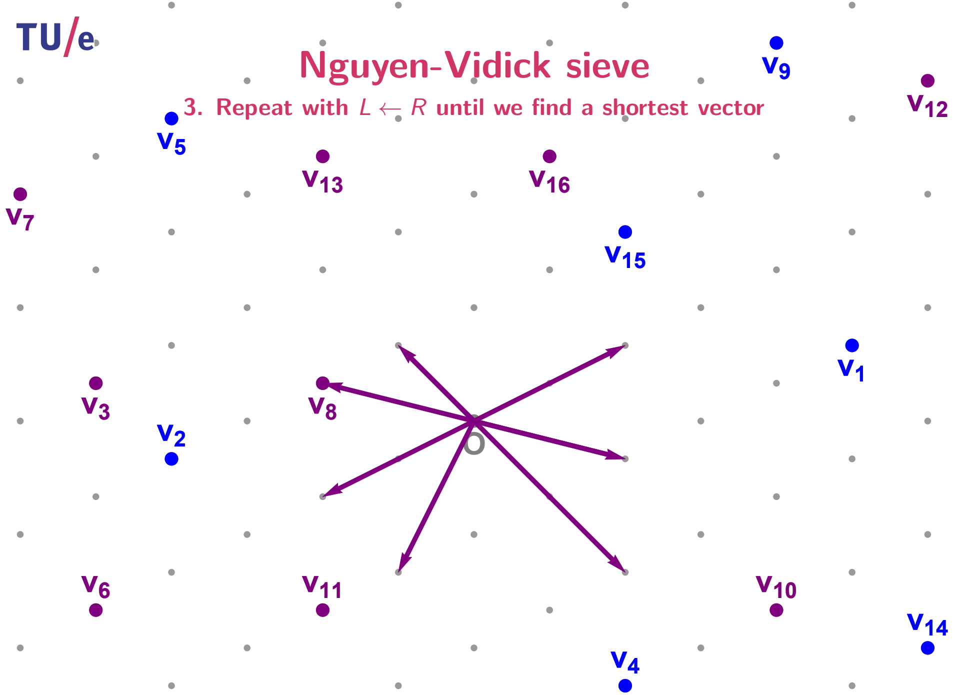

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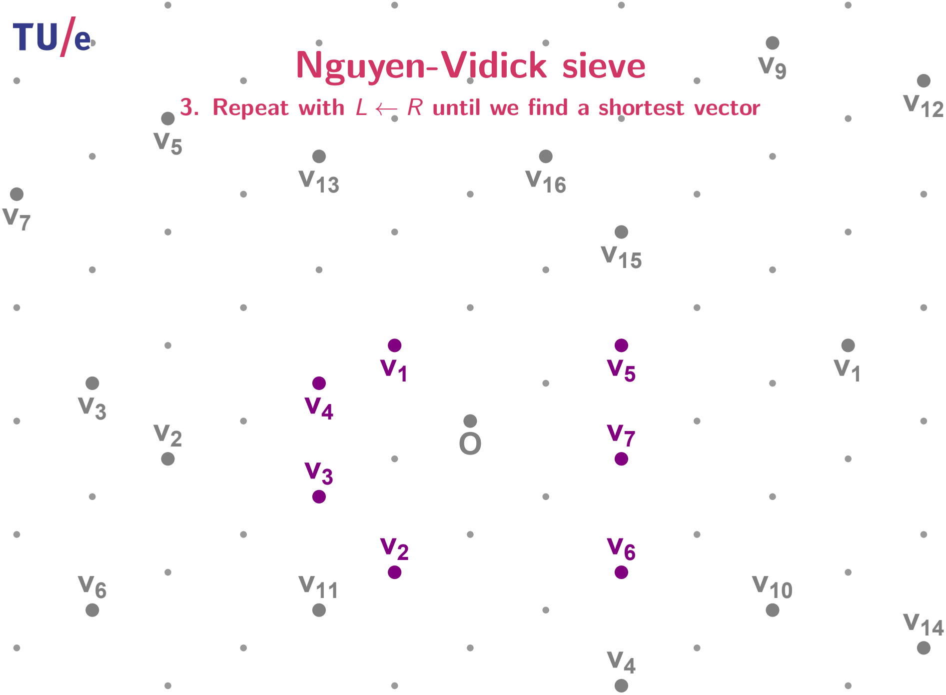

Nguyen-Vidick sieve

Overview

Nguyen-Vidick sieve

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- Space complexity: $\sqrt{4/3}^{n} \approx 2^{0.21n+o(n)}$ vectors
  - Need $\sqrt{4/3}^{n}$ vectors to cover all corners of $\mathbb{R}^n$

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  - Need $\sqrt{4/3}^n$ vectors to cover all corners of $\mathbb{R}^n$
- Time complexity: $(4/3)^n \approx 2^{0.42n+o(n)}$
  - Comparing a target vector to all centers: $2^{0.21n+o(n)}$
  - Repeating this for each list vector: $2^{0.21n+o(n)}$
  - Repeating the whole sieving procedure: $\mathrm{poly}(n)$

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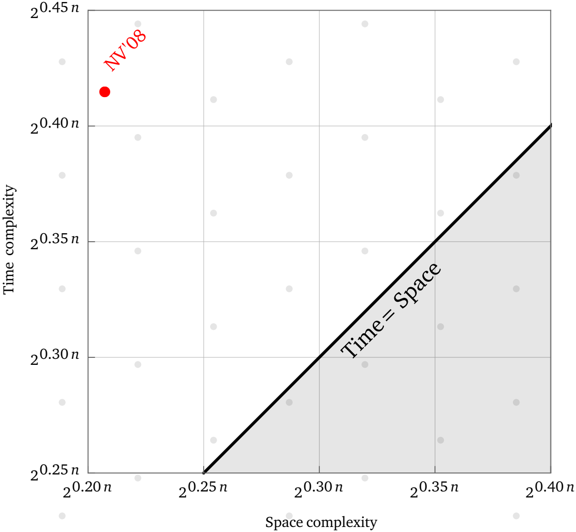
Heuristic (Nguyen and Vidick, J. Math. Crypt. '08)

The NV-sieve runs in time $2^{0.42n+o(n)}$ and space $2^{0.21n+o(n)}$.

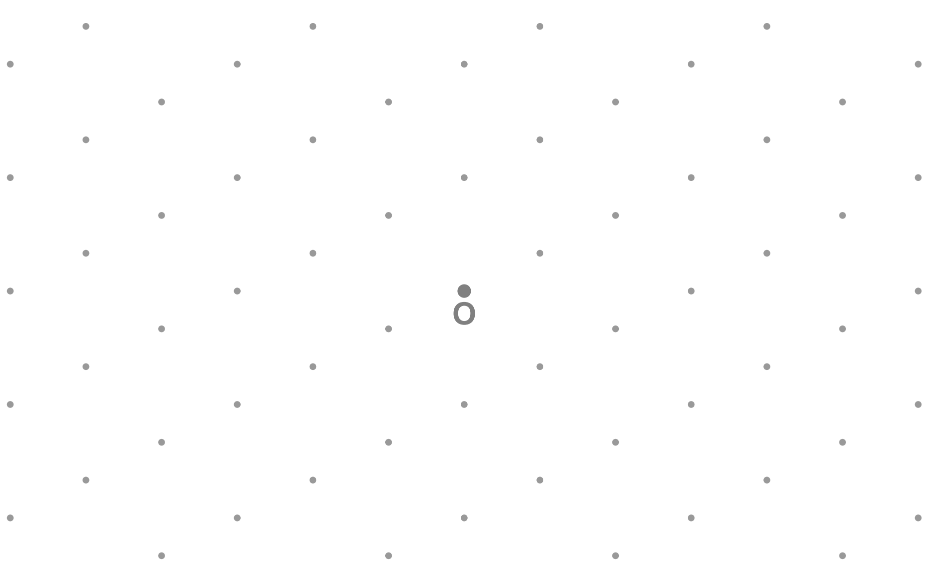
Nguyen-Vidick sieve

Space/time trade-off



GaussSieve

1. Generate random lattice vectors



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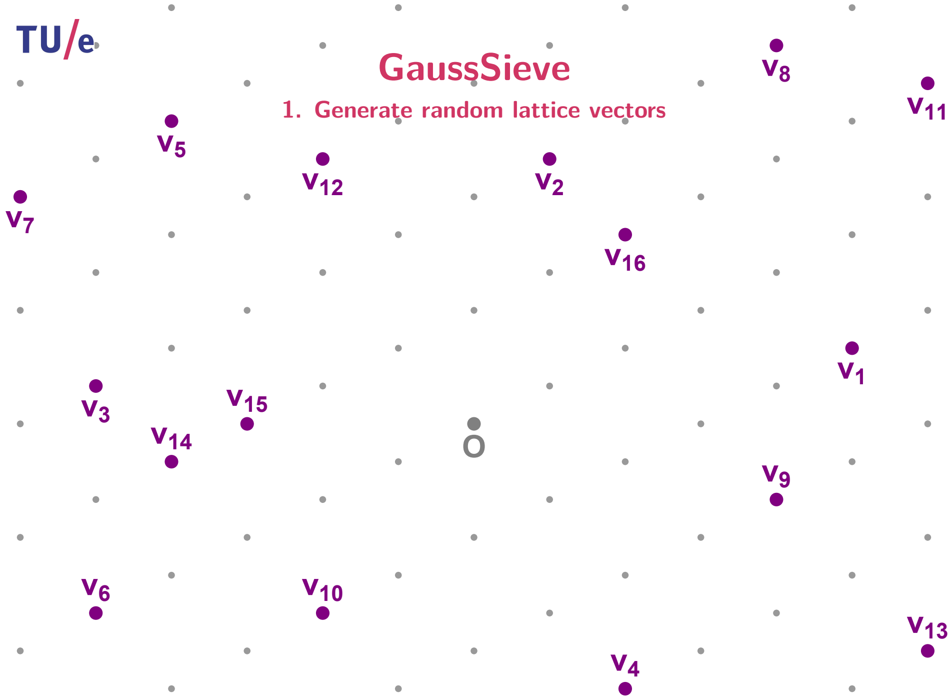

GaussSieve

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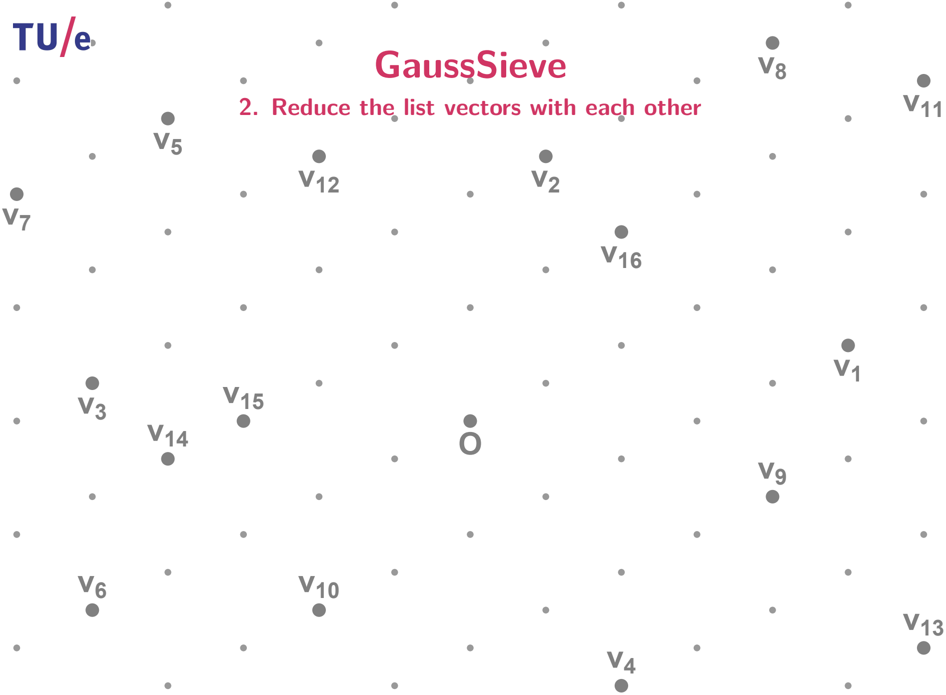

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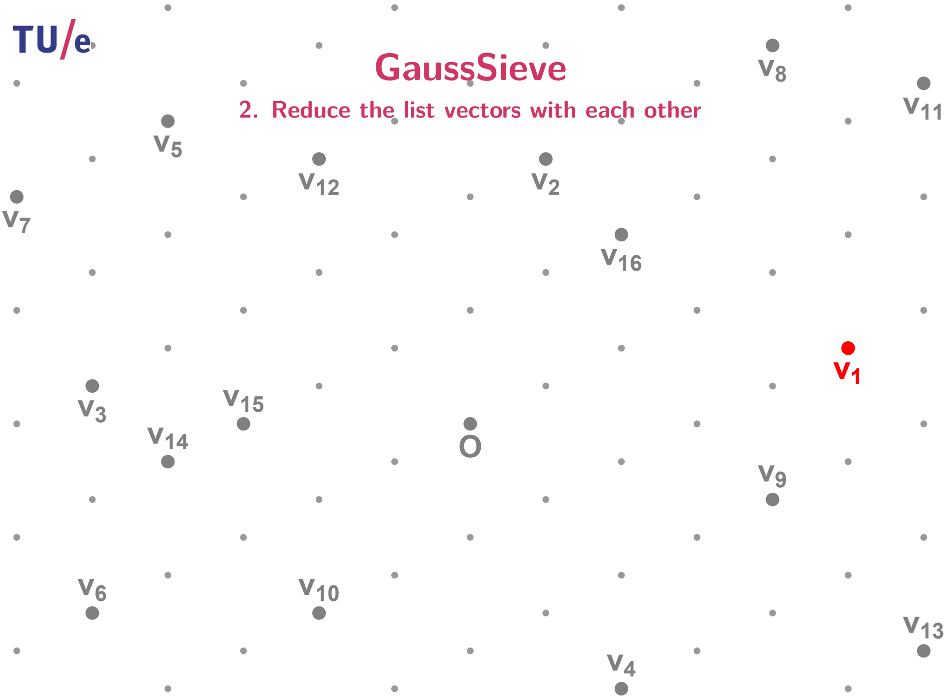

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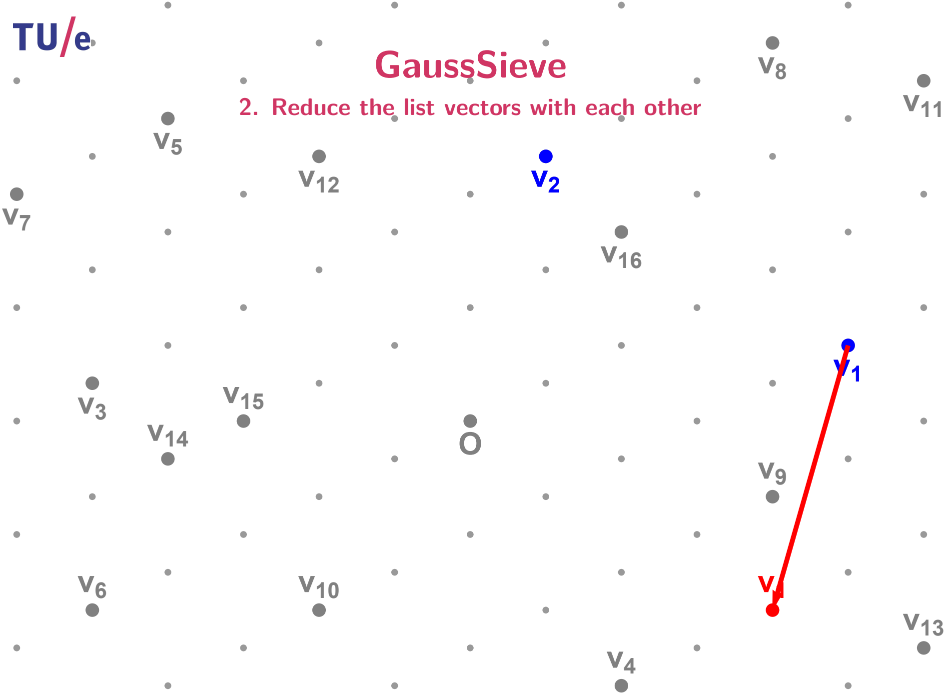

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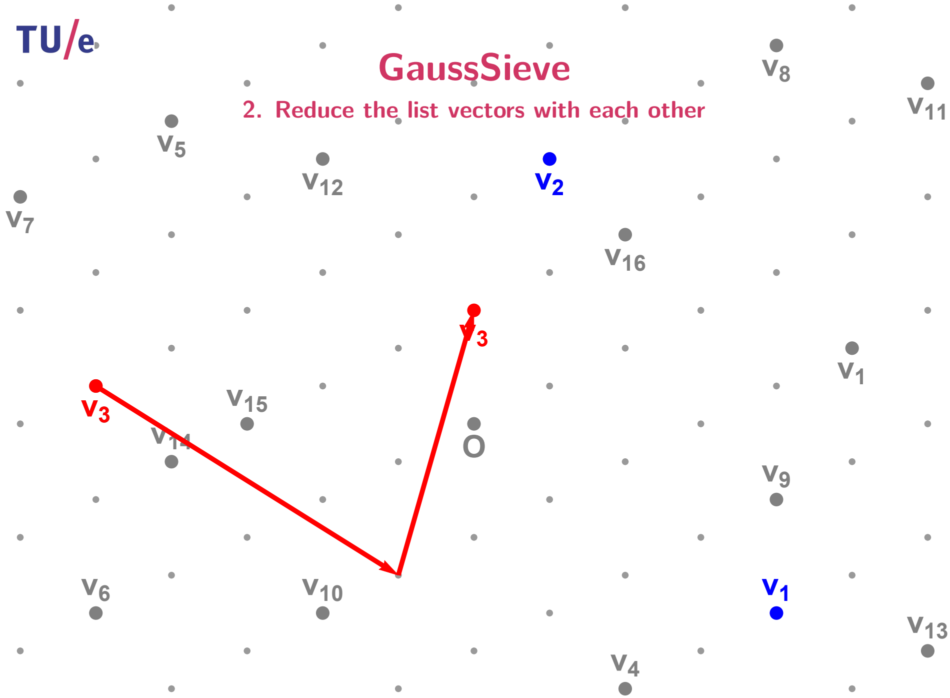

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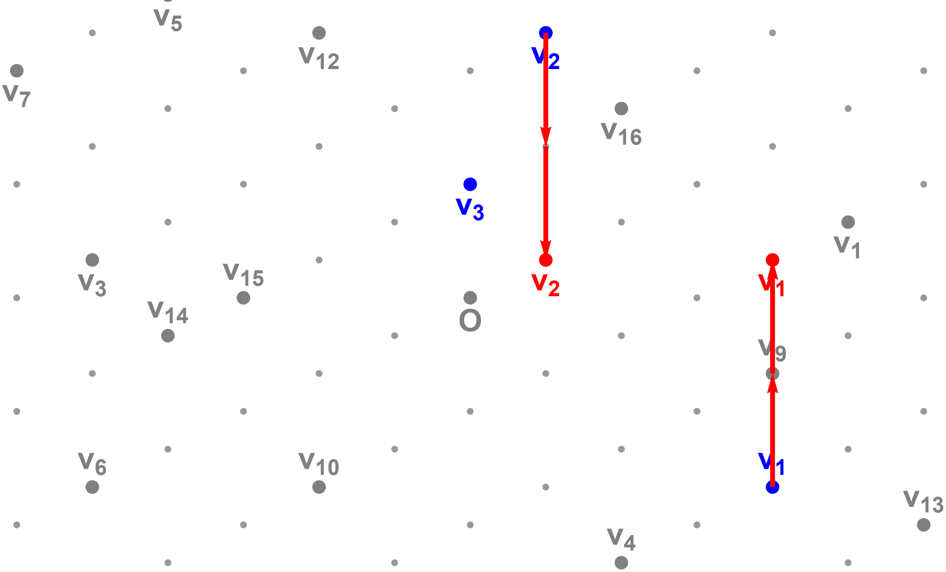

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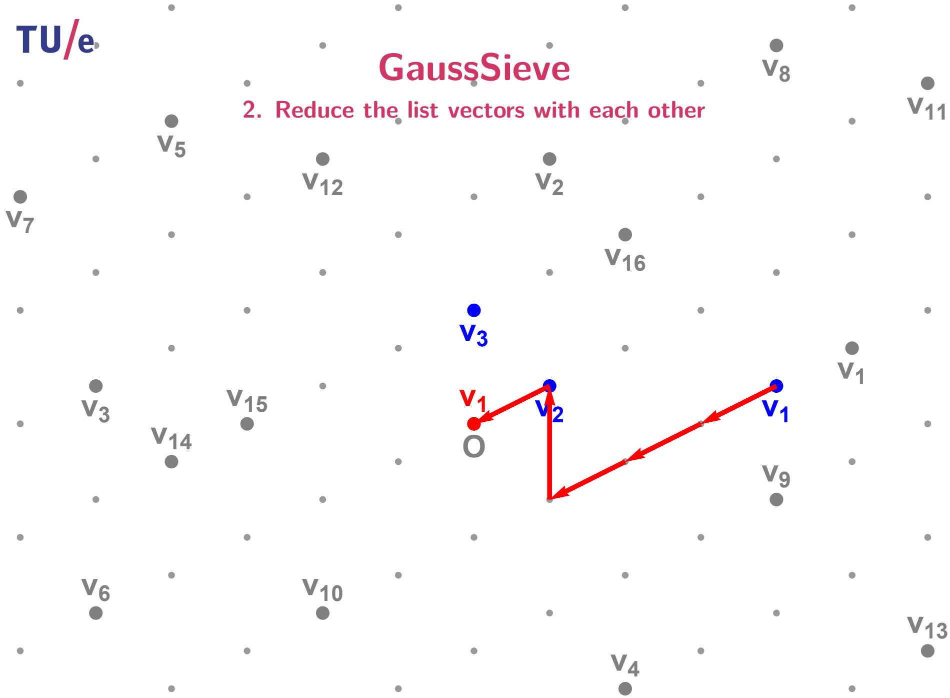

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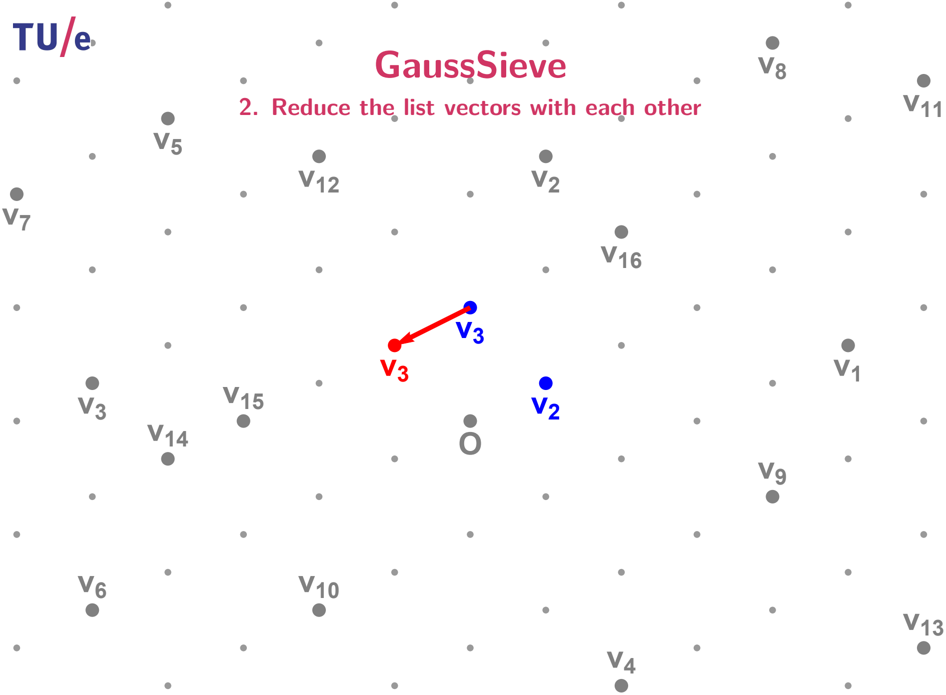

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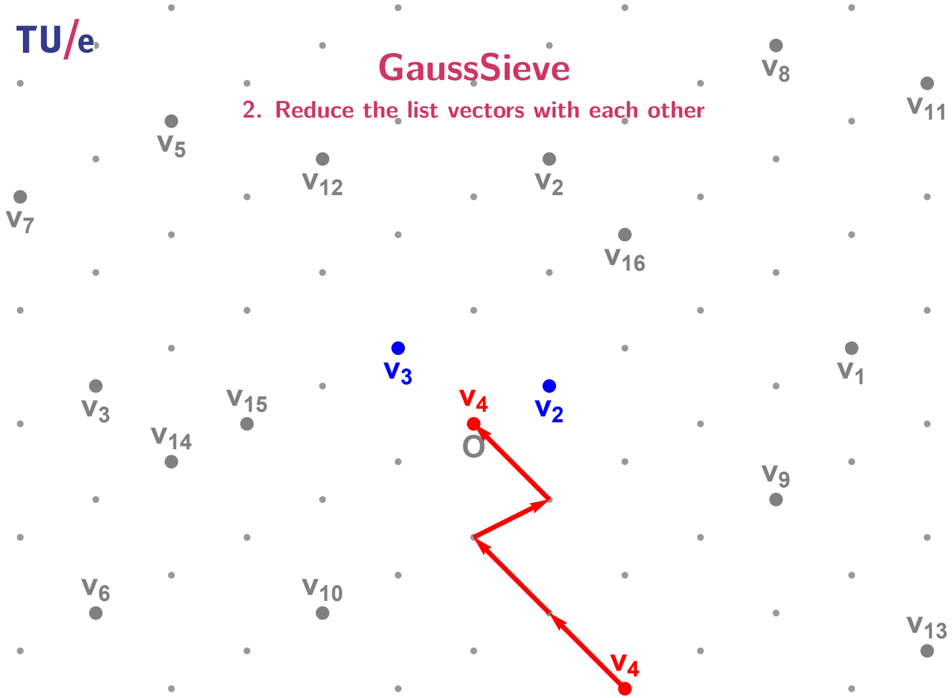

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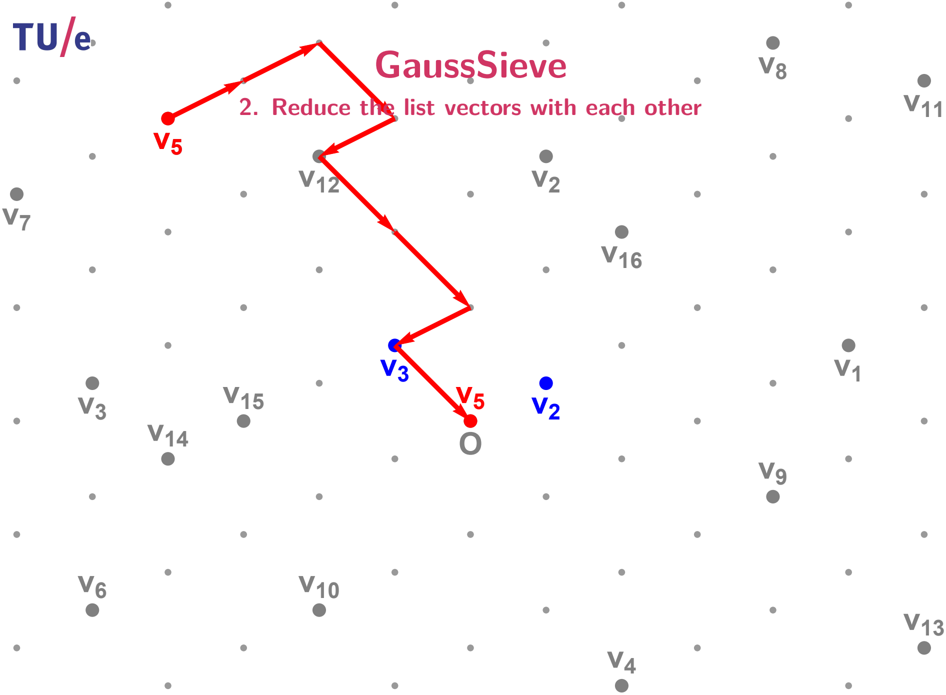

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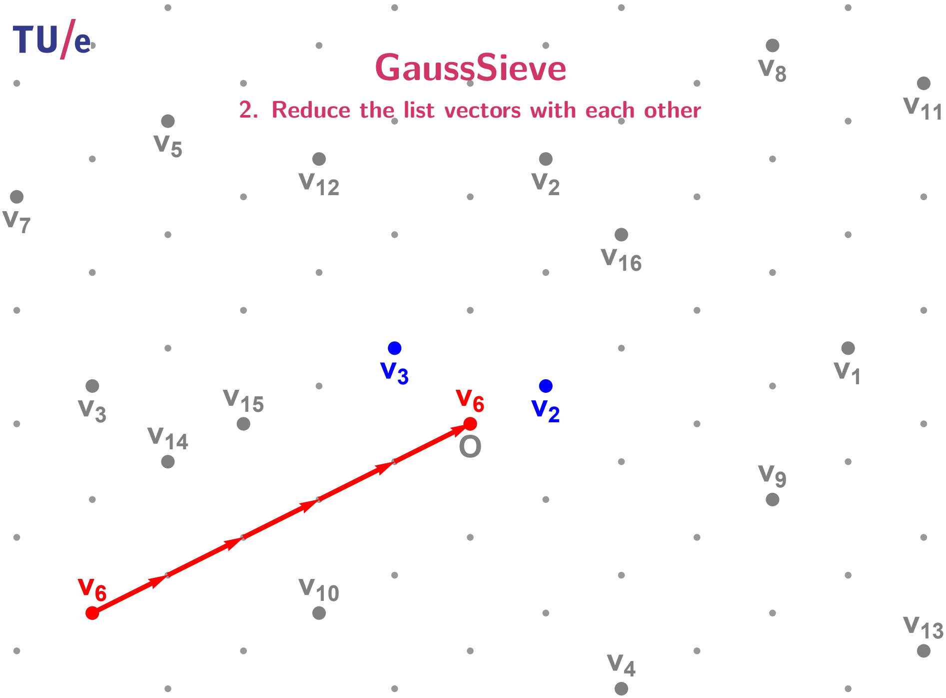

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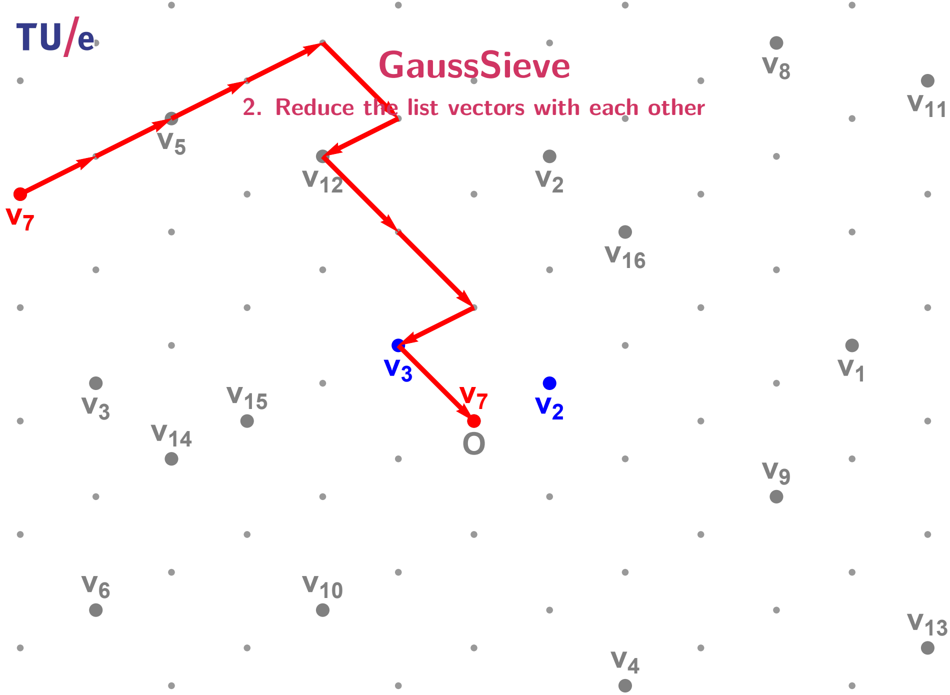

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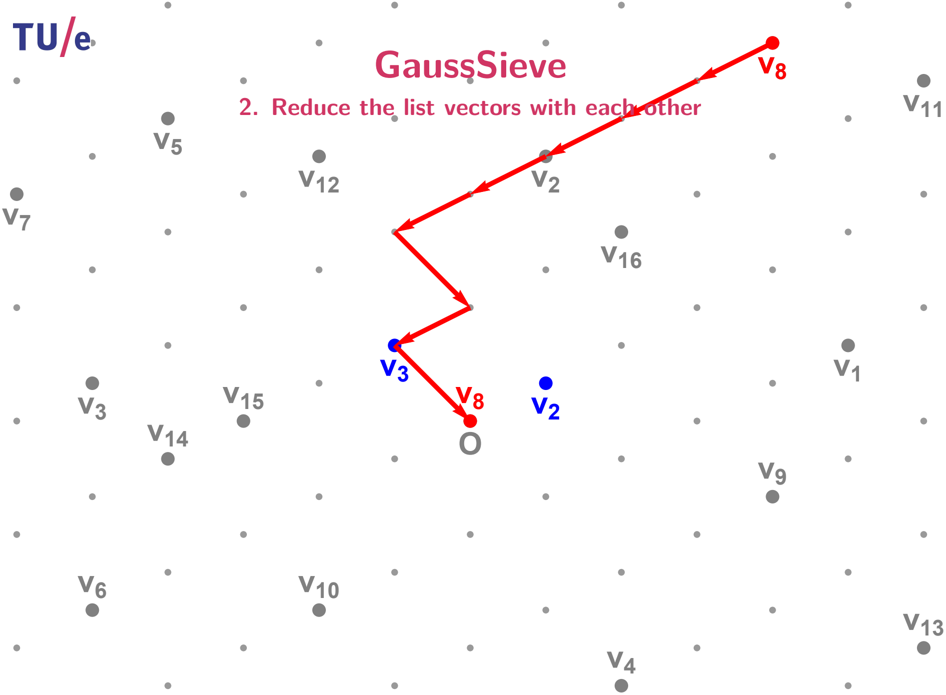

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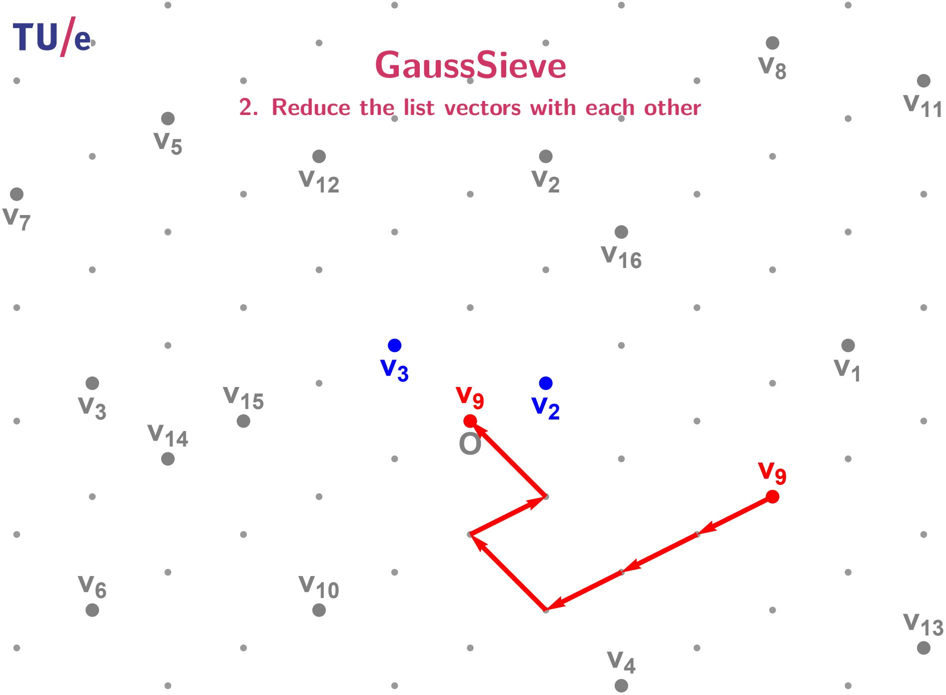

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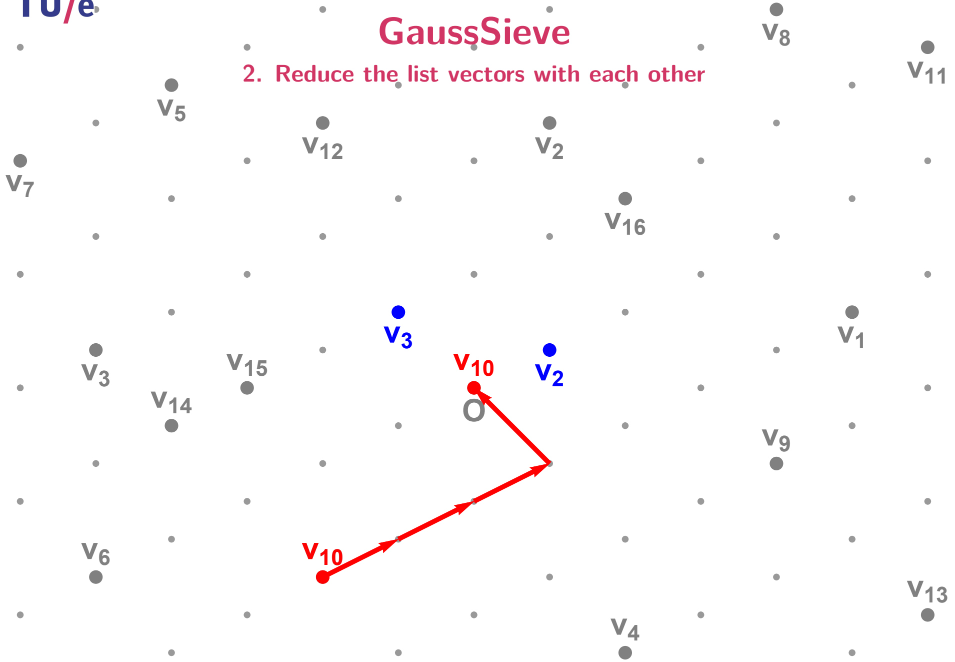

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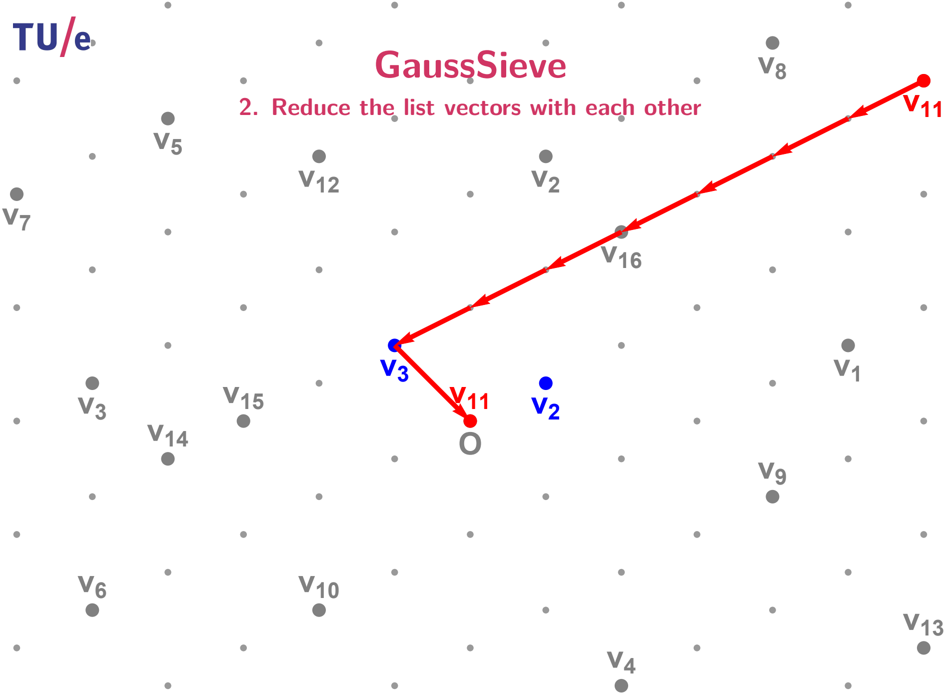

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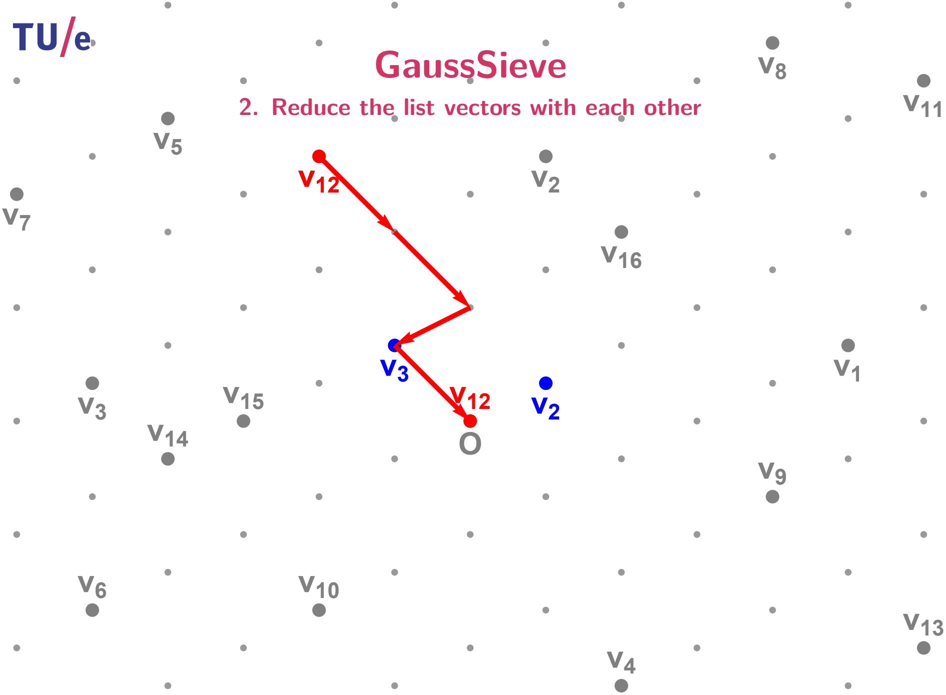

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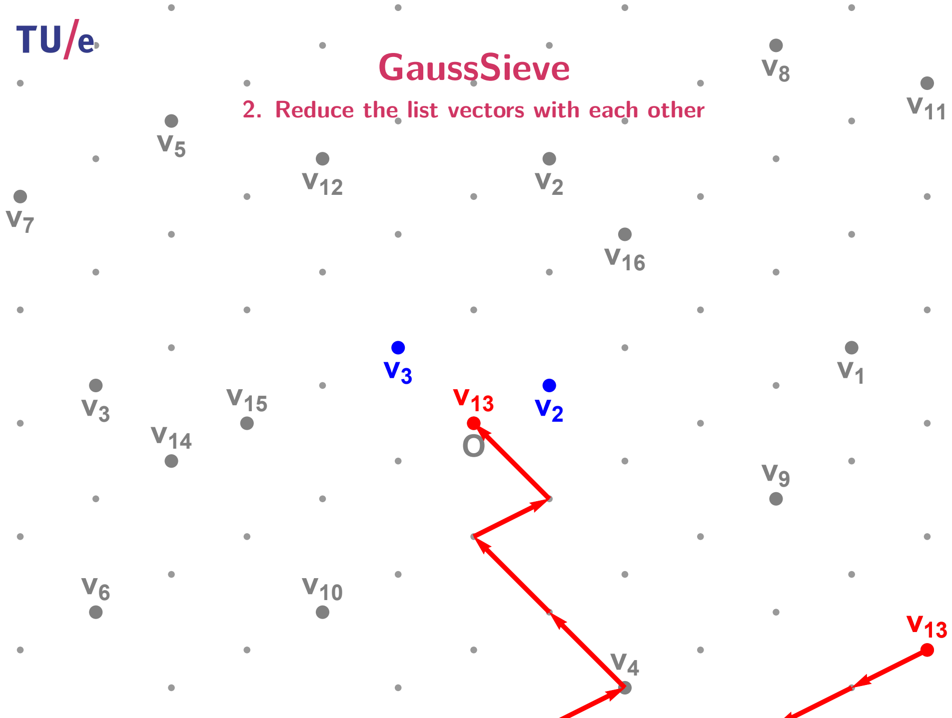

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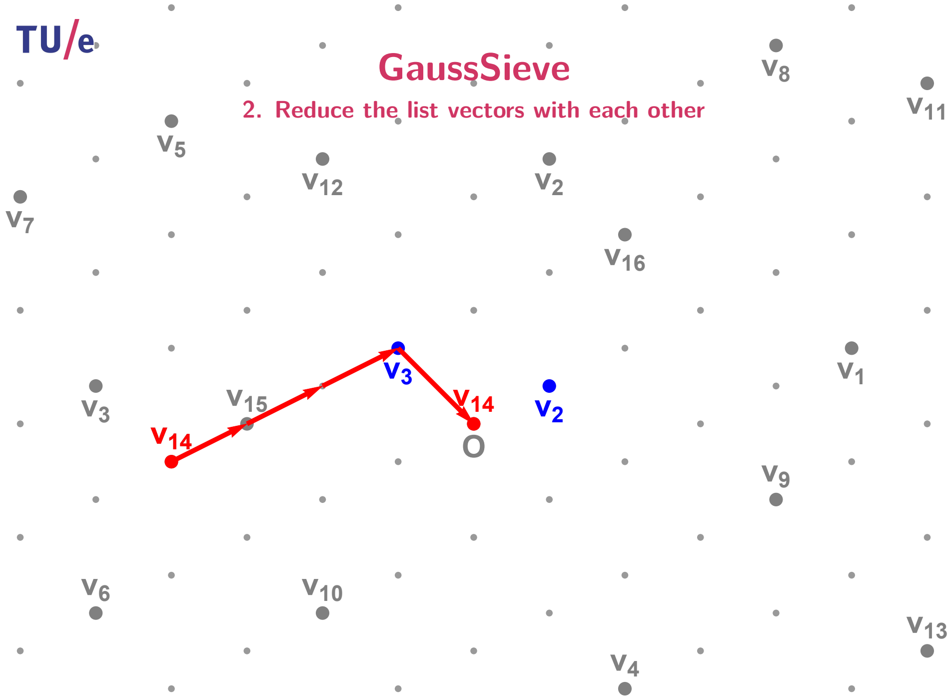

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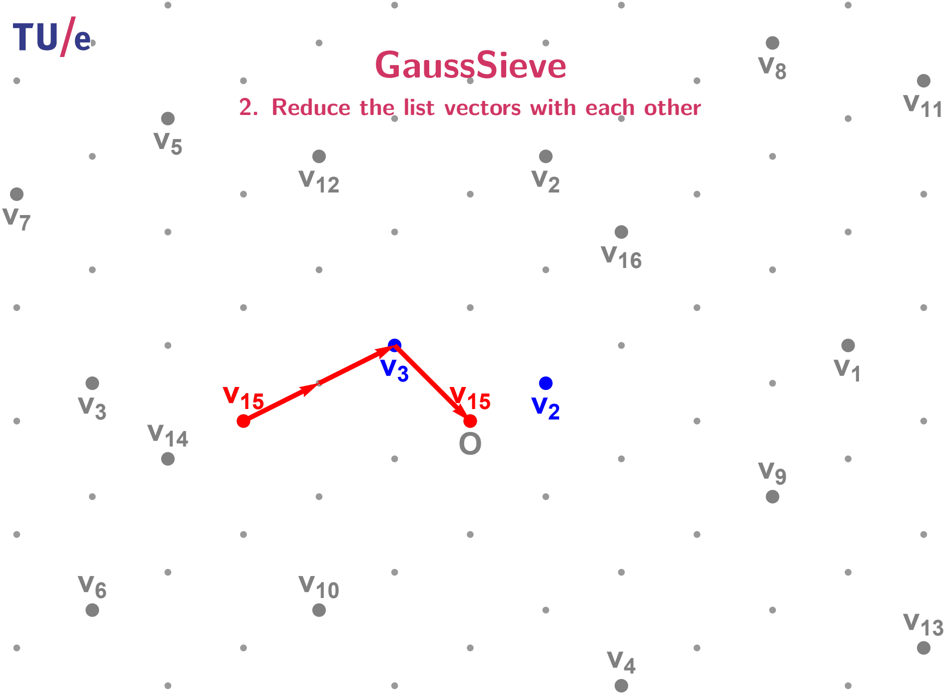

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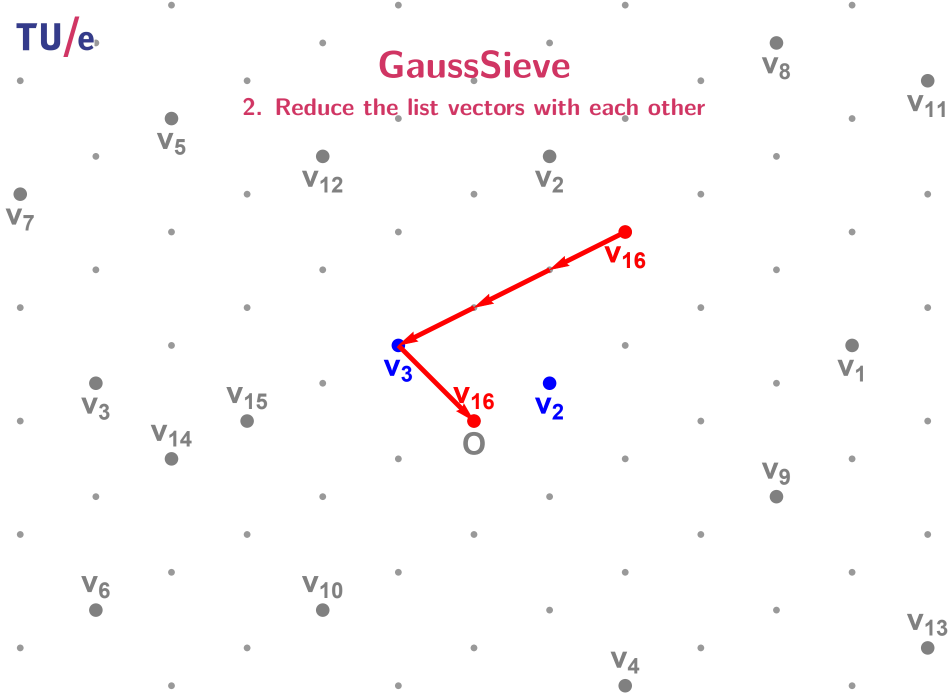

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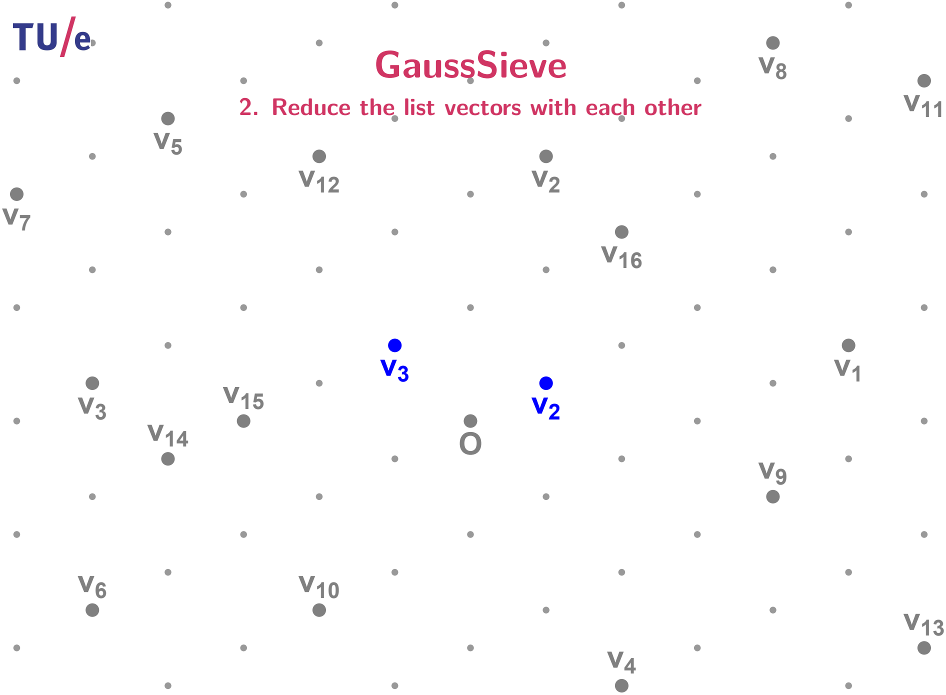

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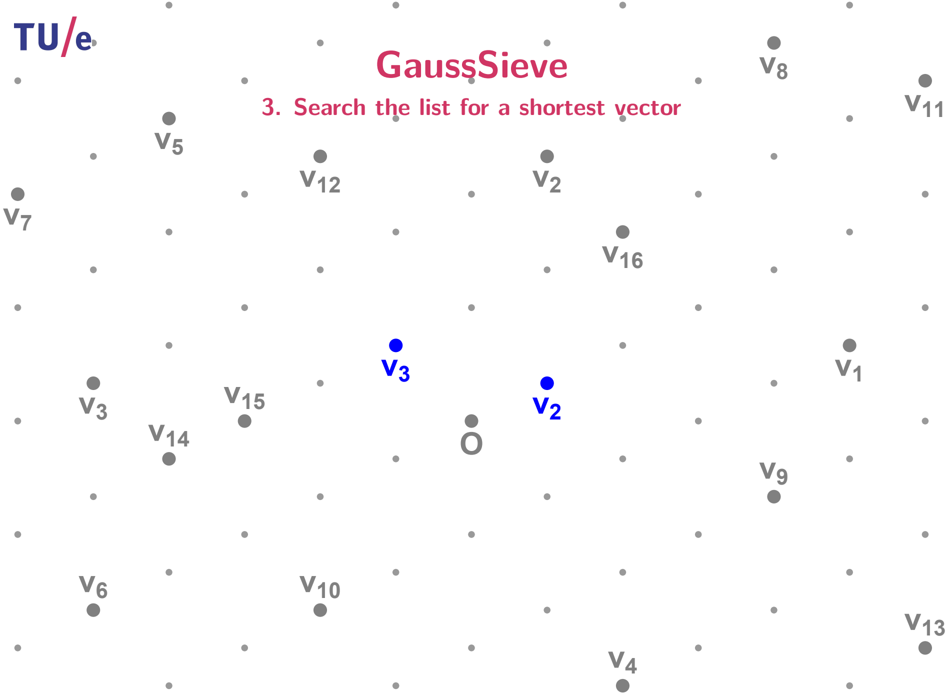
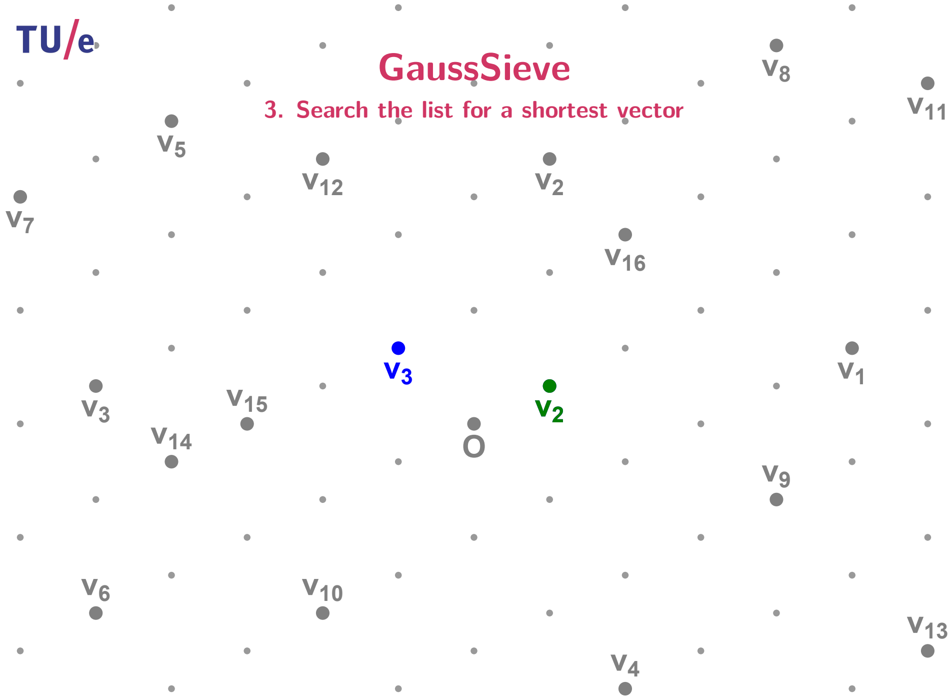


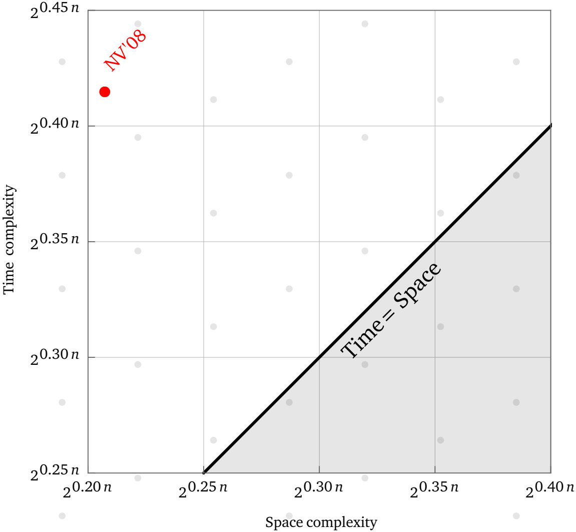
GaussSieve

3. Search the list for a shortest vector



GaussSieve

Space/time trade-off



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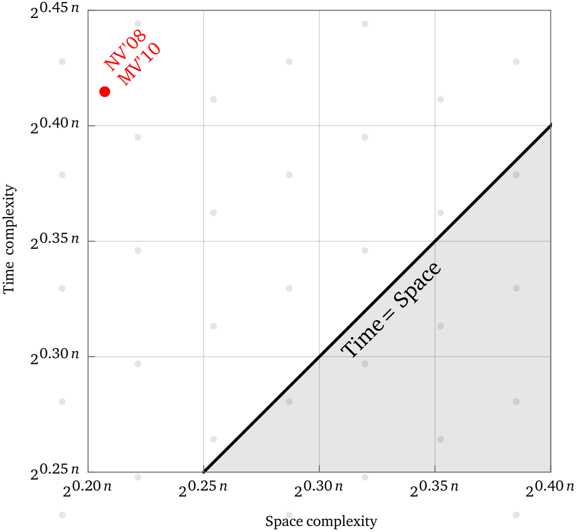

Two-level sieve

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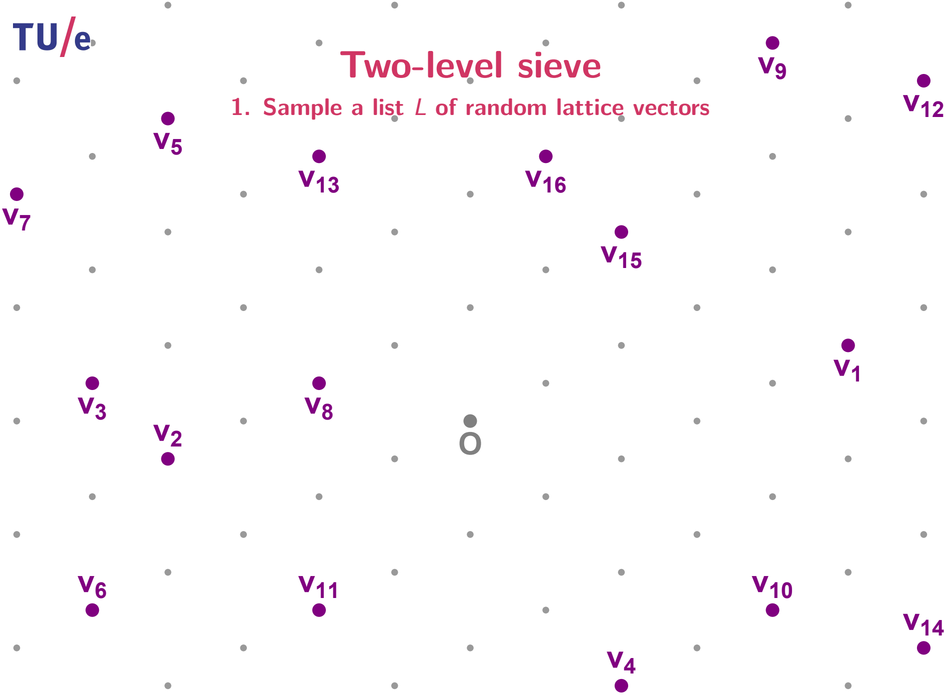

Two-level sieve

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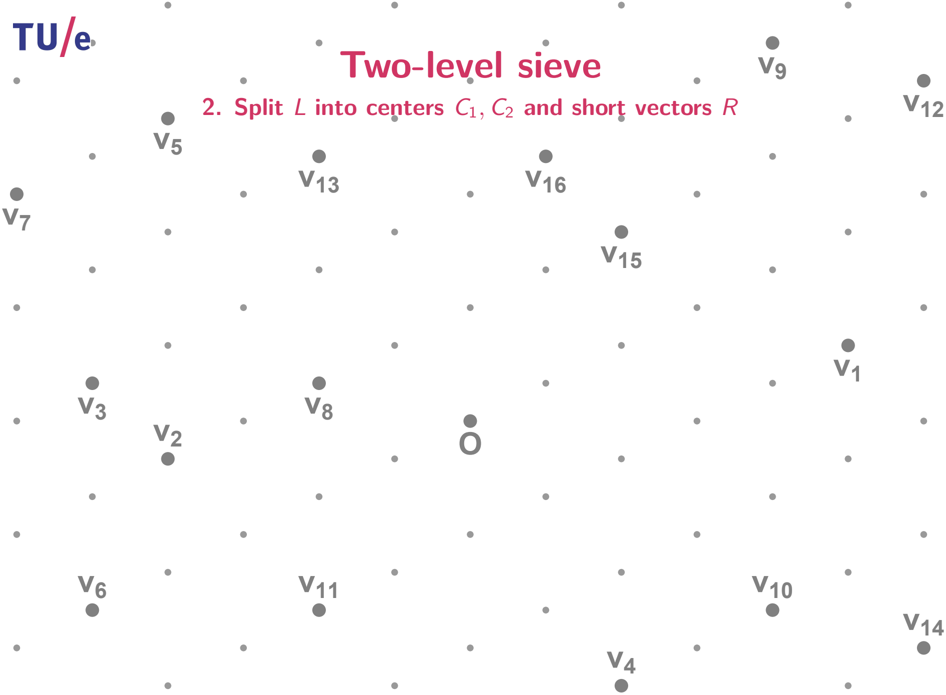

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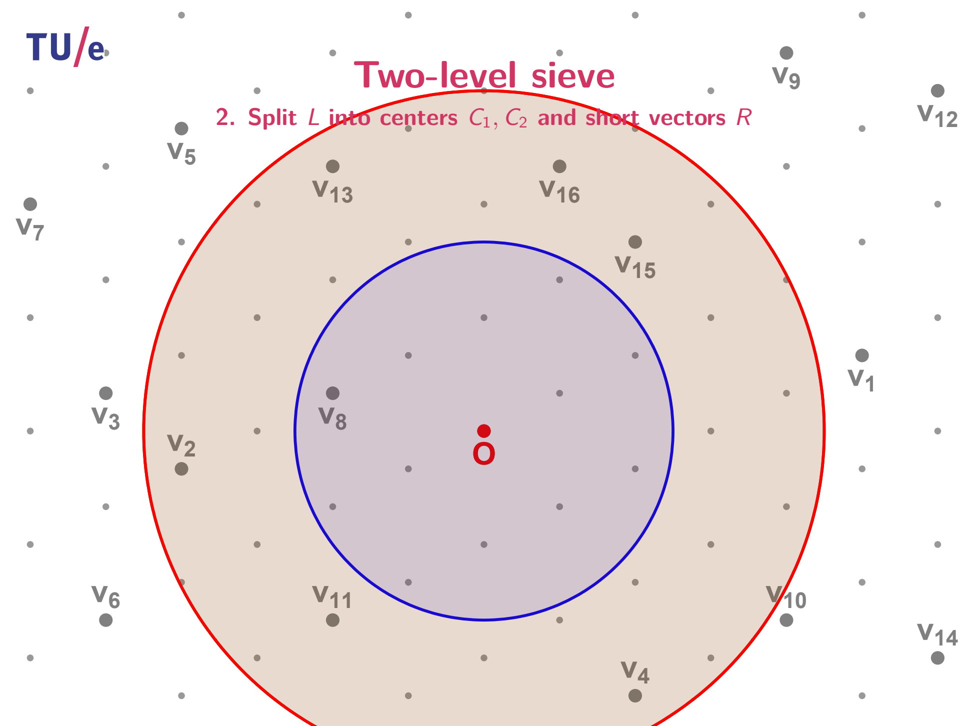

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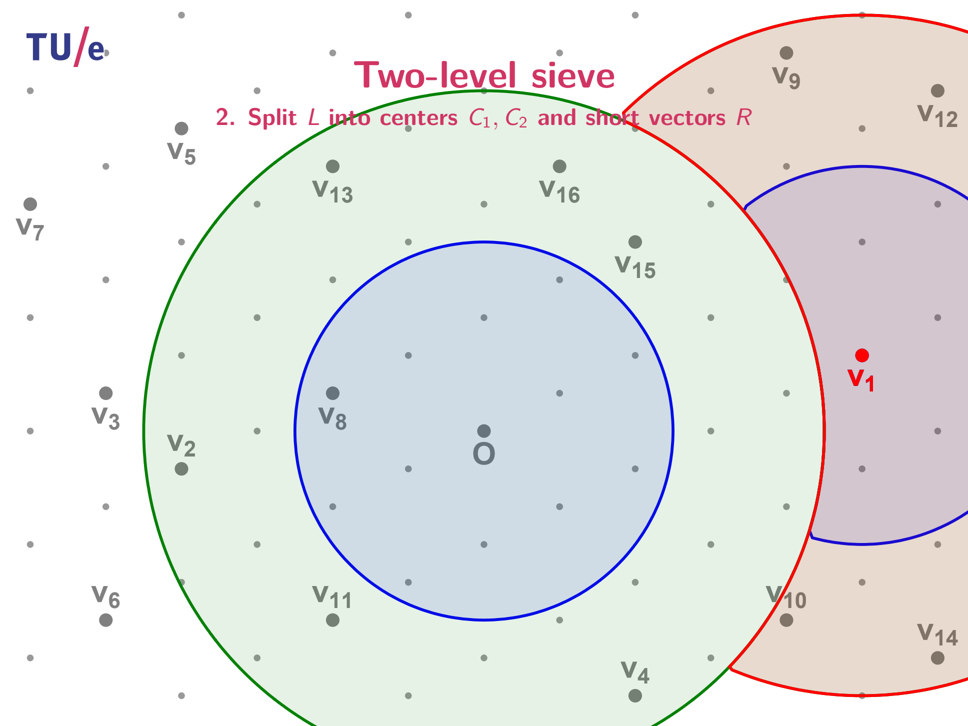

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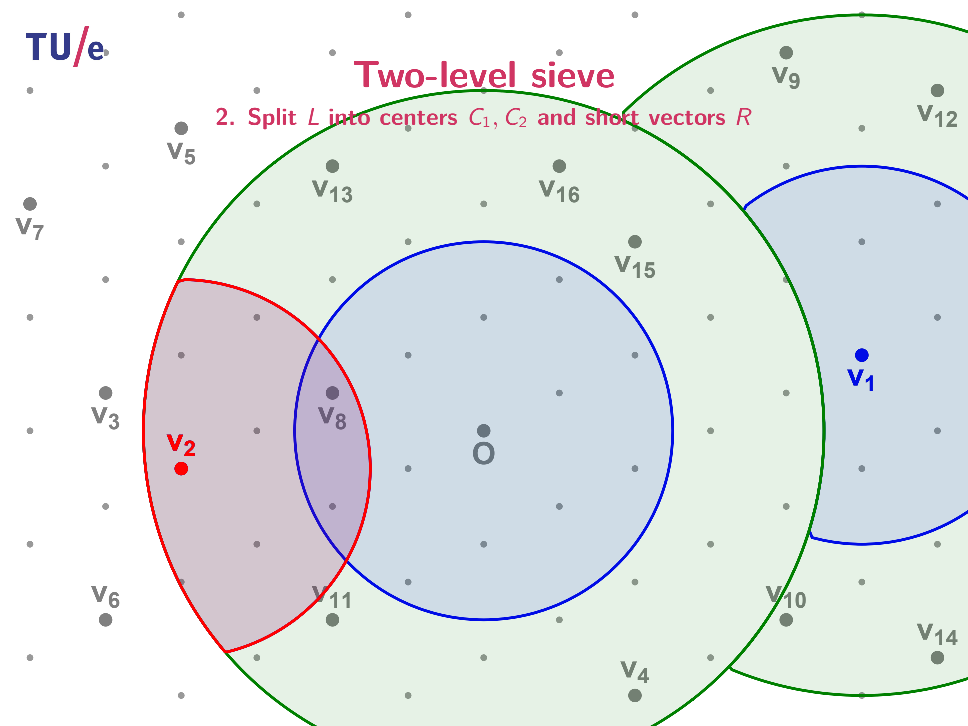

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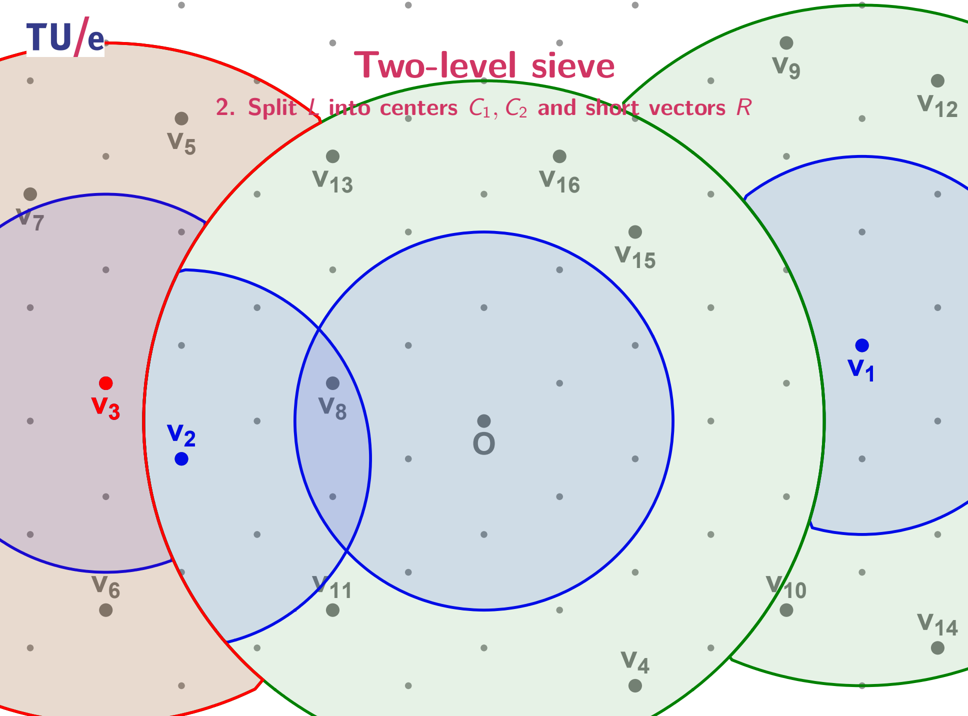

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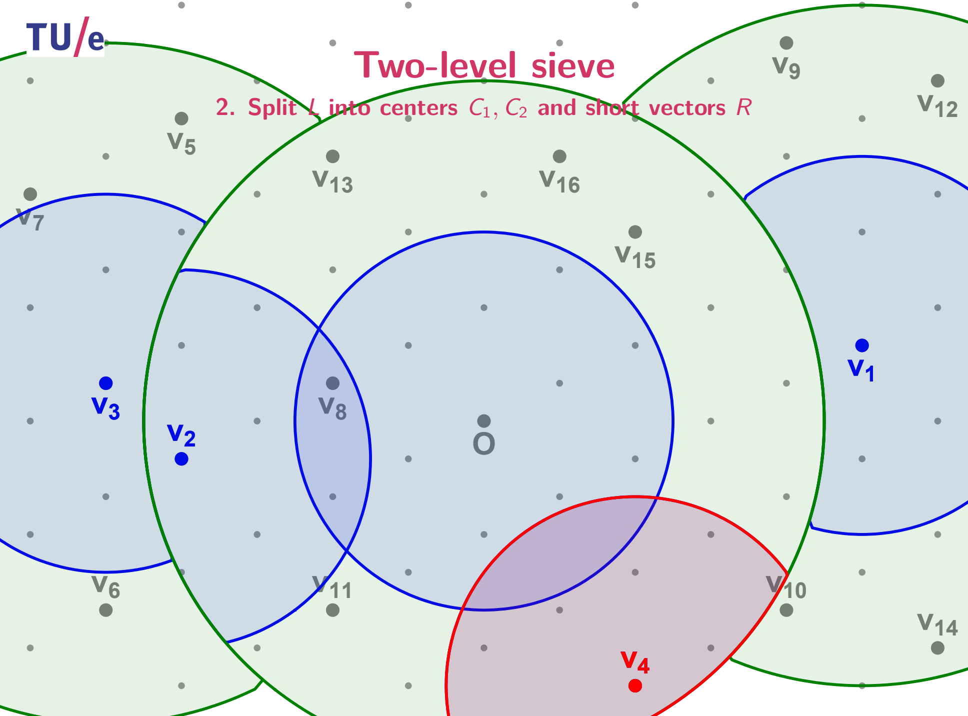

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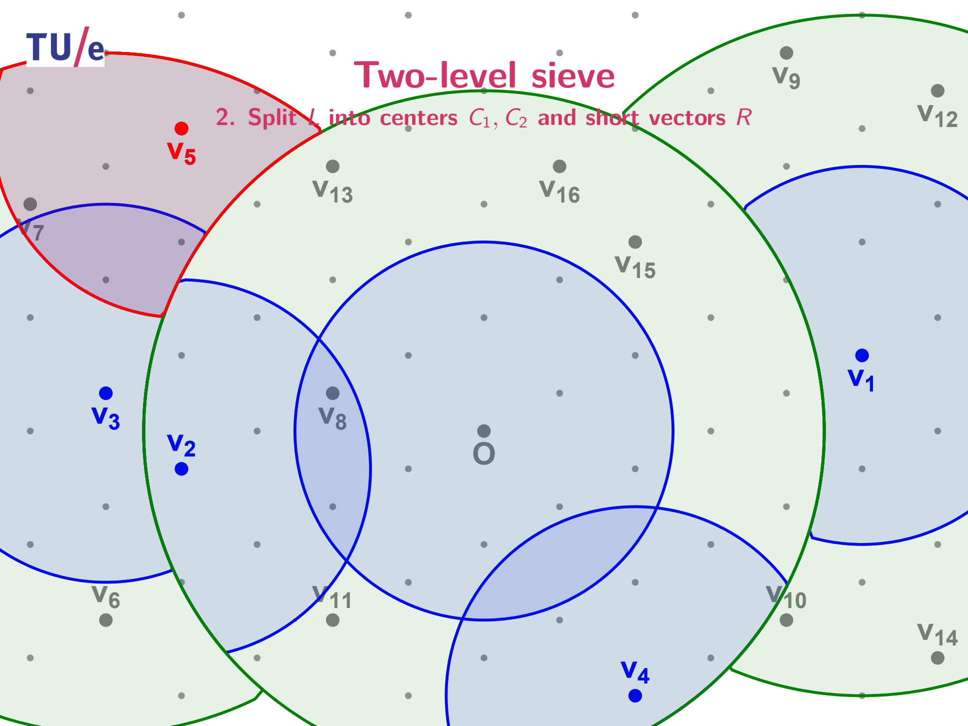

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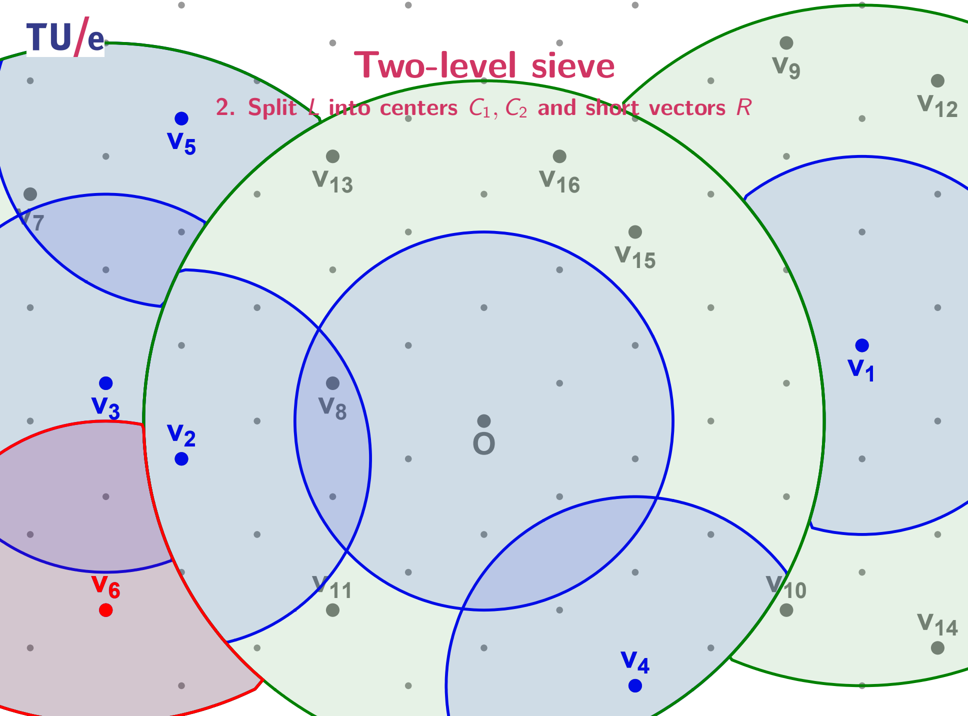

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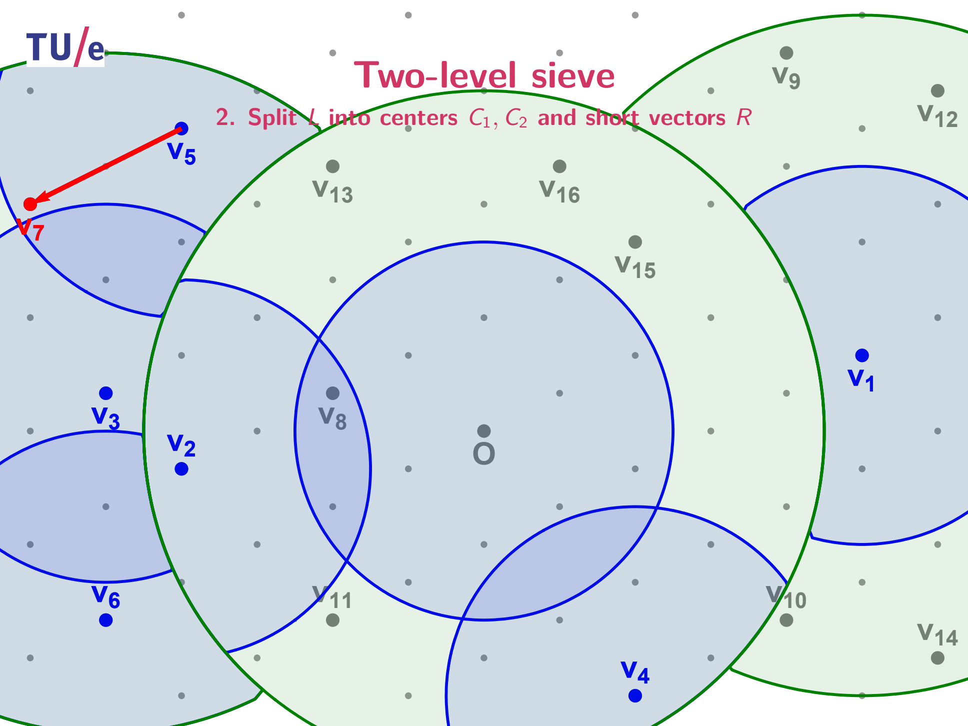

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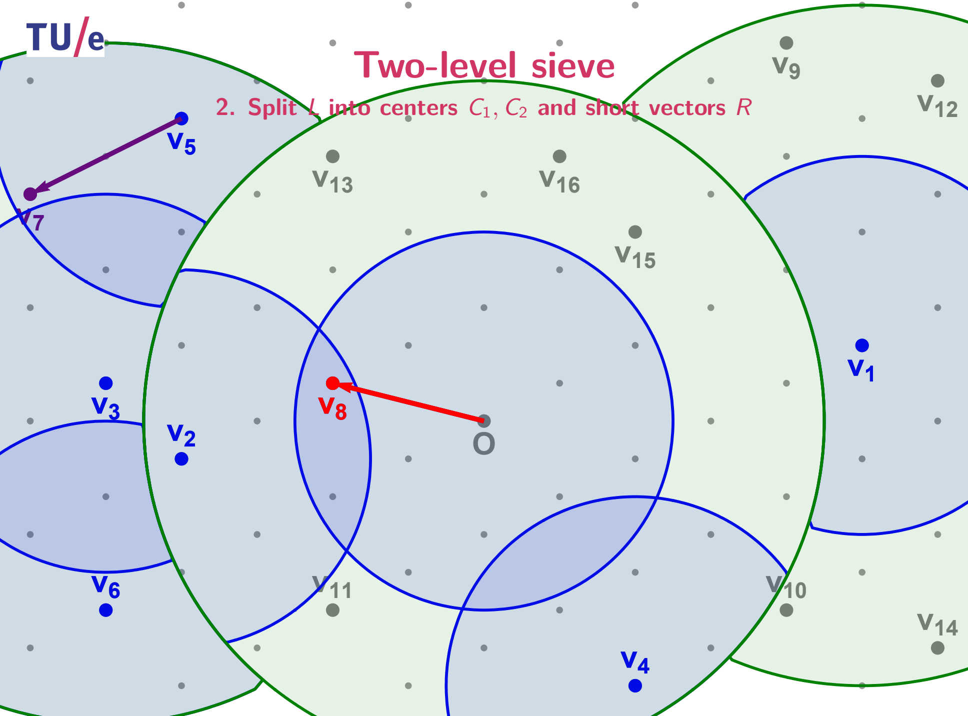

TU/e

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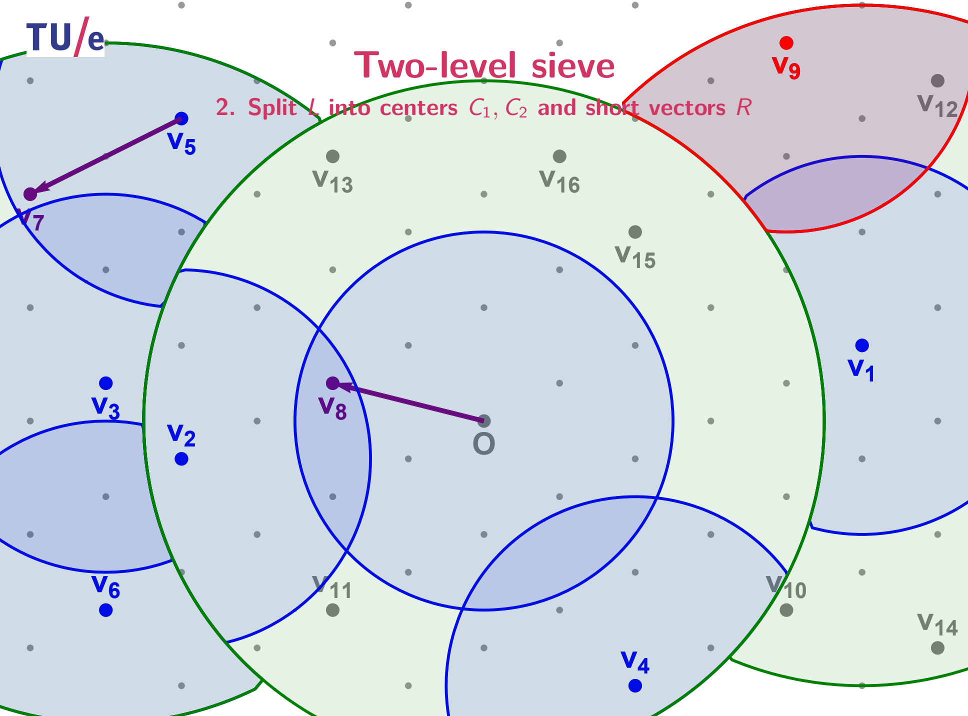

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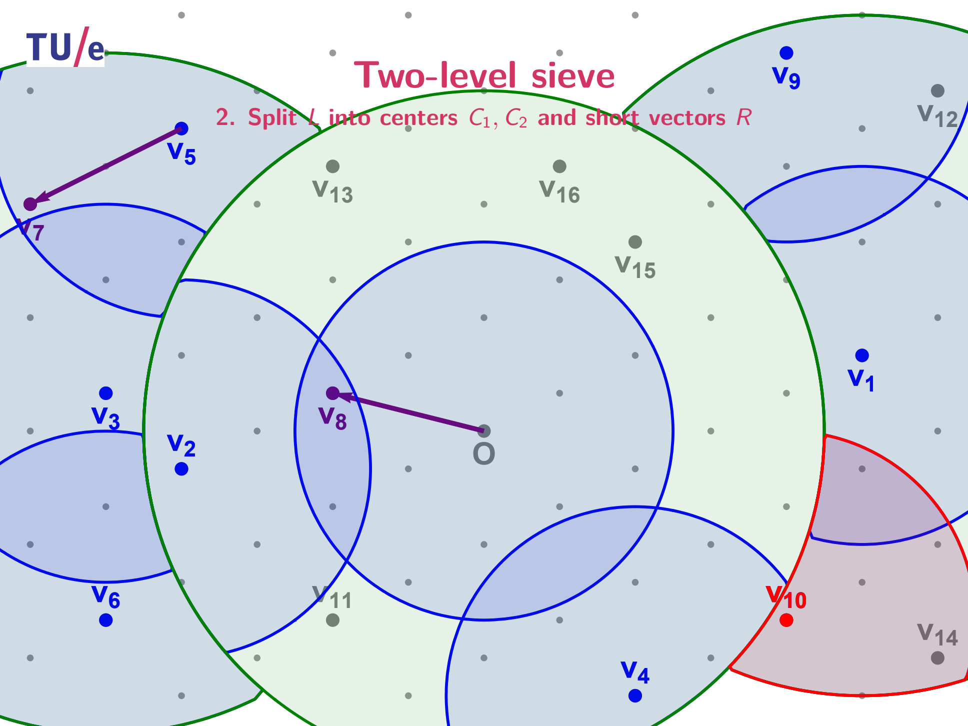

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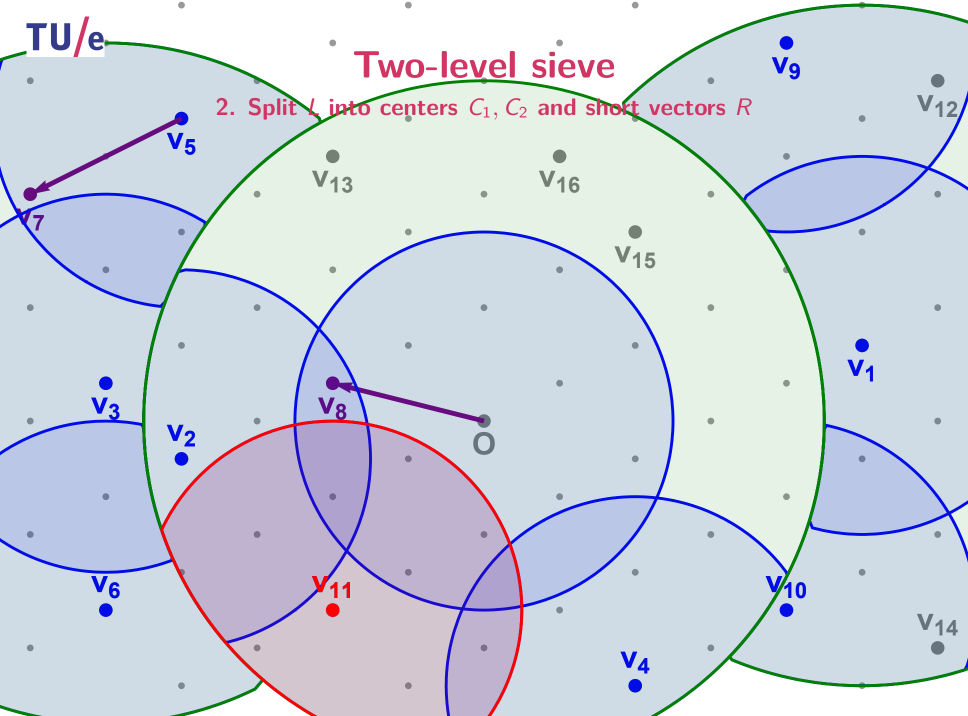

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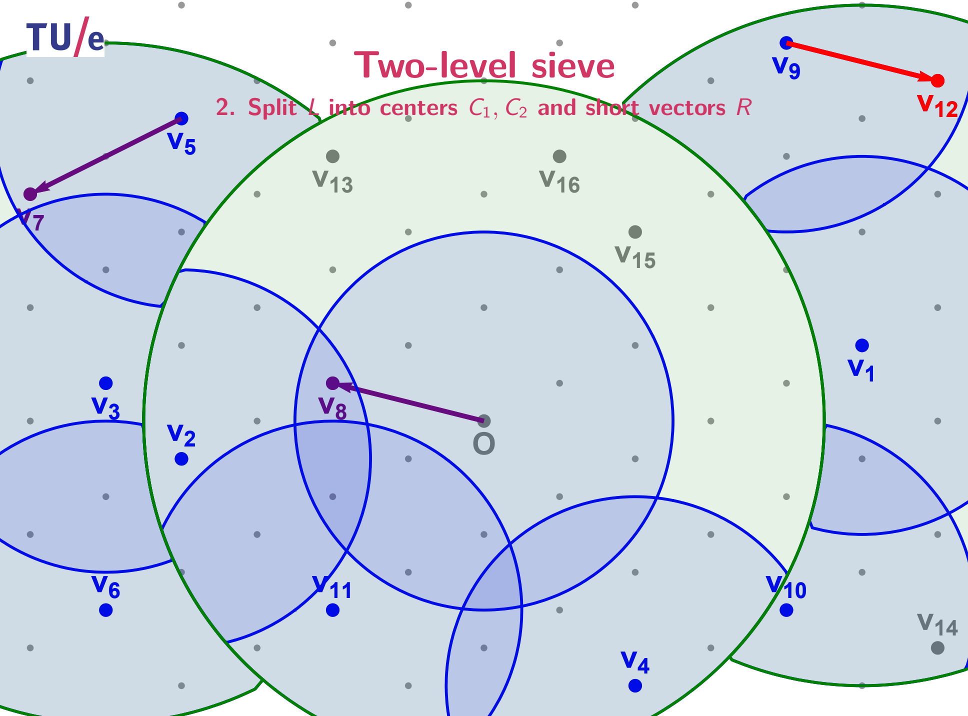

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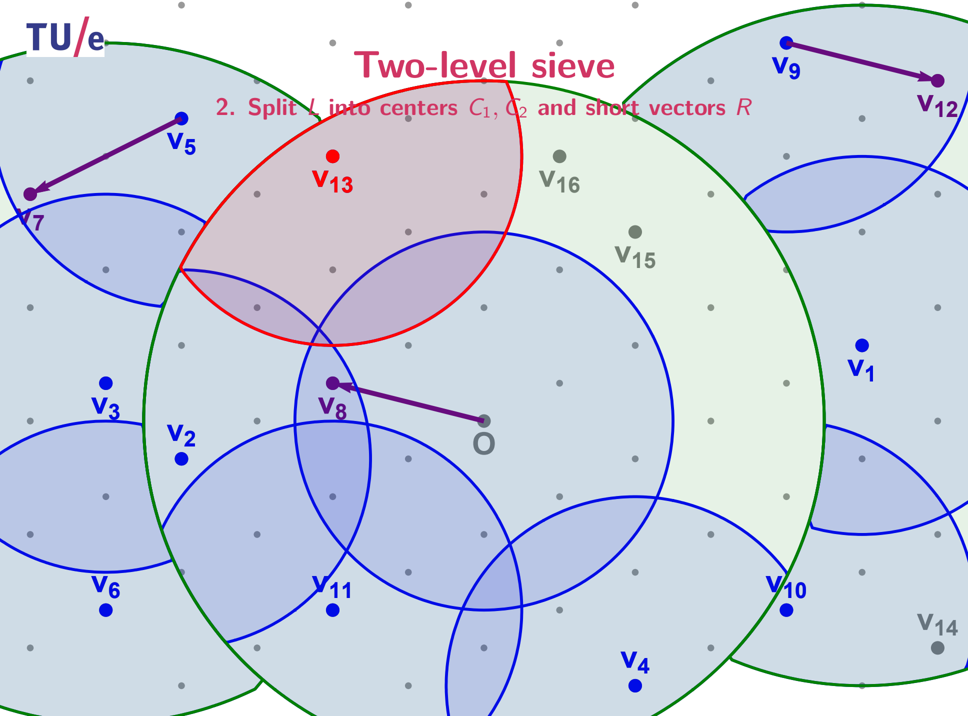

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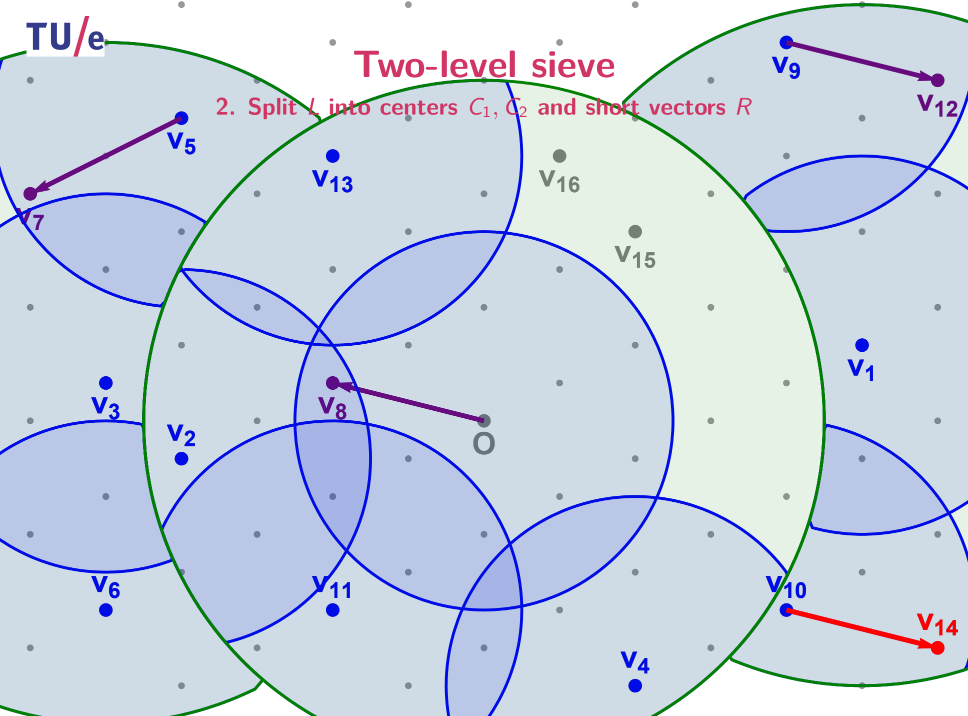

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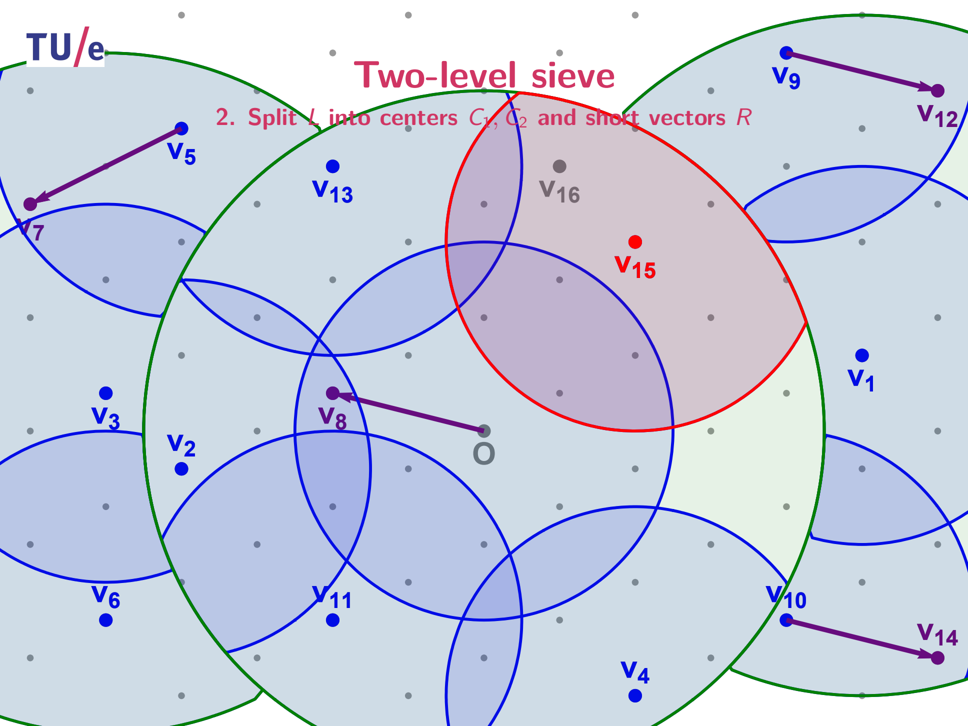

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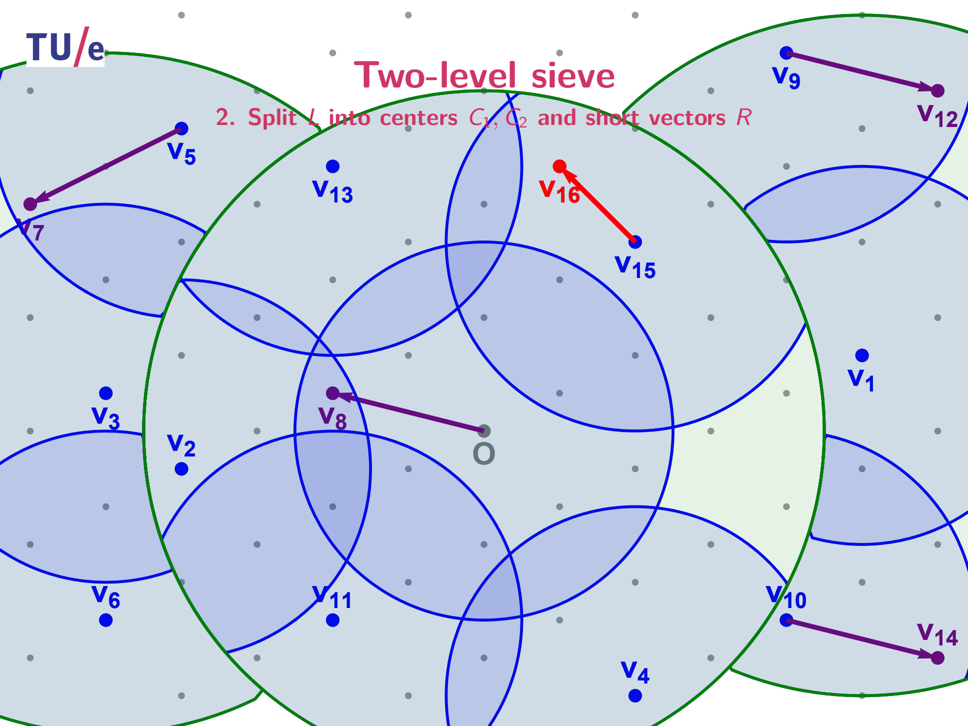

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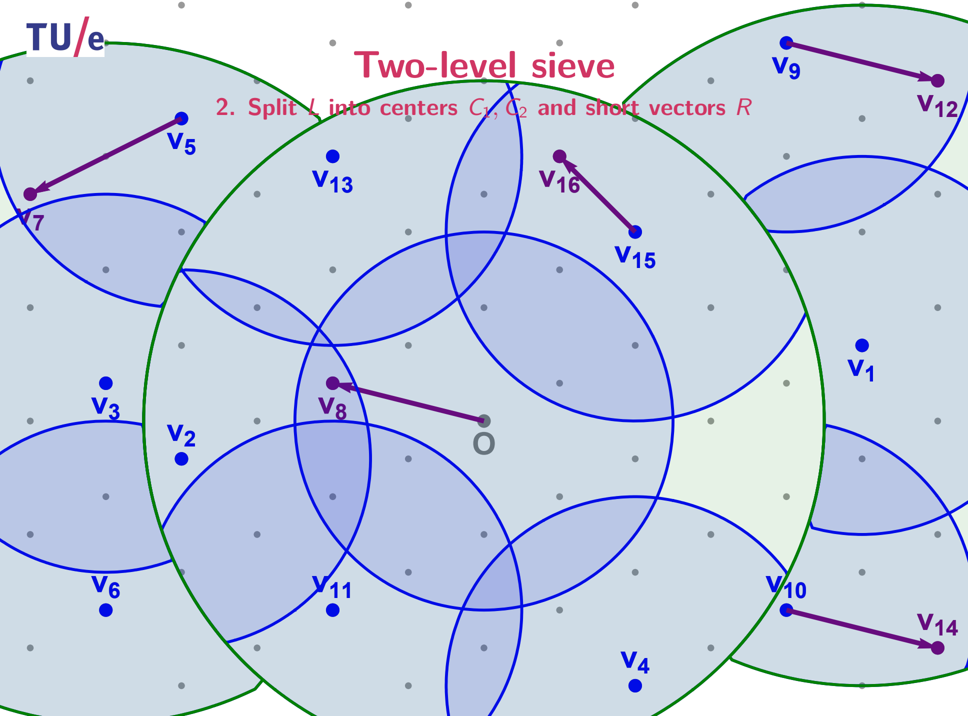

TU/e

Two-level sieve

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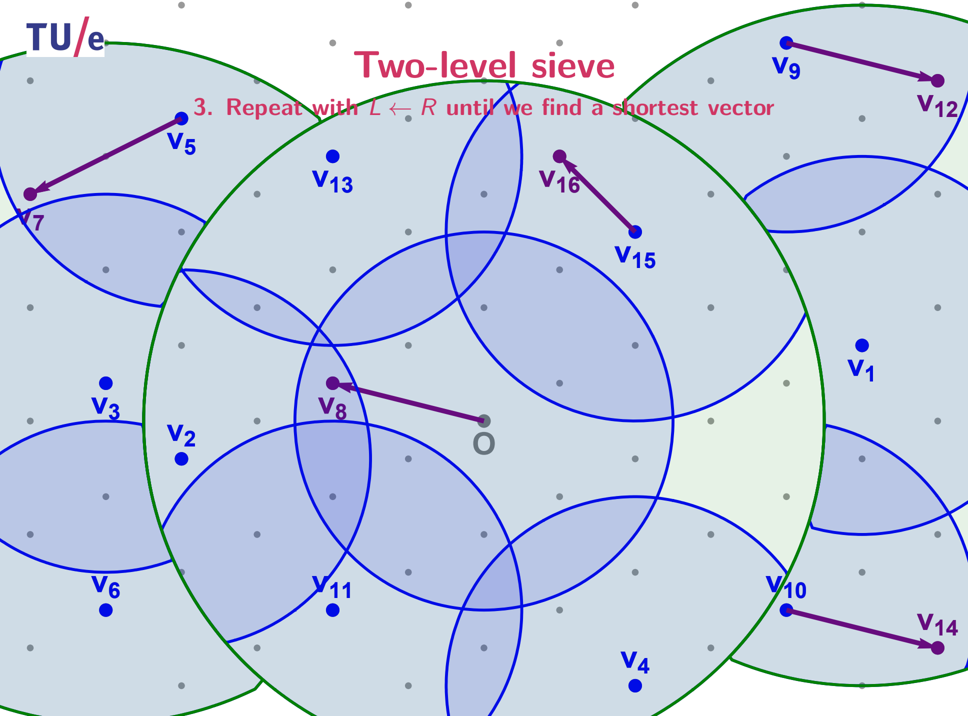

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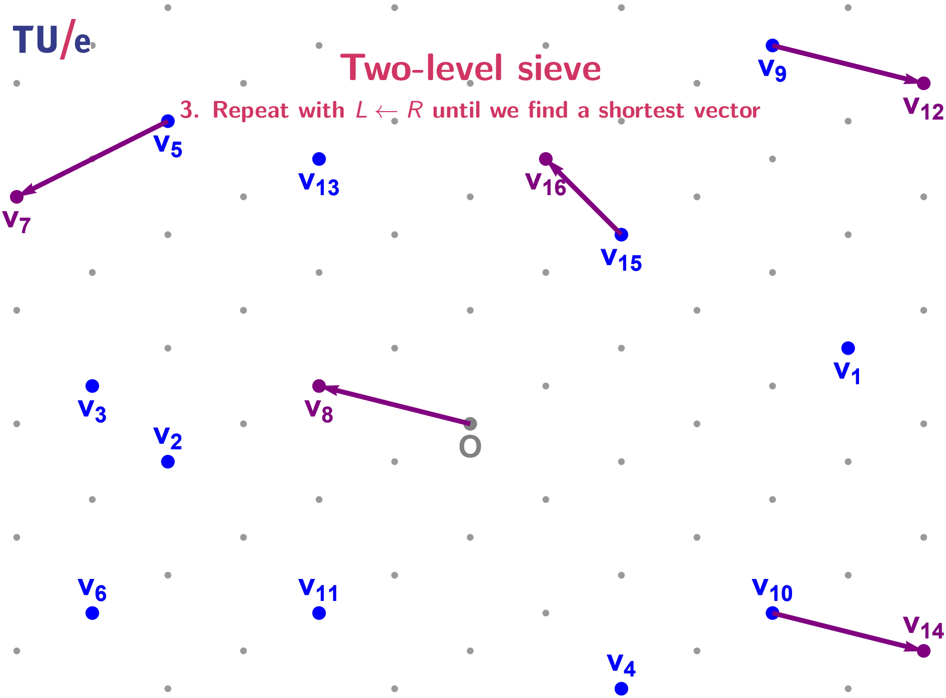

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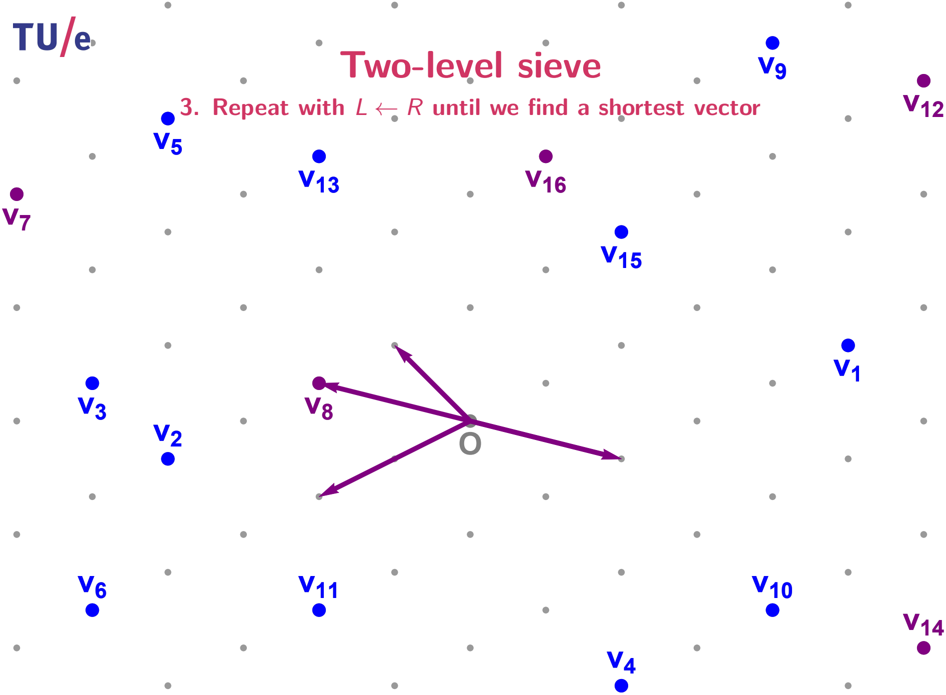

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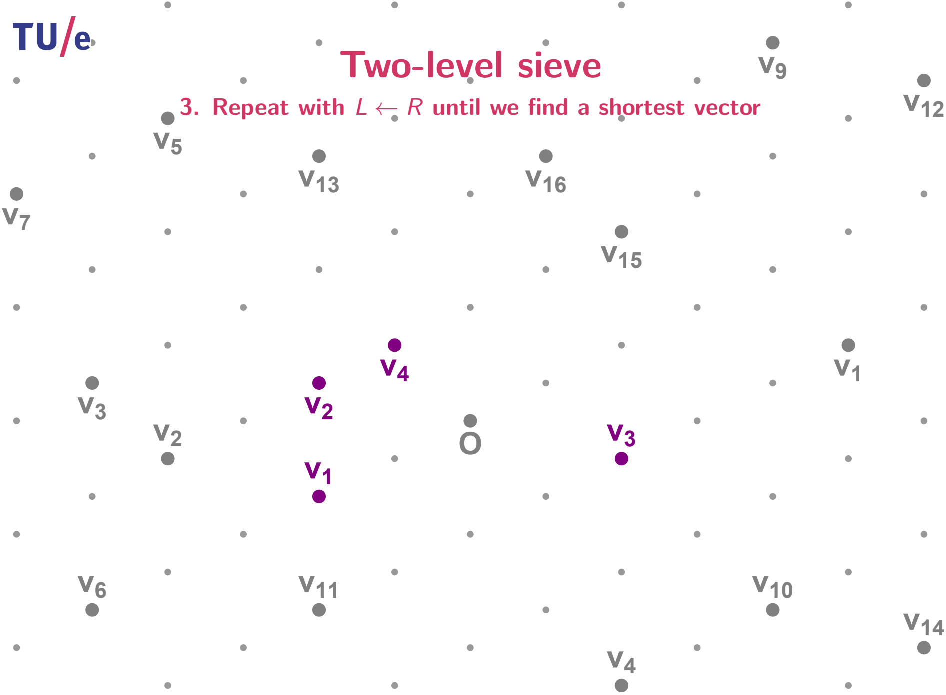
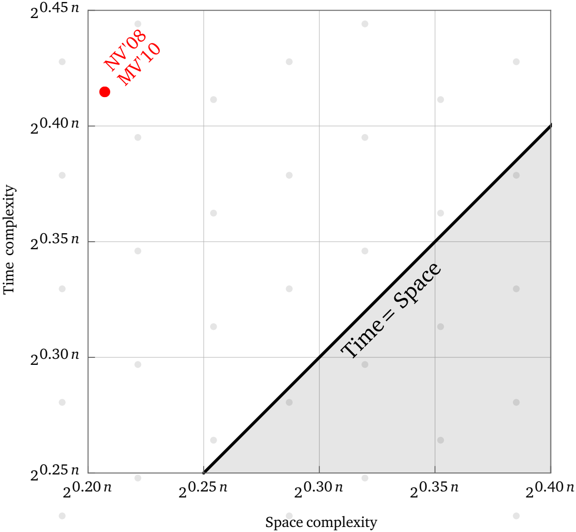


Two-level sieve

Space/time trade-off



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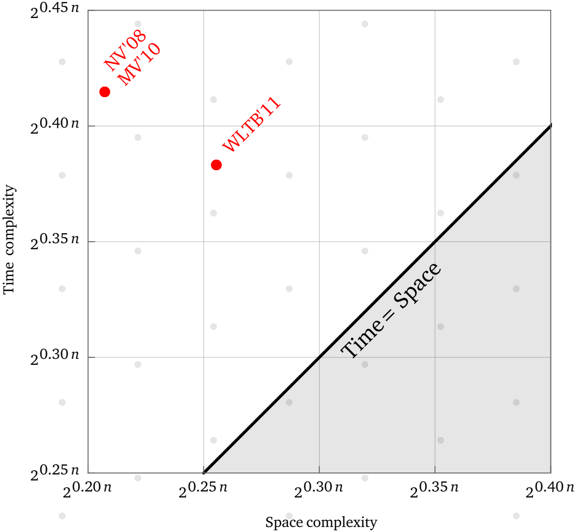

Three-level sieve

Overview

Heuristic (Nguyen and Vidick, J. Math. Crypt. '08)

The one-level sieve runs in time $2^{0.4150n}$ and space $2^{0.2075n}$.

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TU/e

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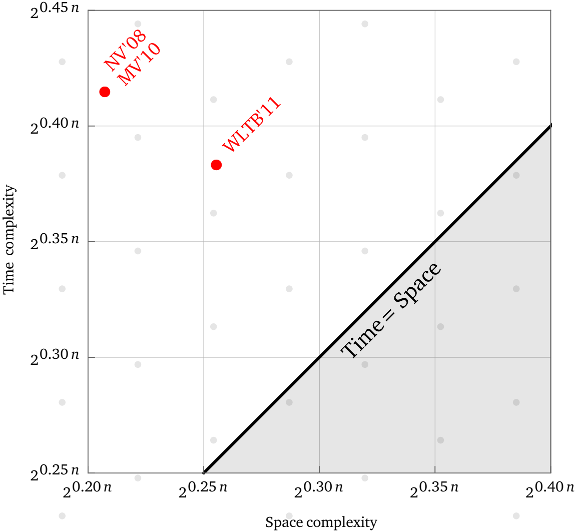
Conjecture

The four-level sieve runs in time $2^{0.3774n}$ and space $2^{0.2925n}$, and higher-level sieves are not faster than this.

Three-level sieve

Space/time trade-off



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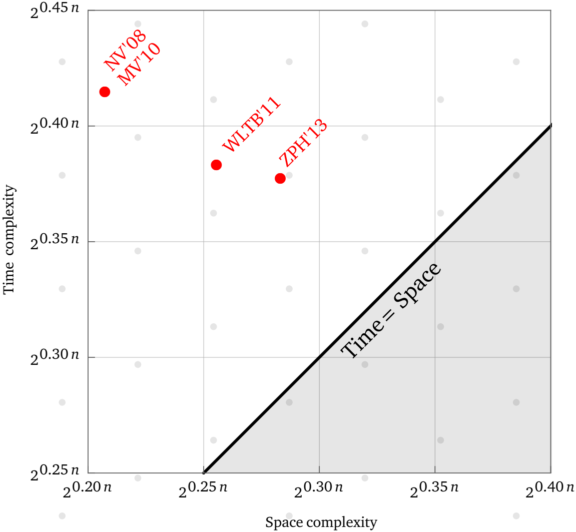
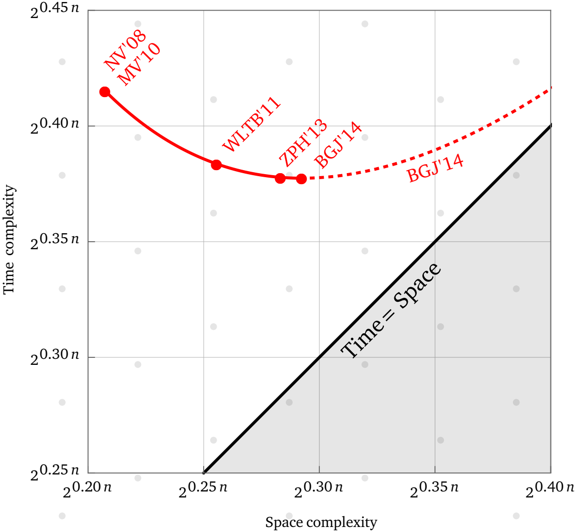


Decomposition approach

Space/time trade-off



HashSieve (GS)

1. Generate random lattice vectors

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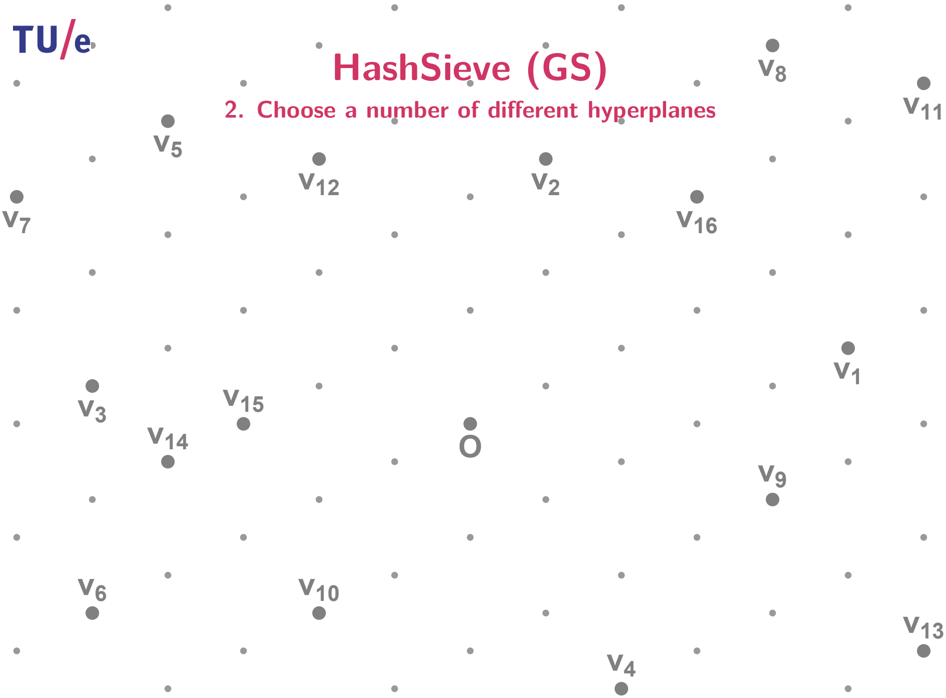
2. Choose a number of different hyperplanes



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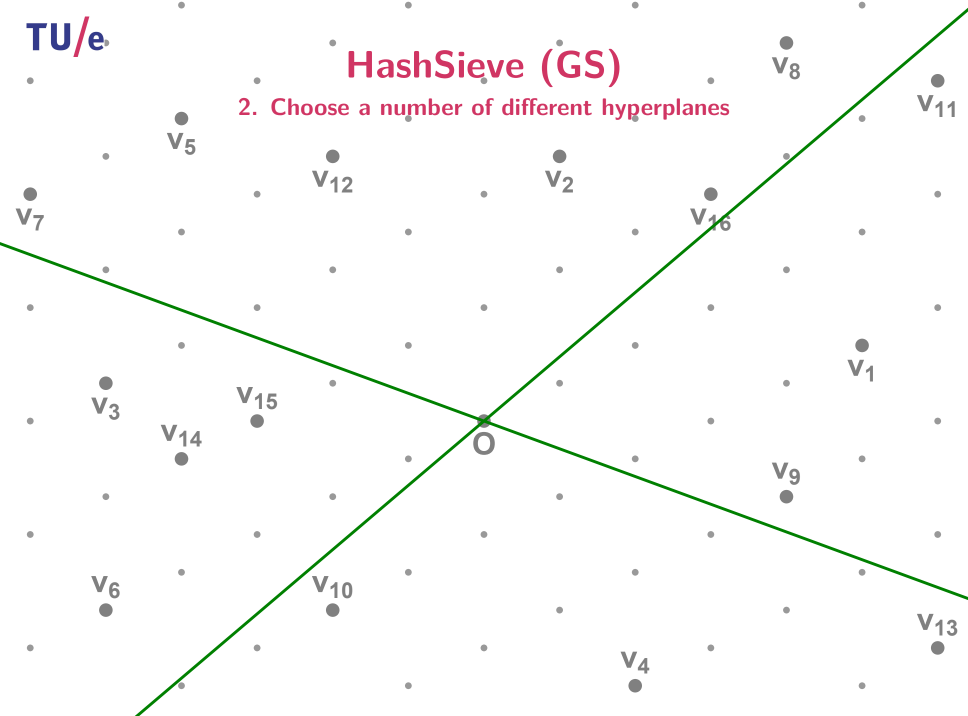
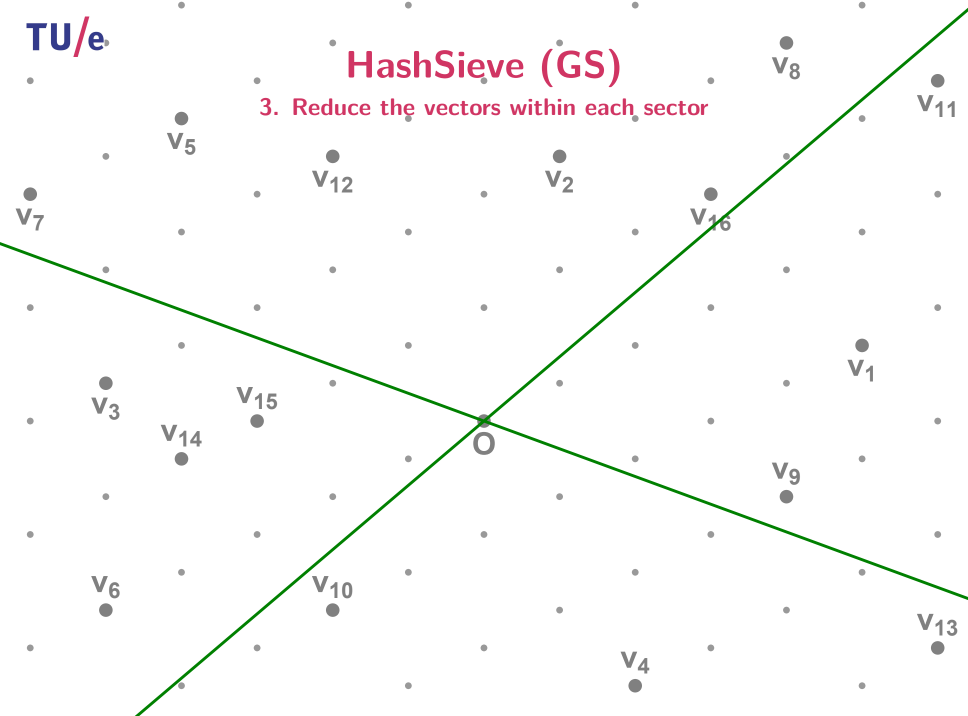


HashSieve (GS)

3. Reduce the vectors within each sector



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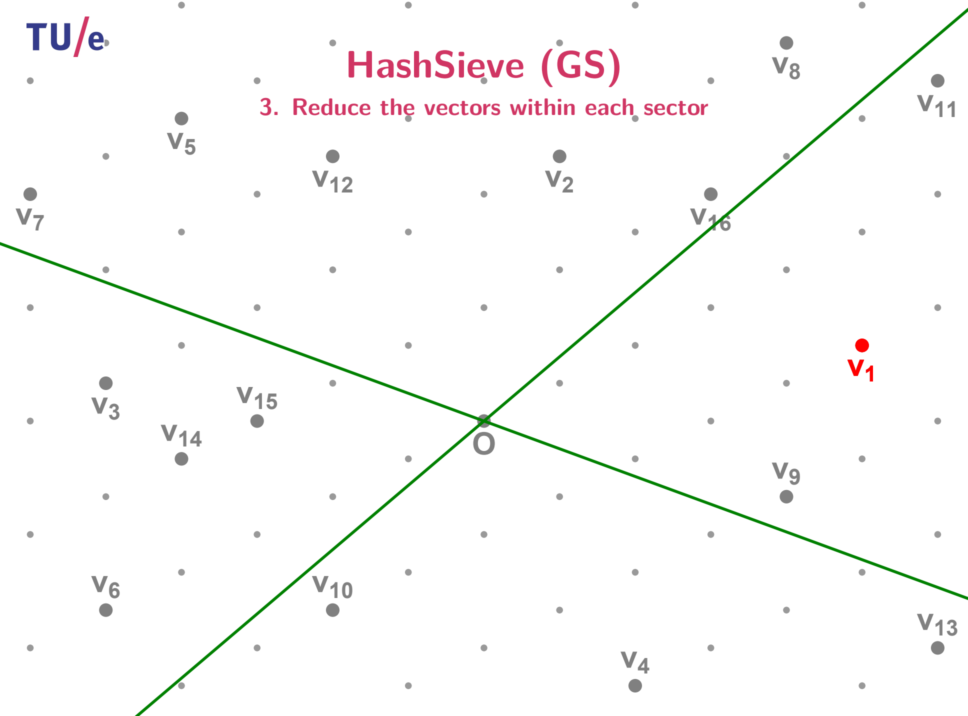

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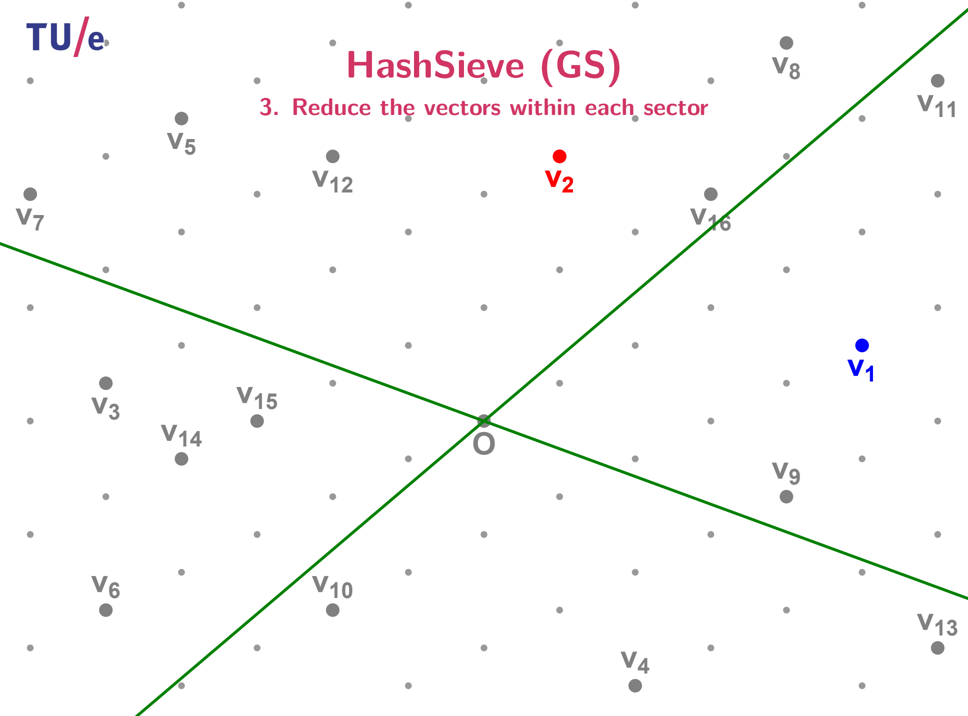

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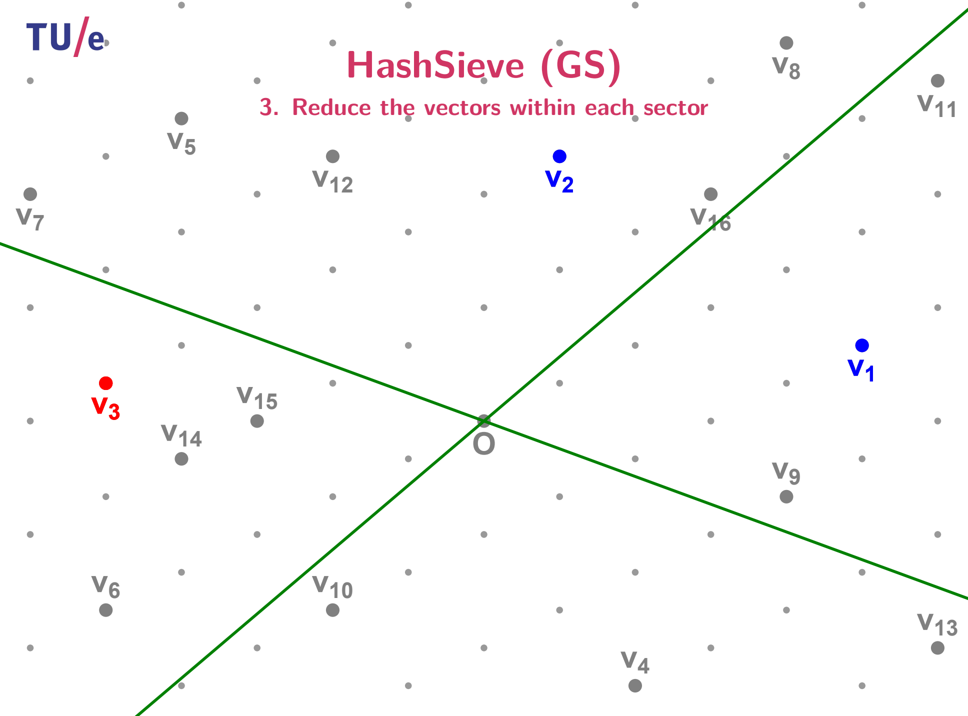

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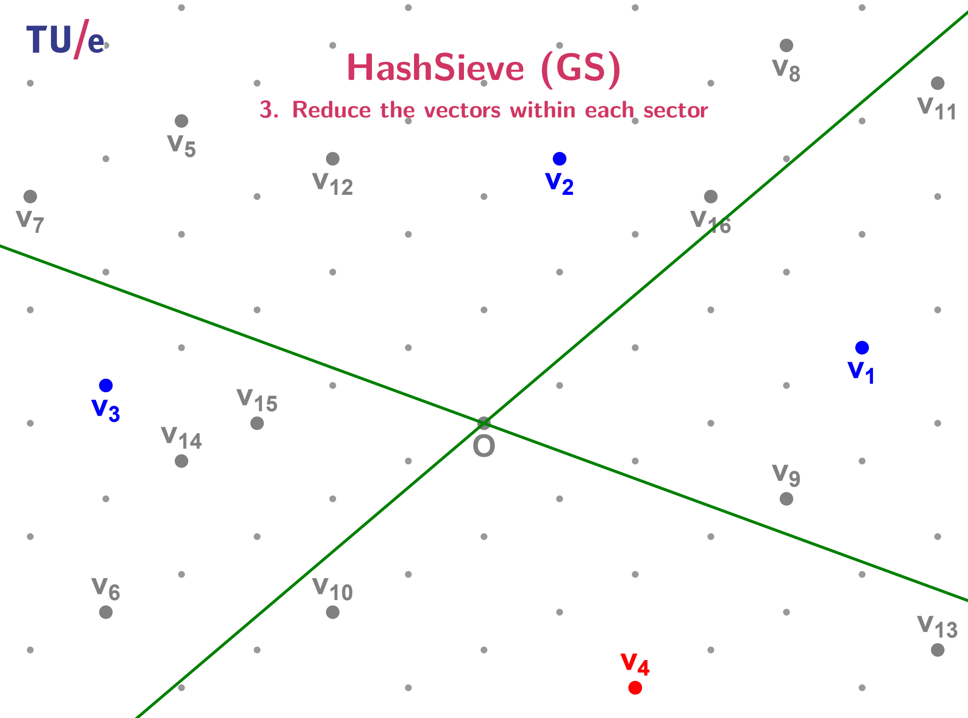

TU/e

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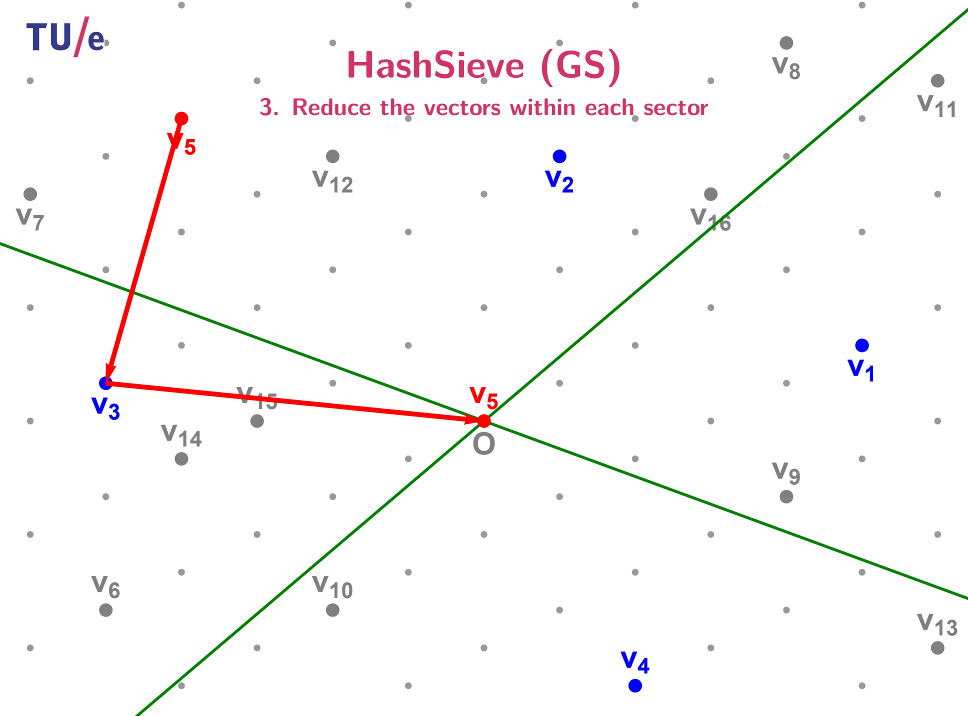

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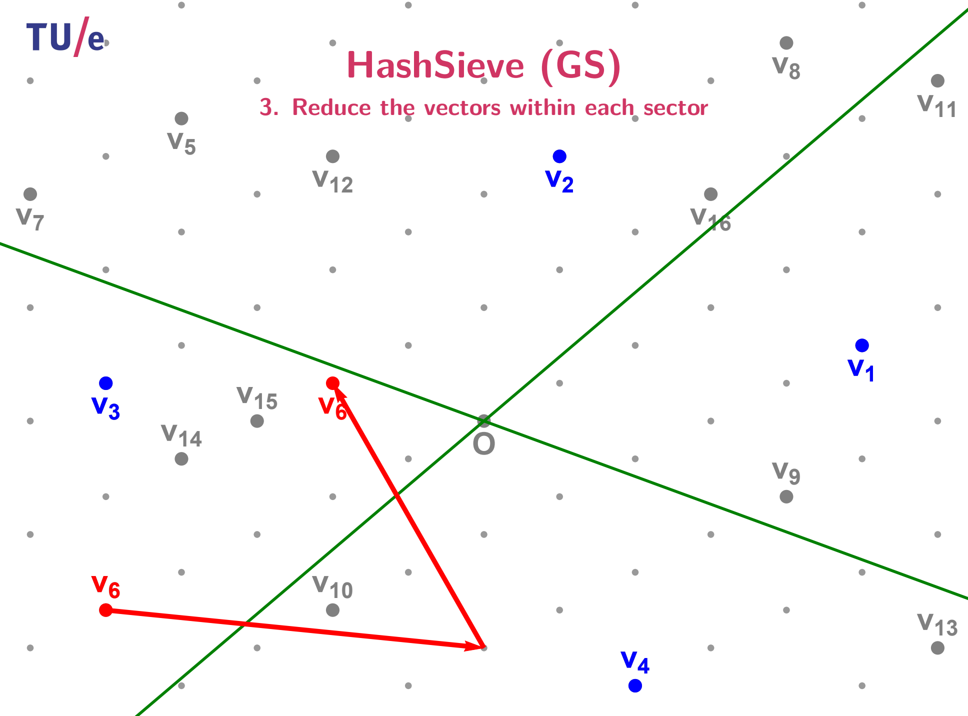

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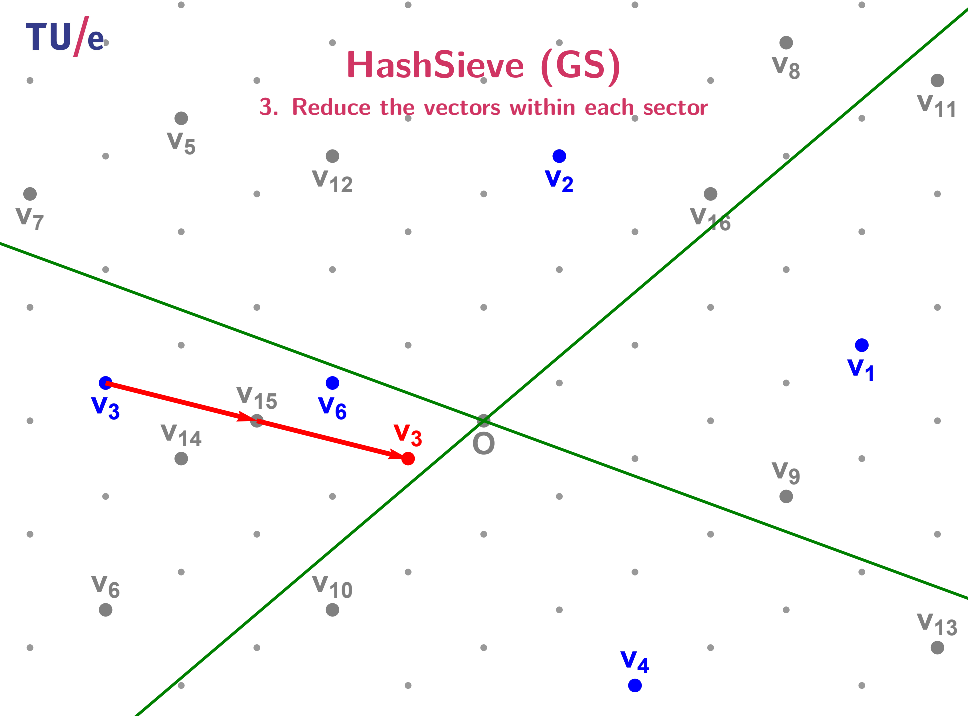

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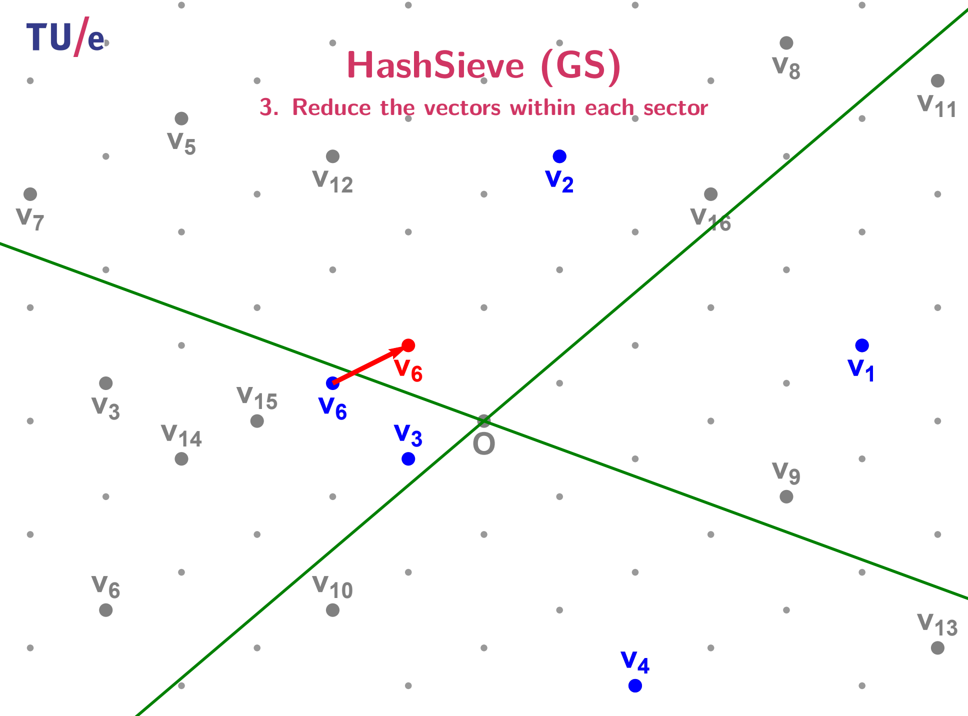

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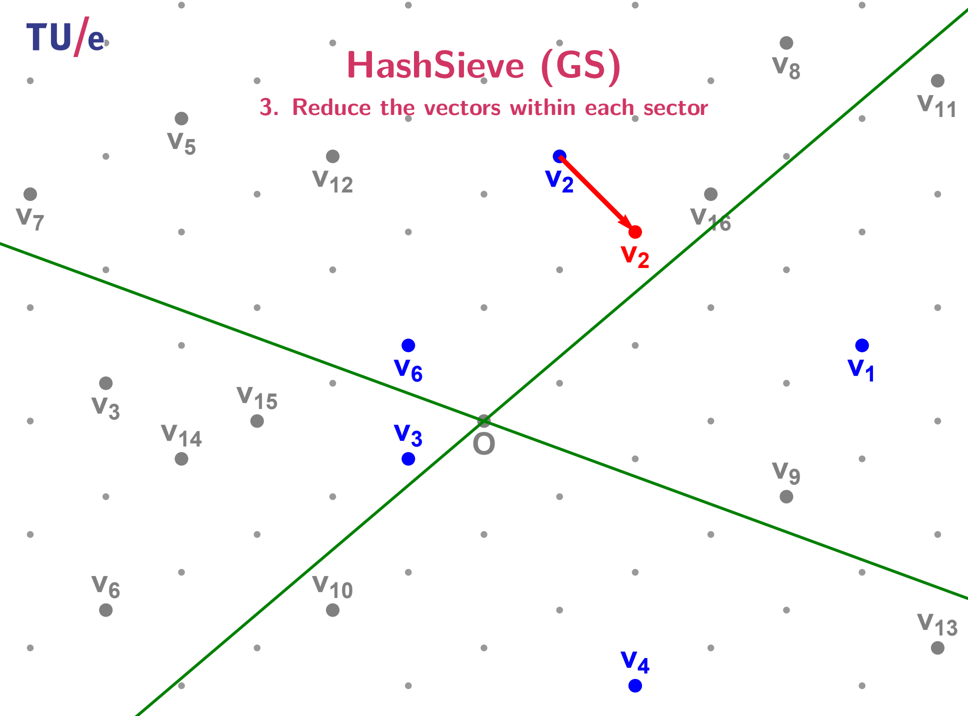

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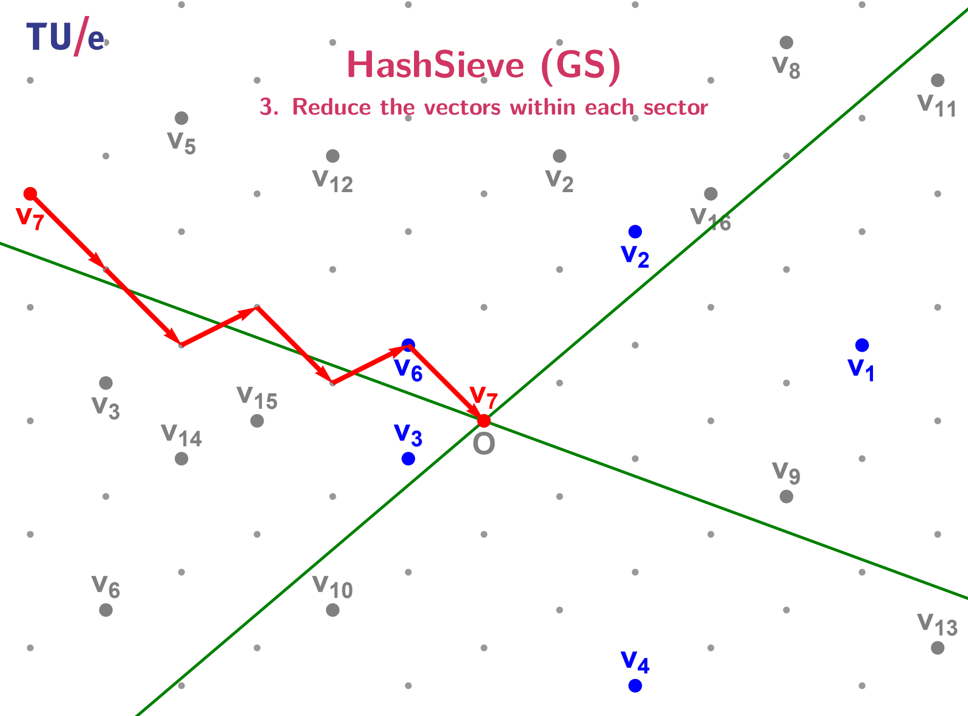

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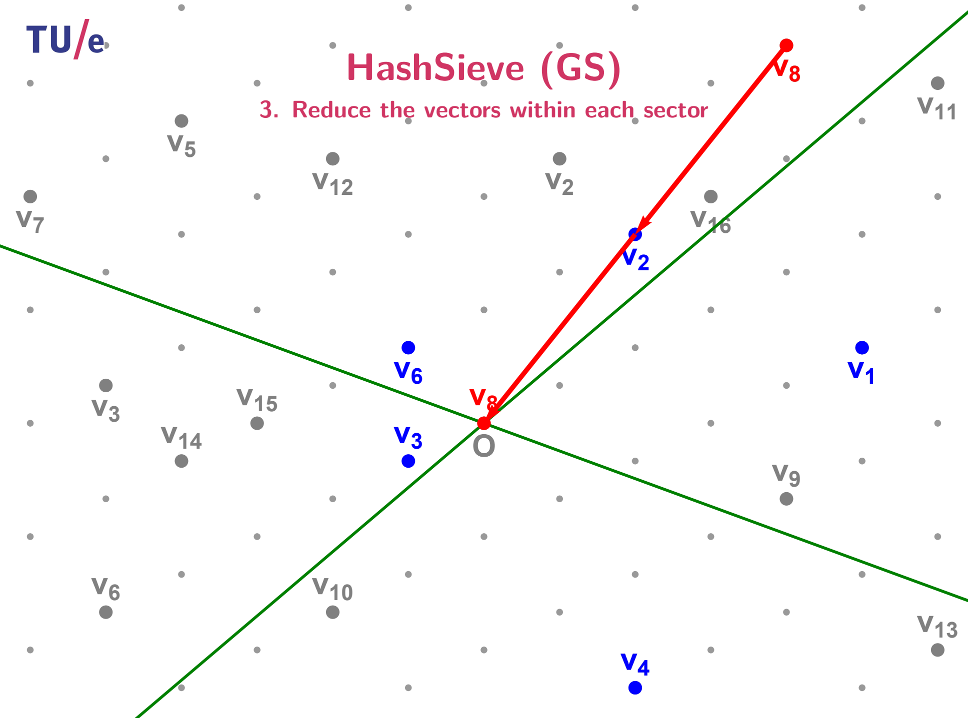

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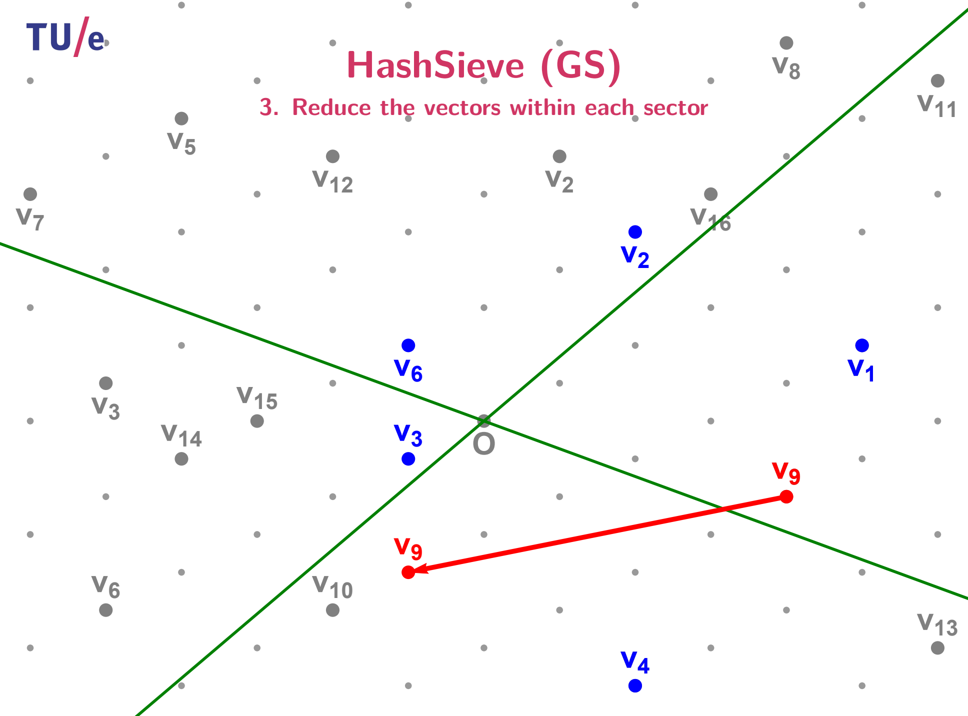

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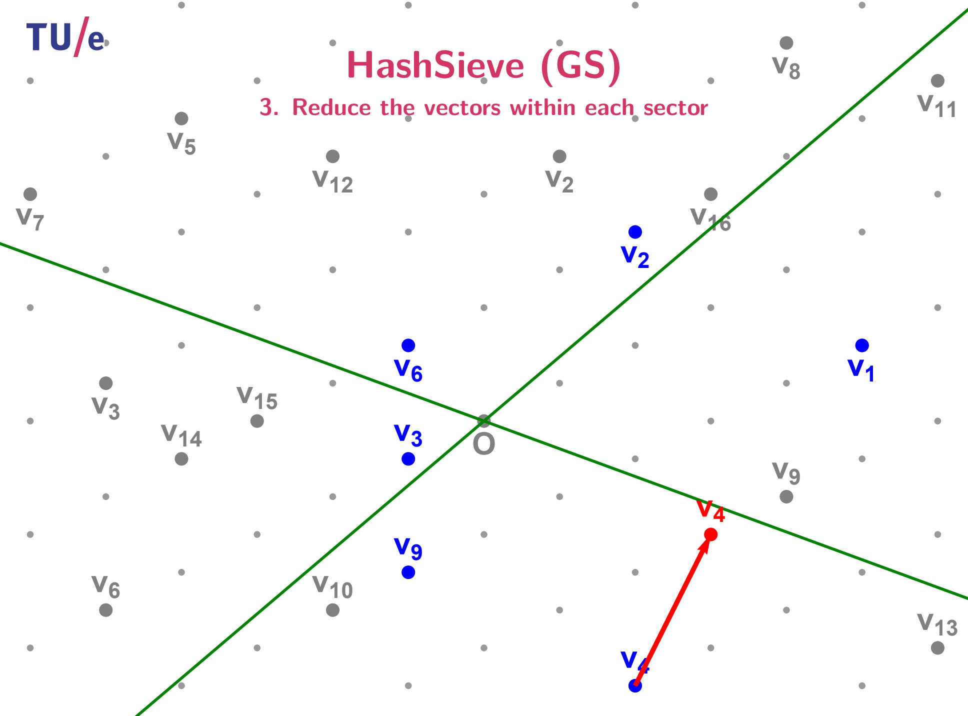

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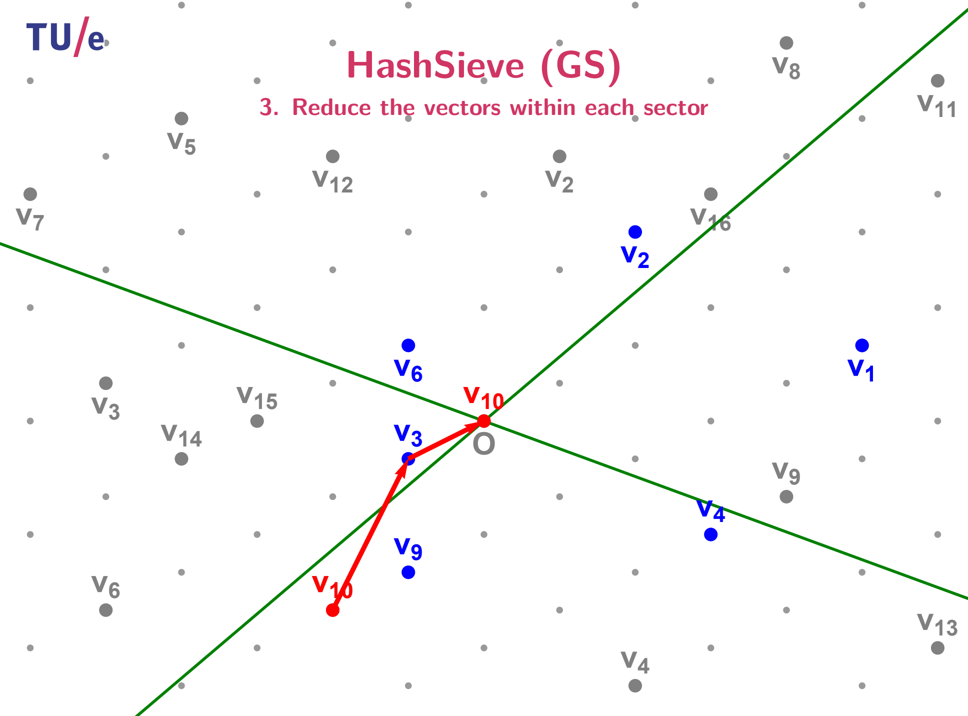

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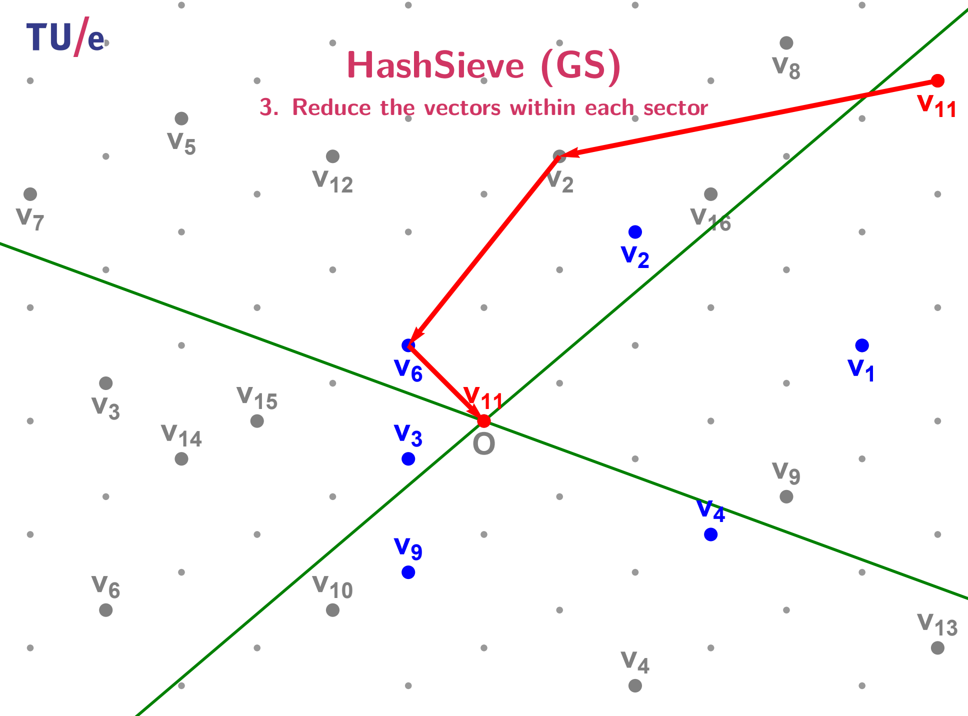

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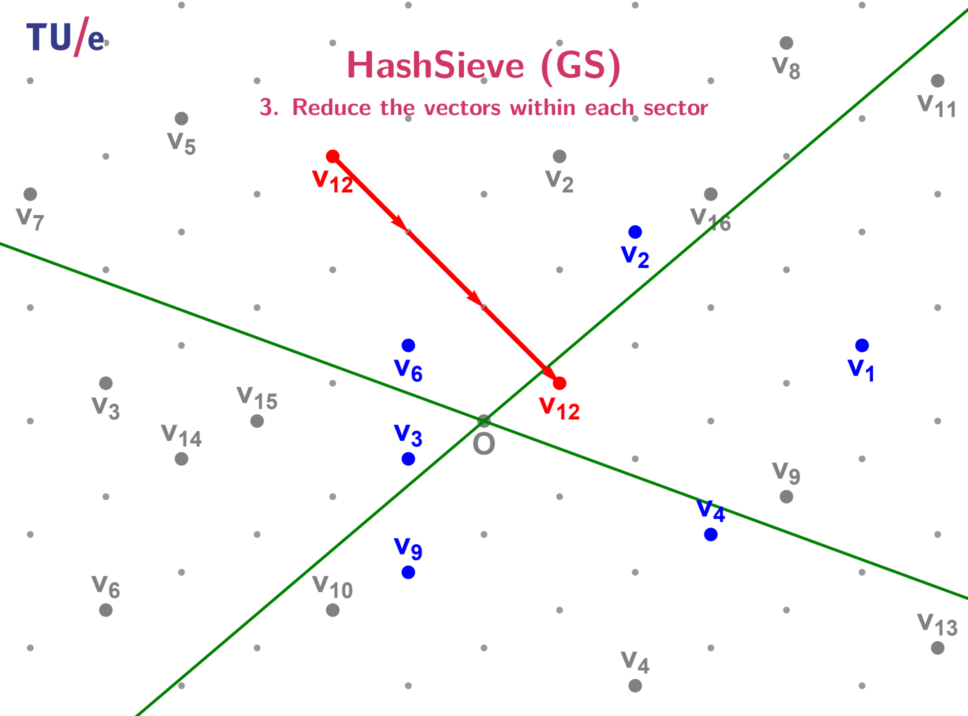

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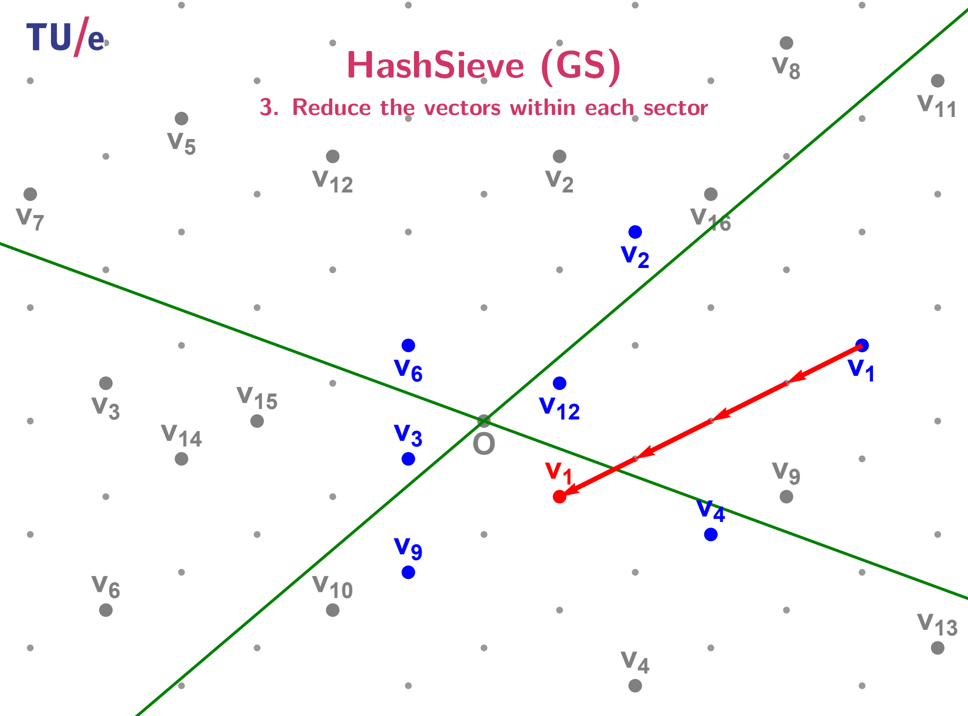

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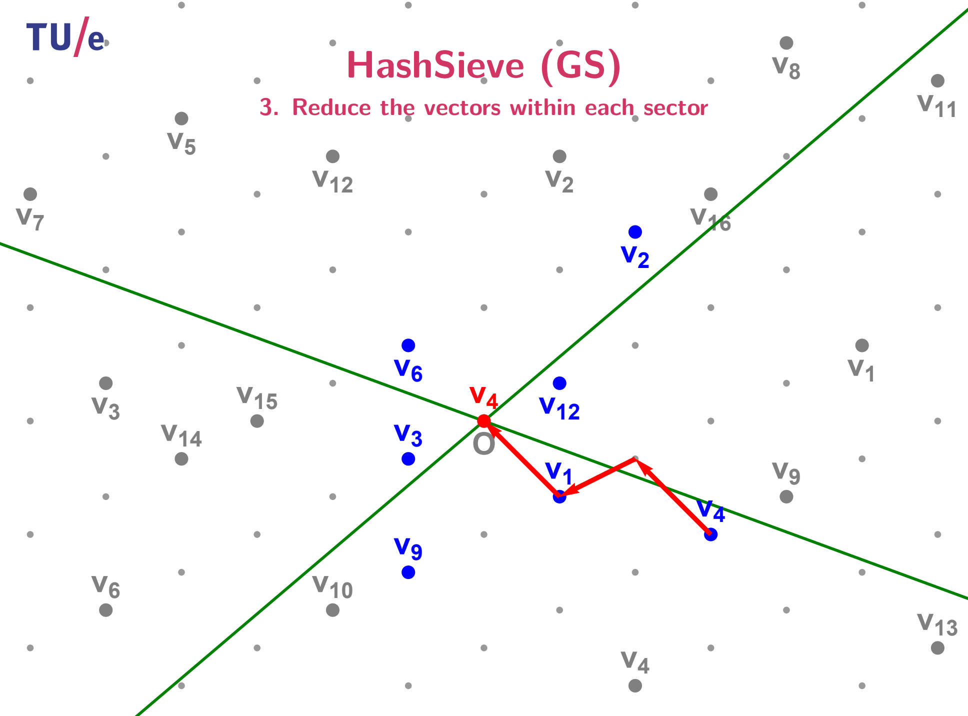

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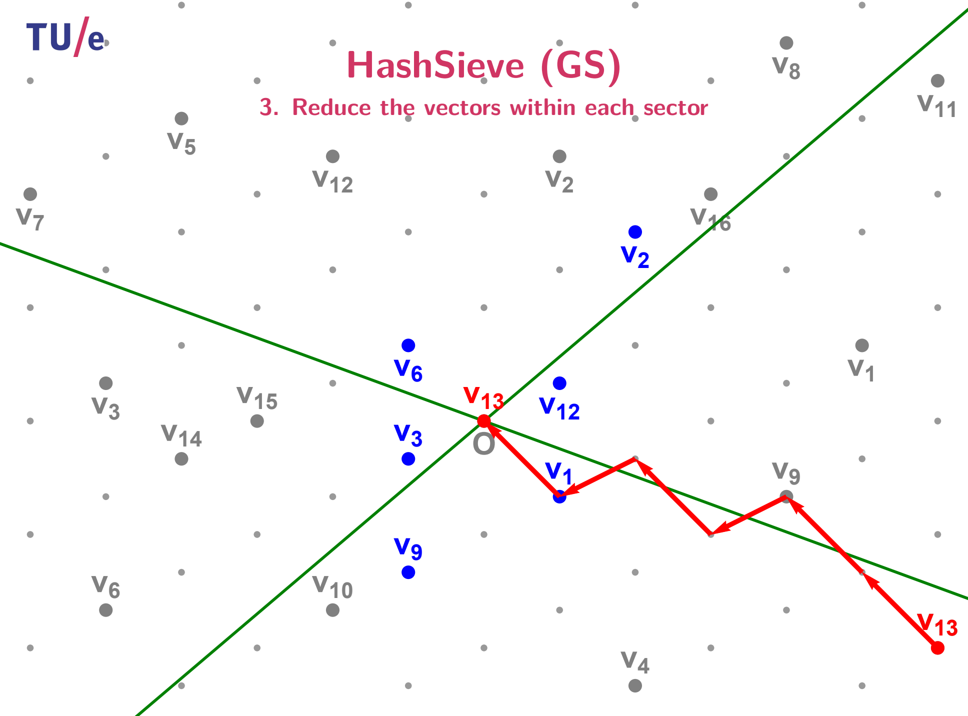

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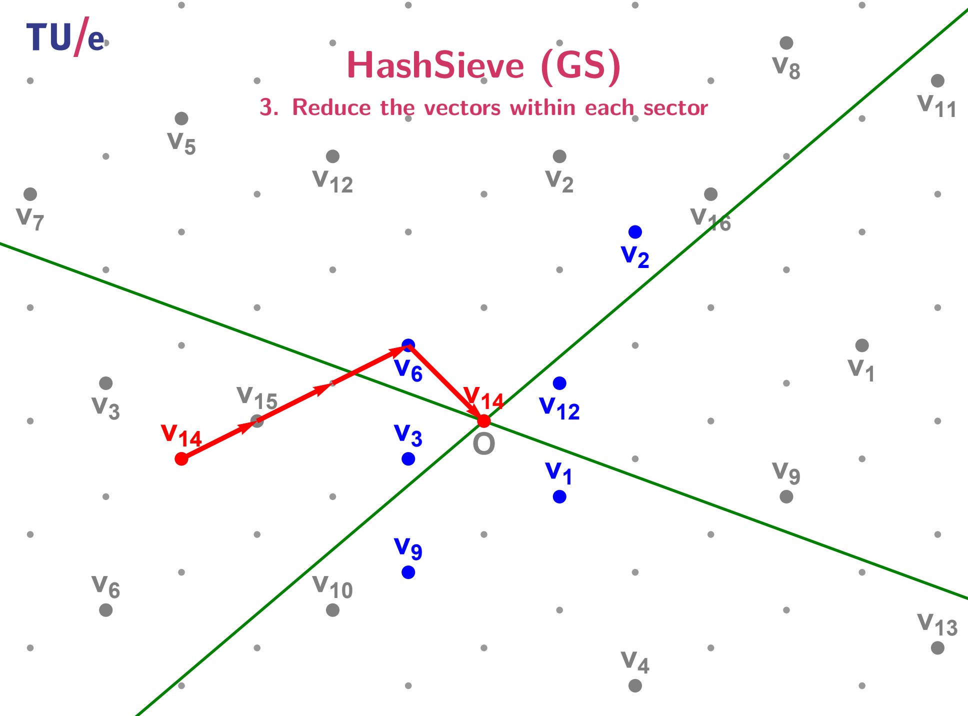

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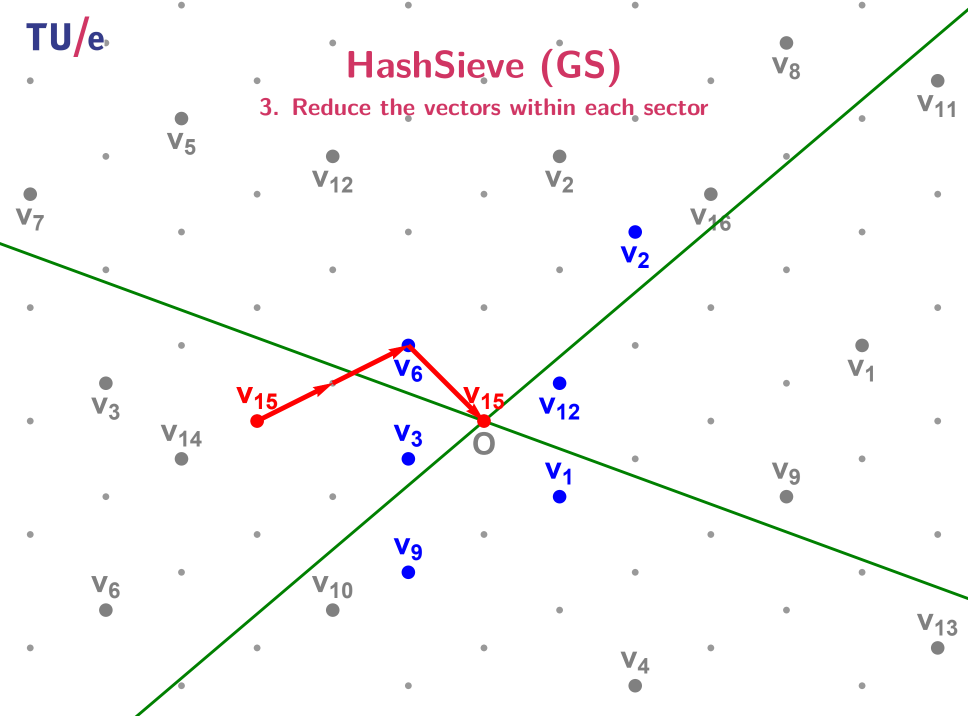

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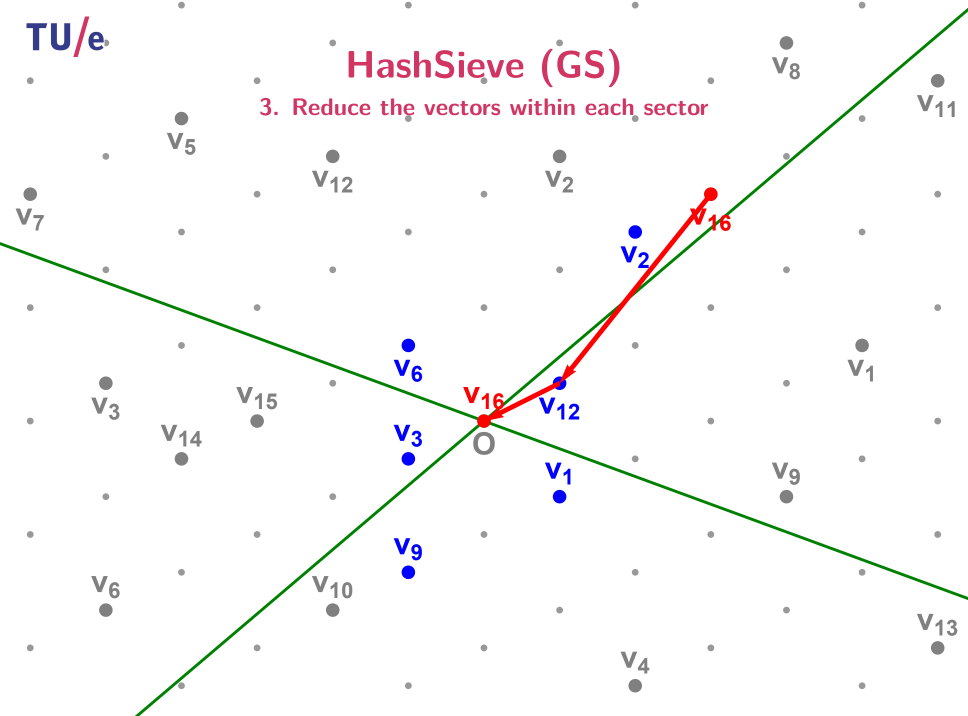

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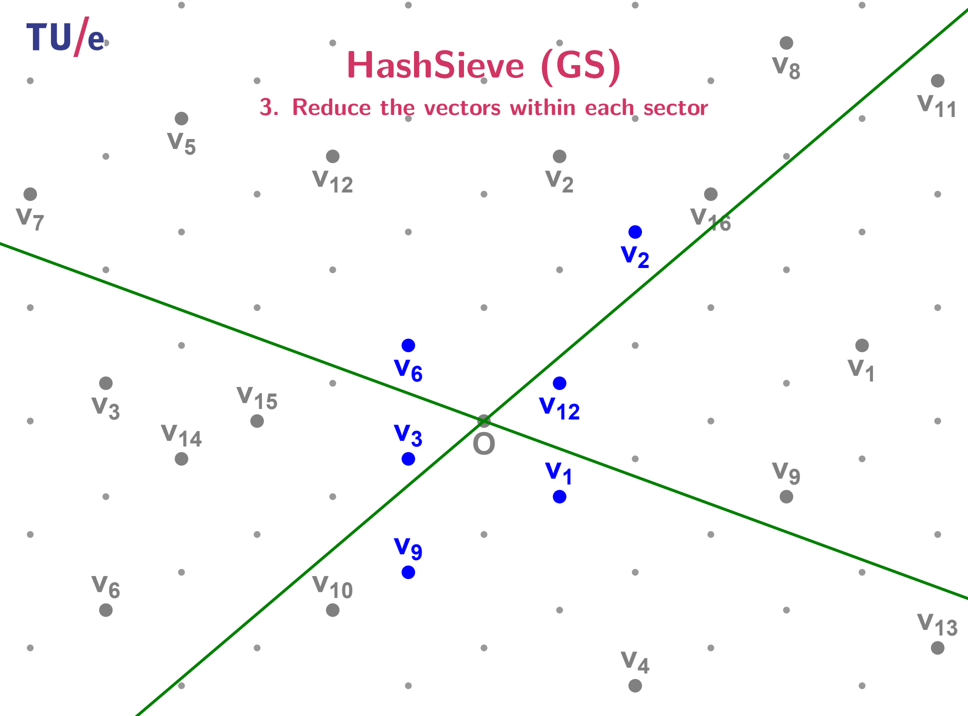

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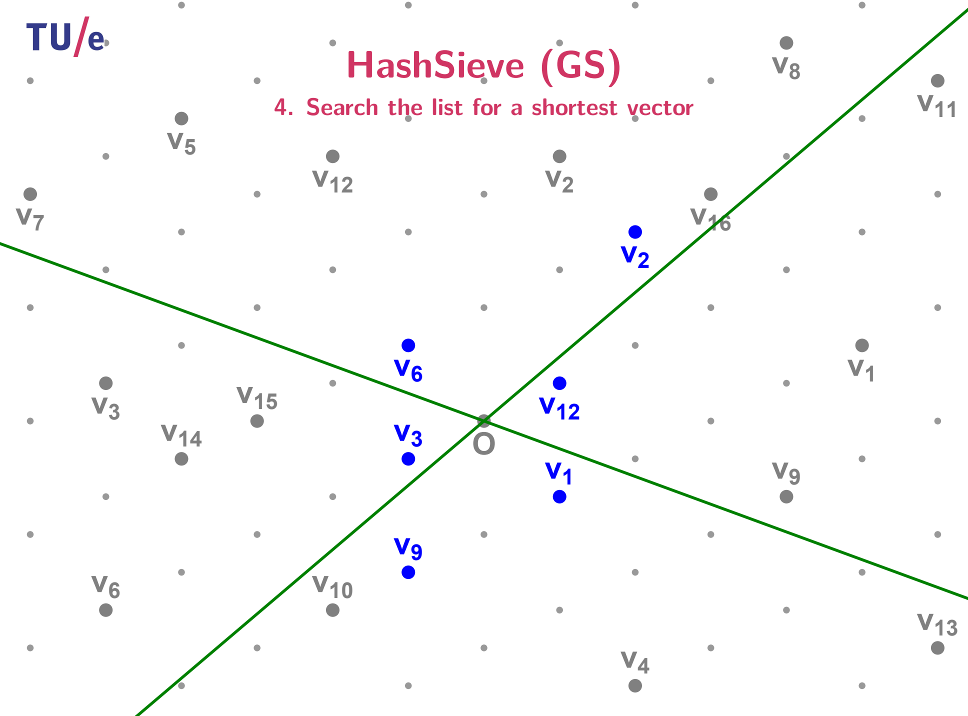

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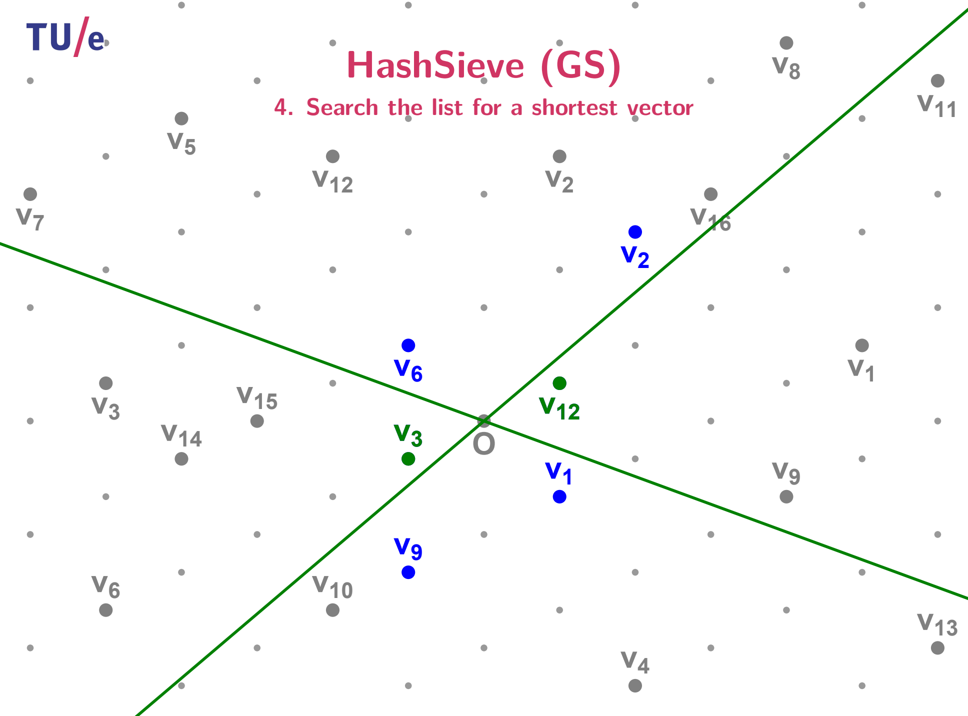

HashSieve (GS)

Overview

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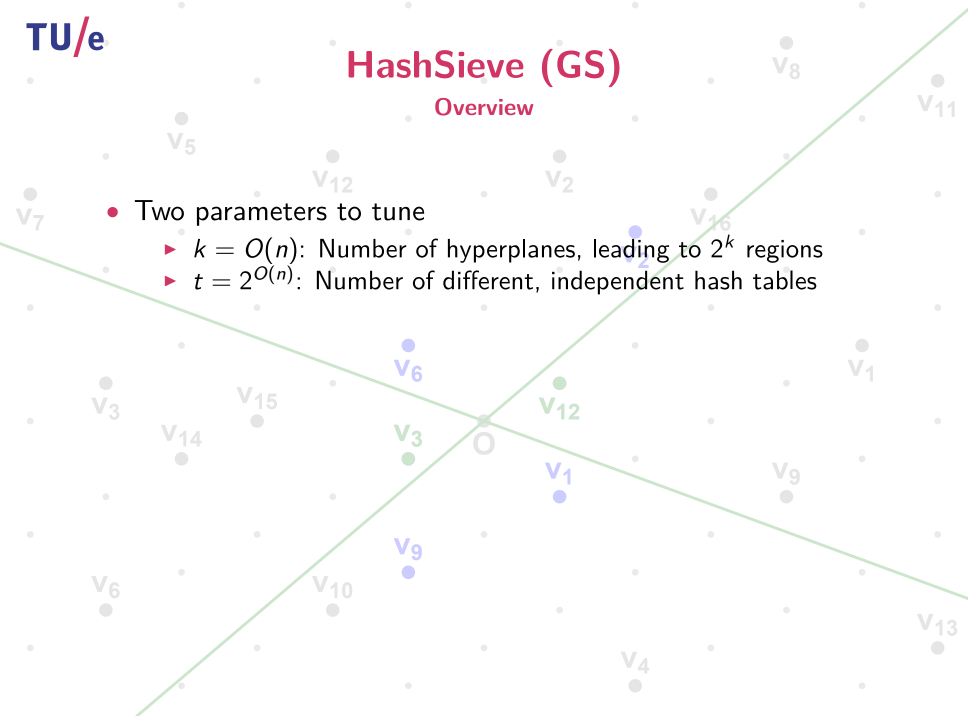
Overview

- Two parameters to tune
  - $k = O(n)$: Number of hyperplanes, leading to 2^k regions
  - $t = 2^{O(n)}$: Number of different, independent hash tables



HashSieve (GS)

Overview

- Two parameters to tune
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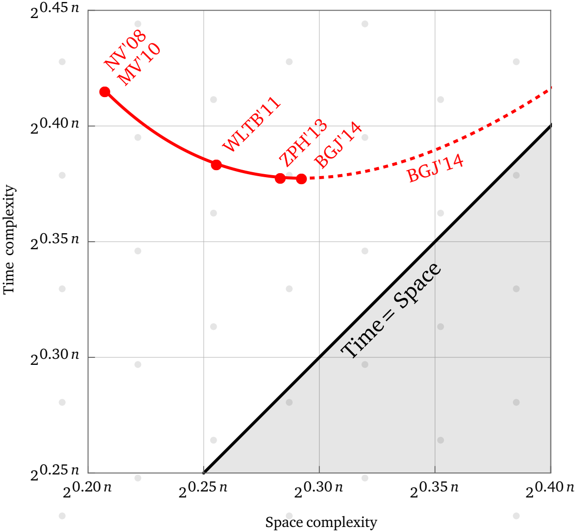
Heuristic

The HashSieve (GS) runs in time and space $2^{0.34n+o(n)}$.

HashSieve (GS)

Space/time trade-off



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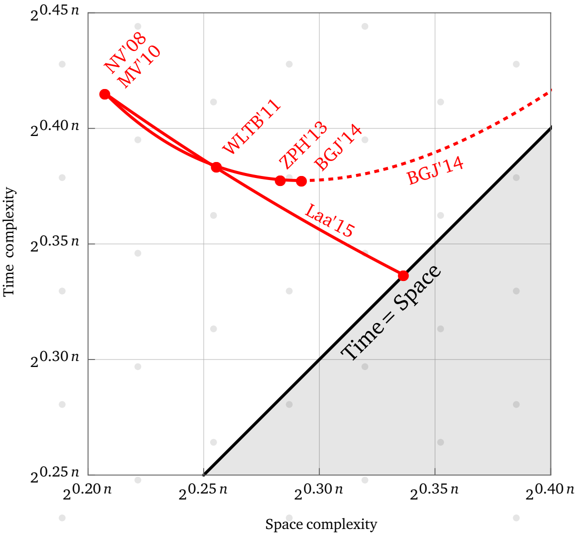

HashSieve (NV)

1. Sample a list L of random lattice vectors

TU/e

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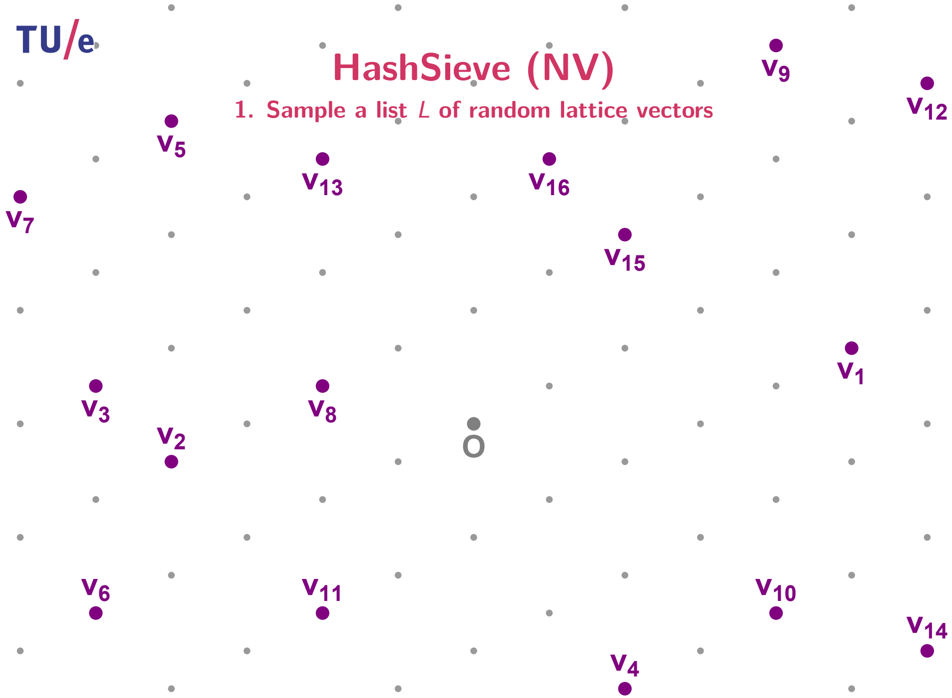

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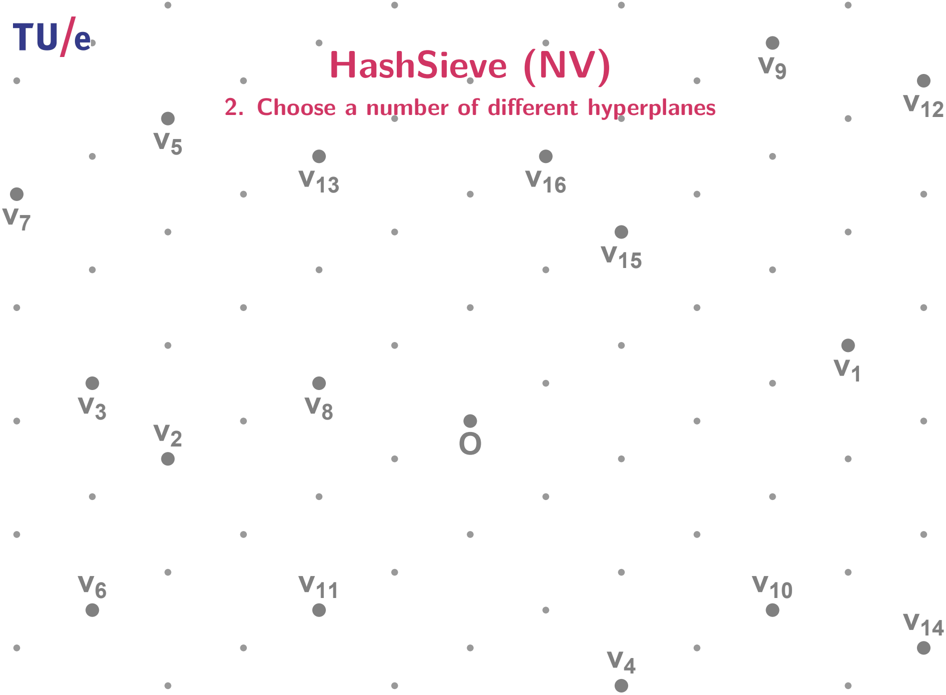

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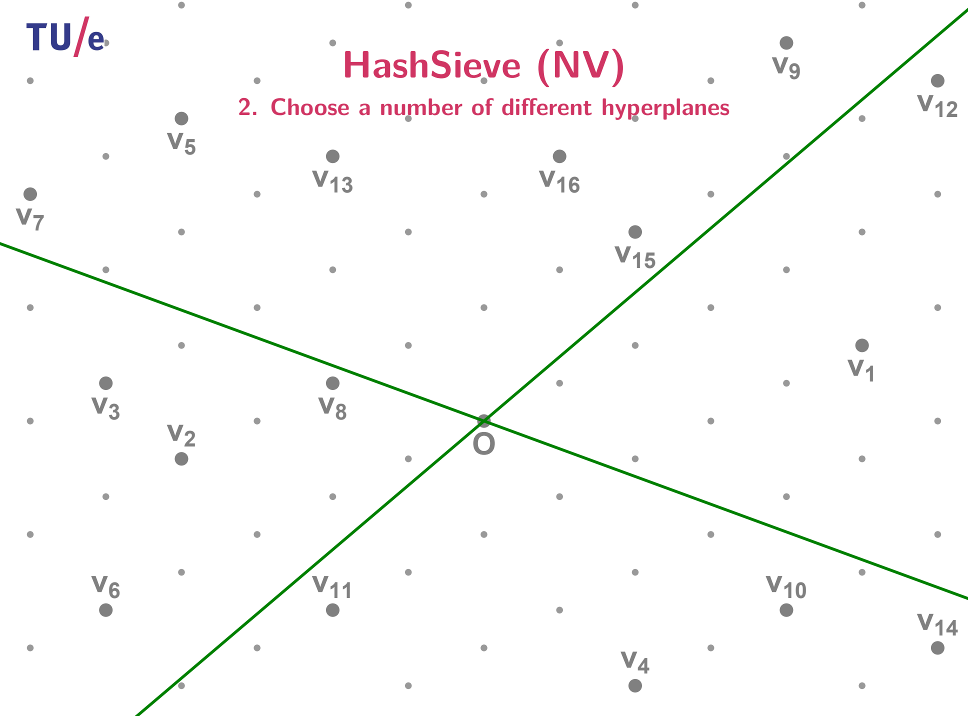

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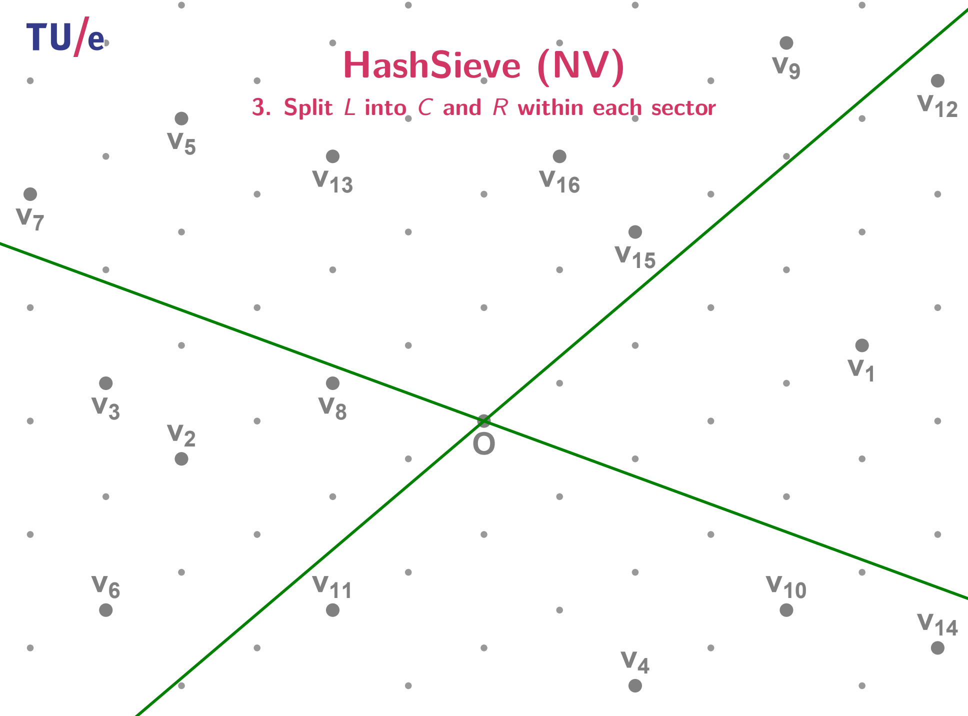

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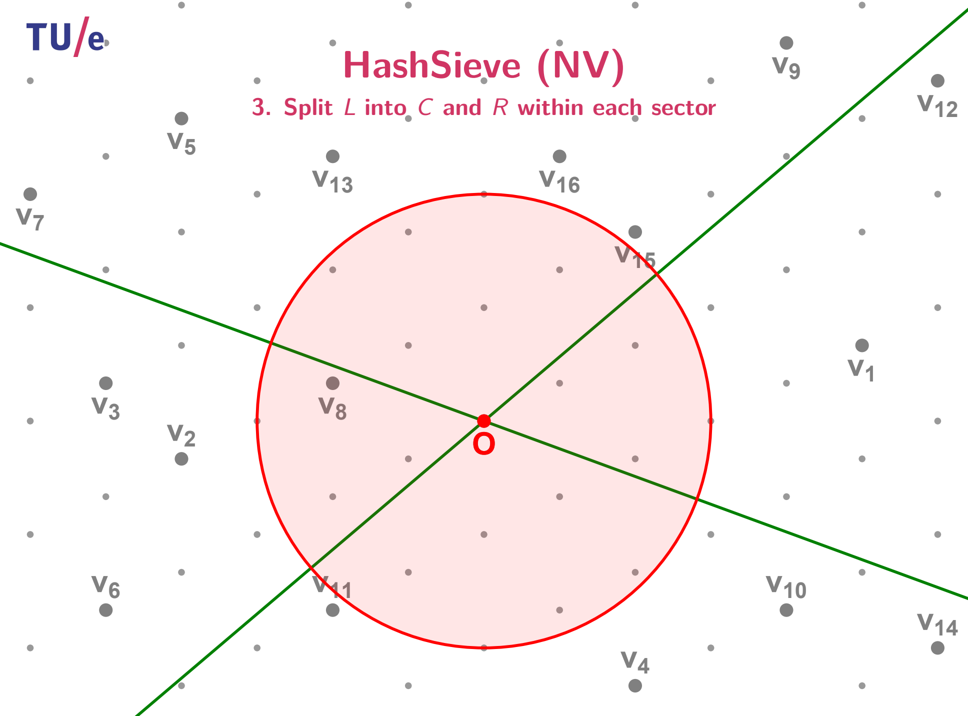

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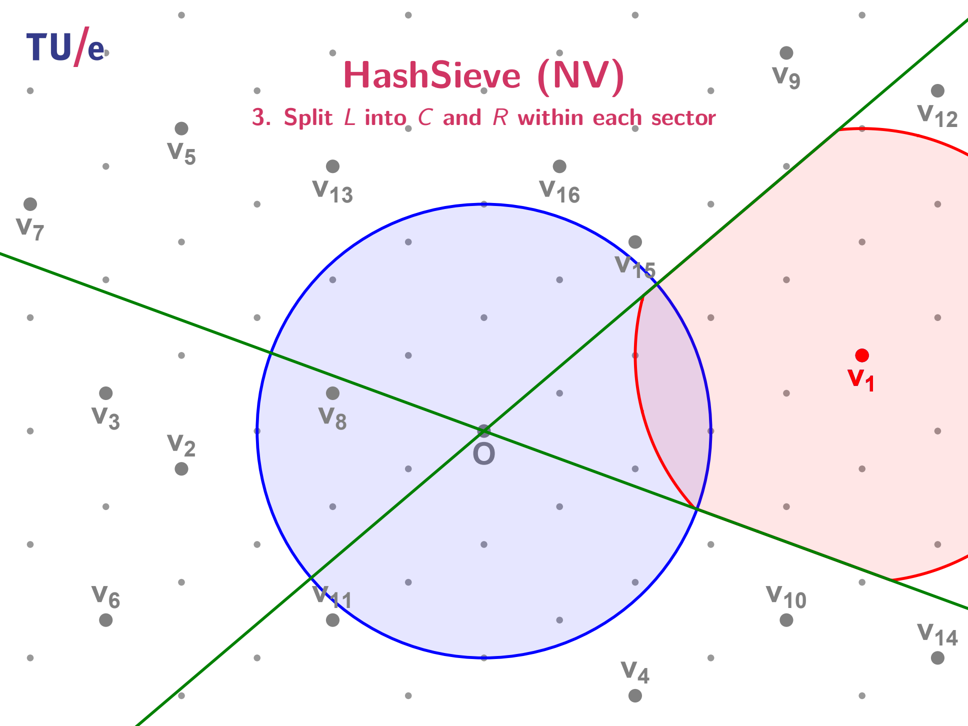

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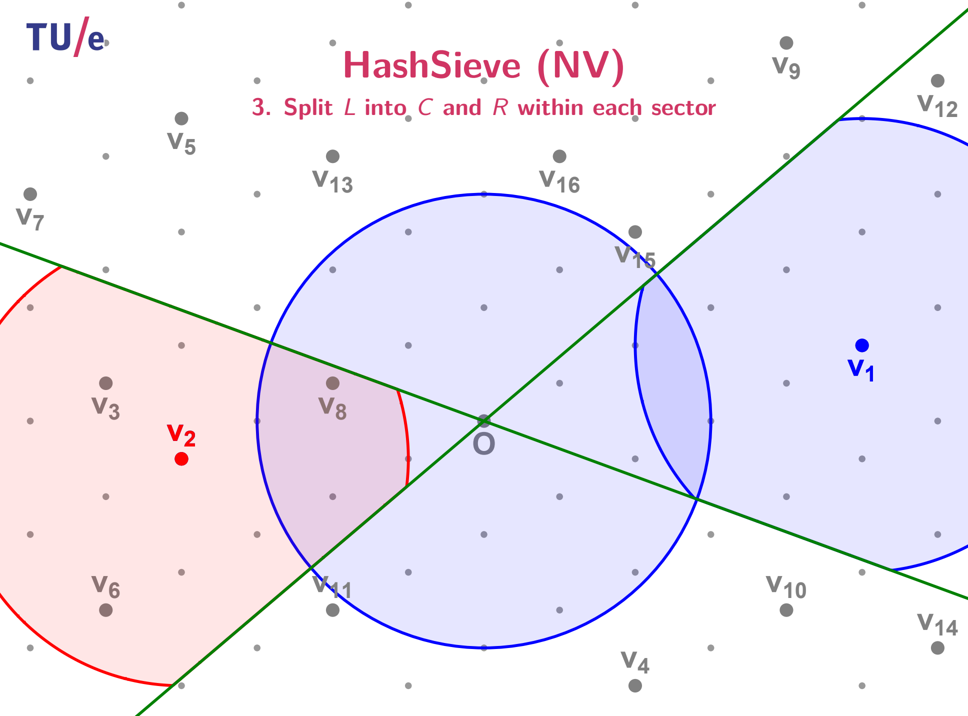

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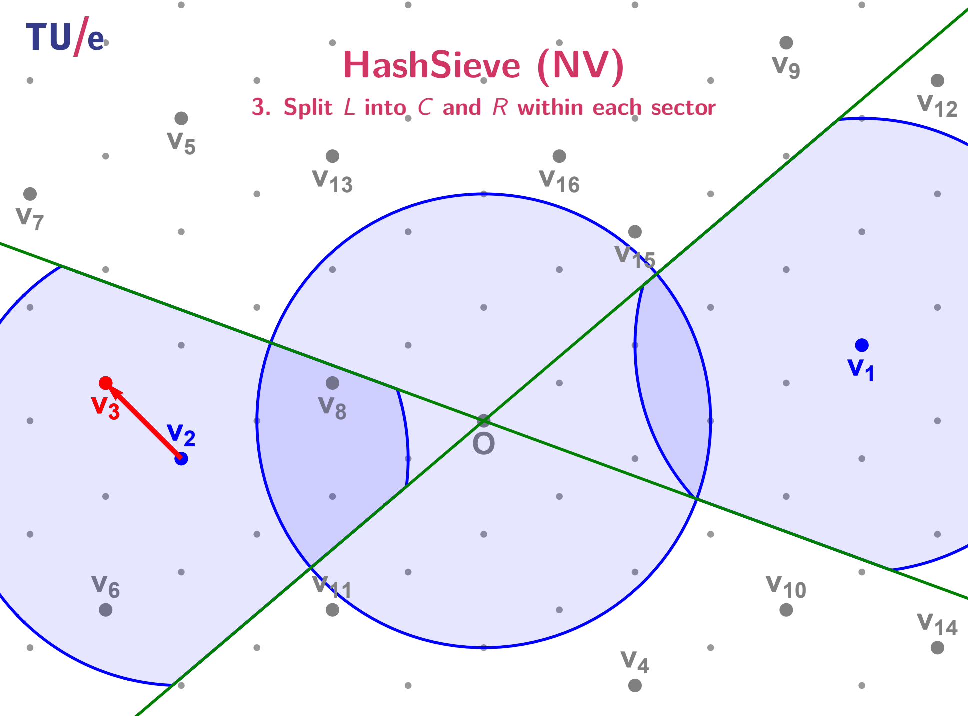

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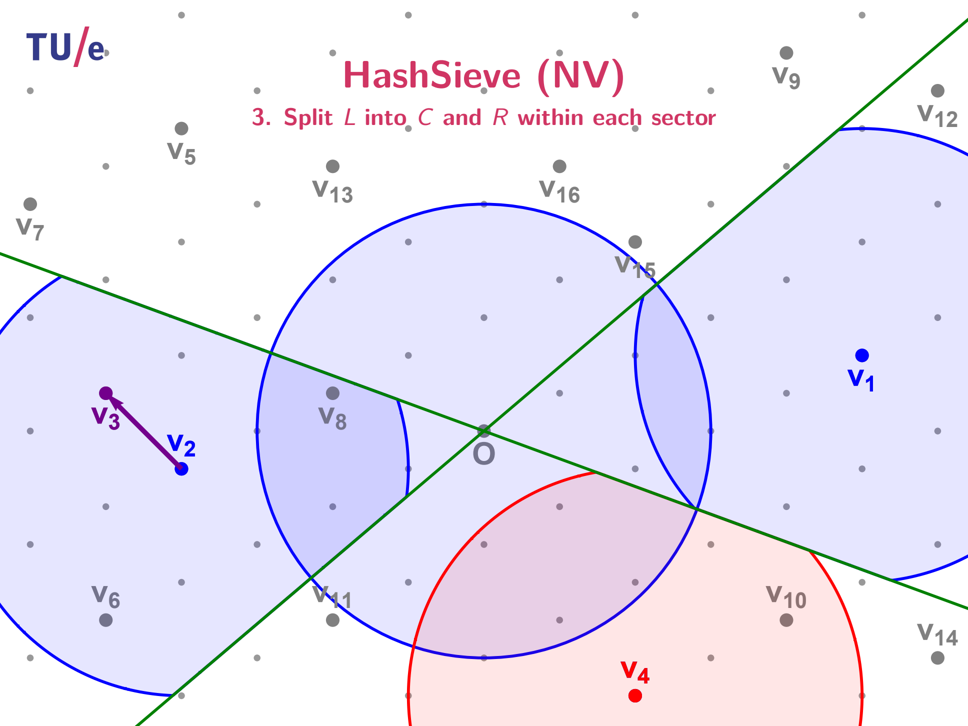

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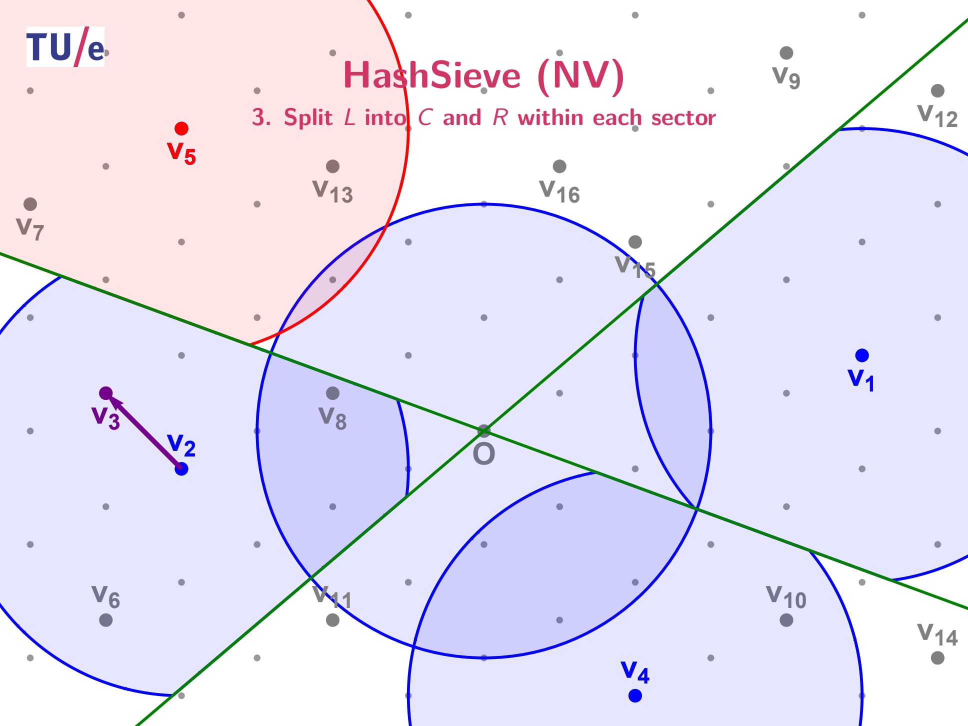

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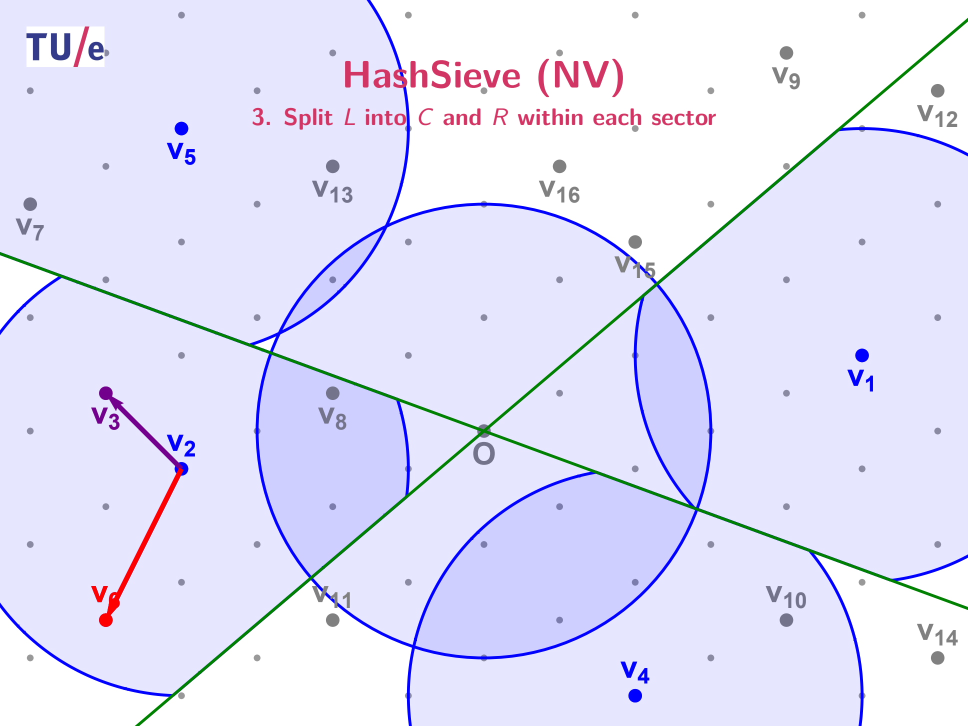

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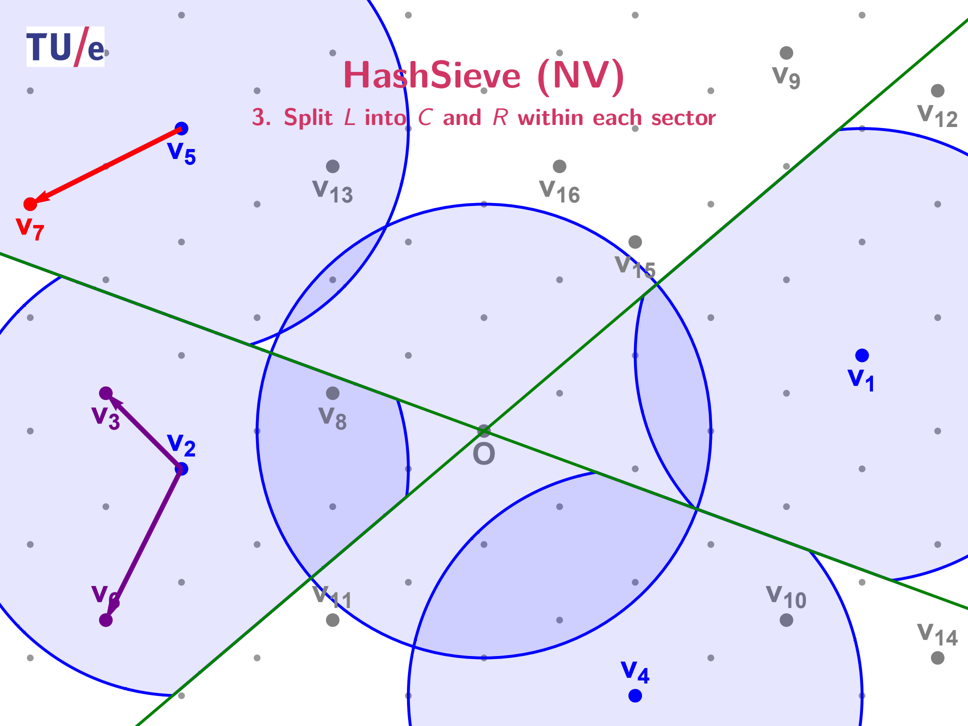

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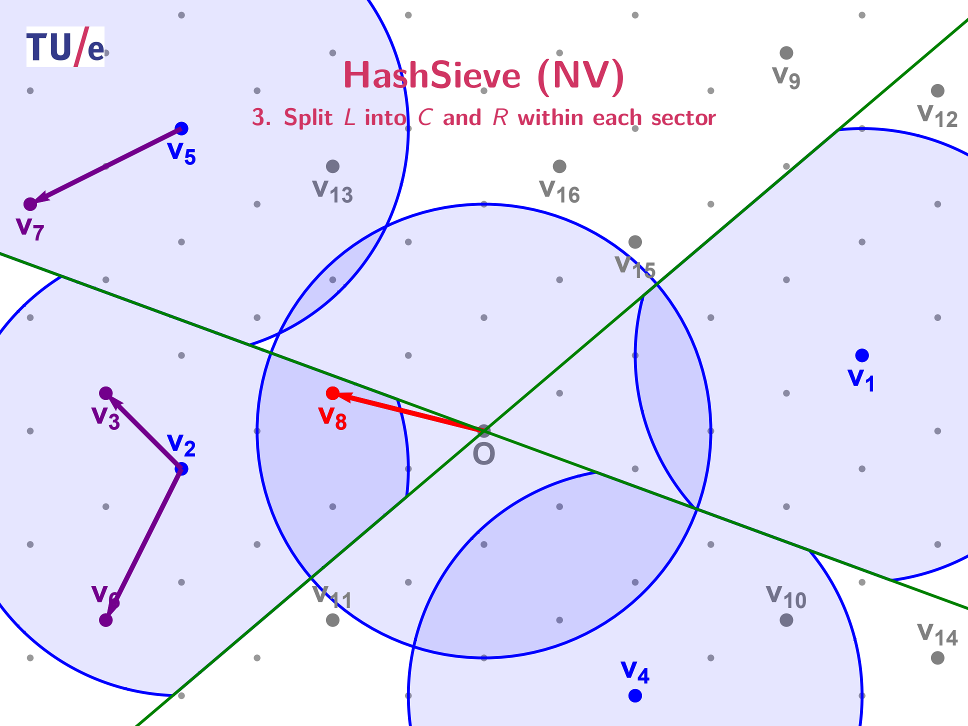

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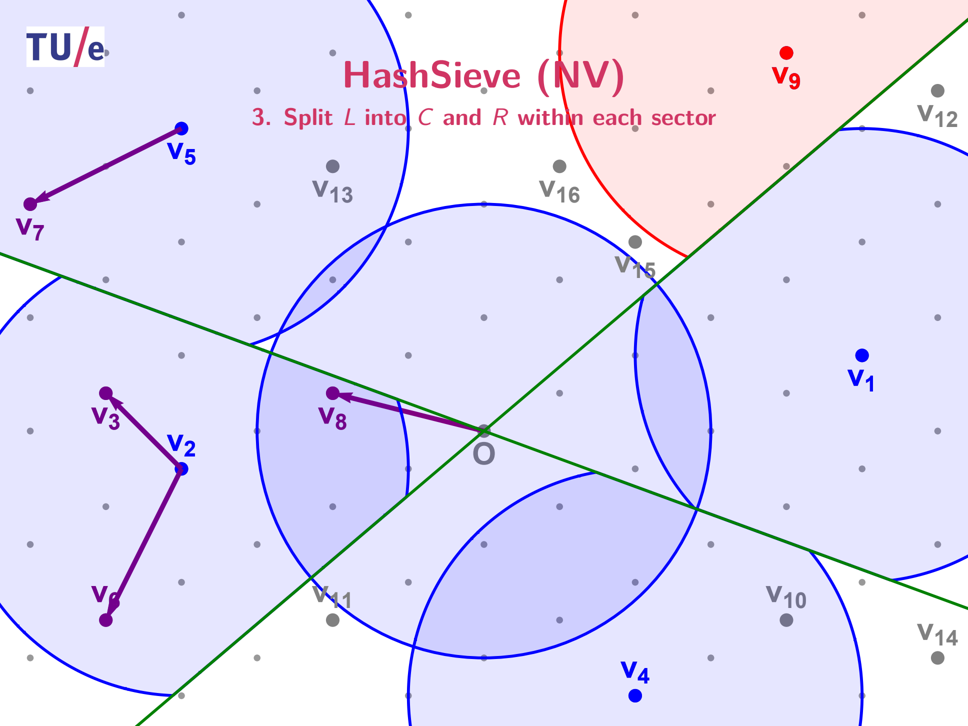

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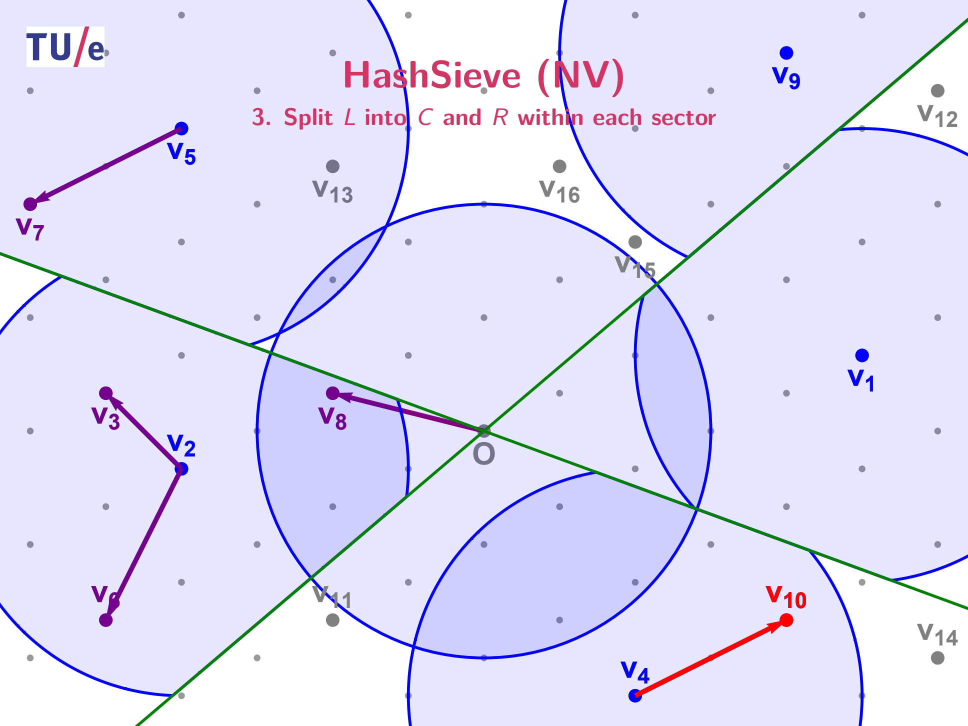

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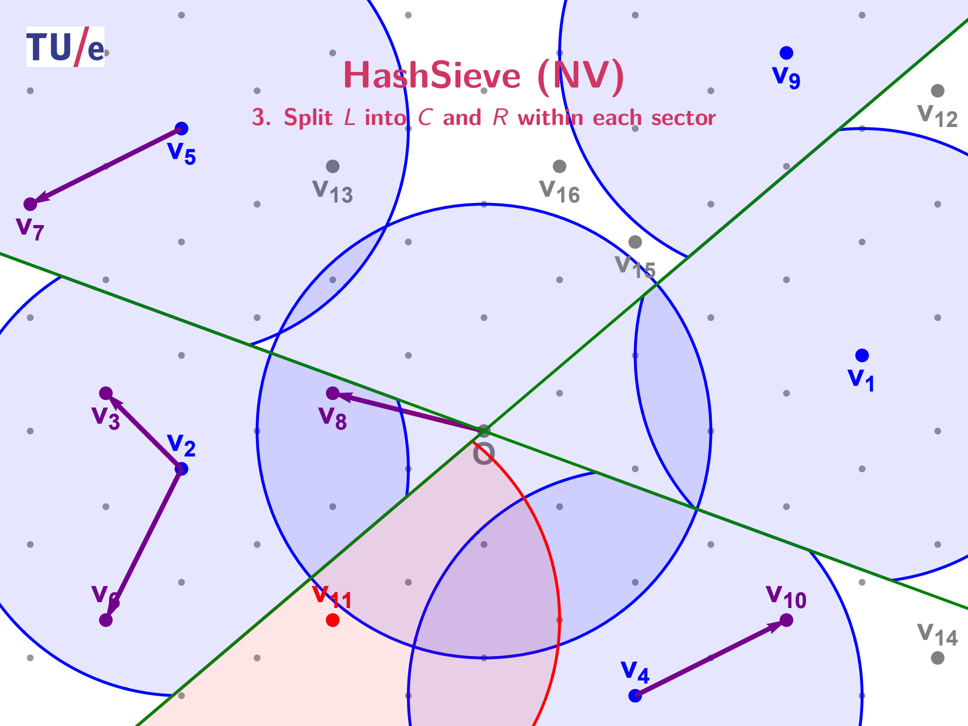

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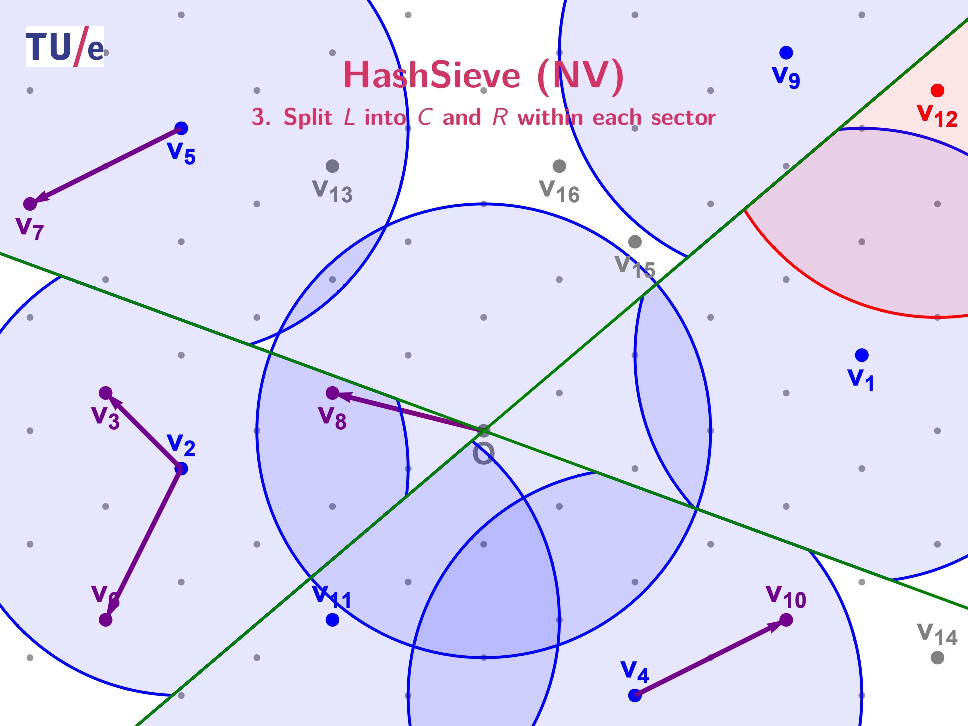

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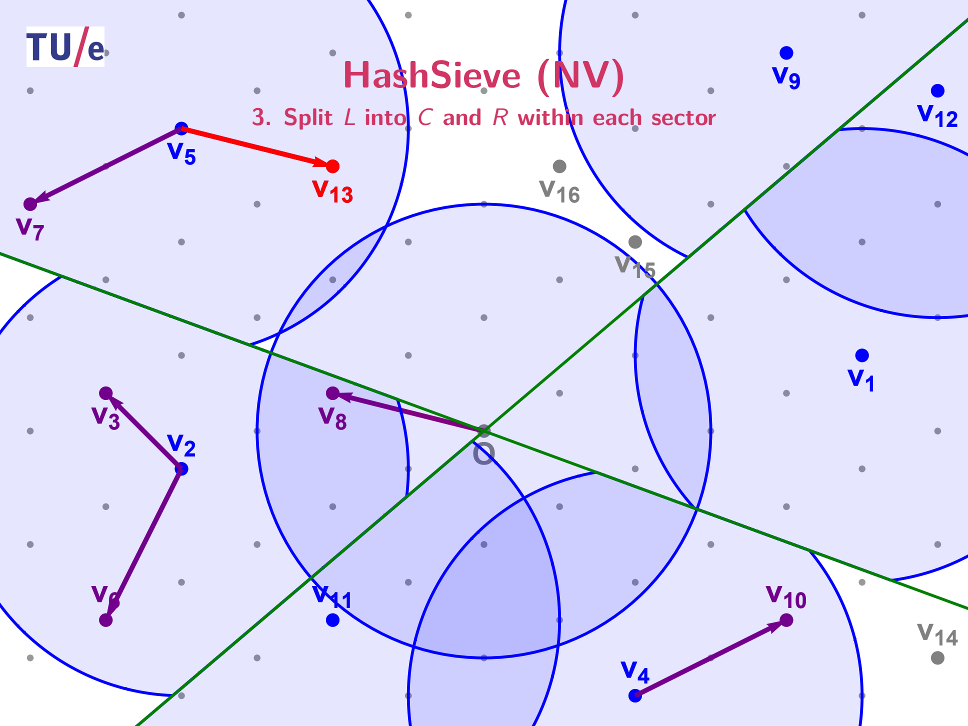

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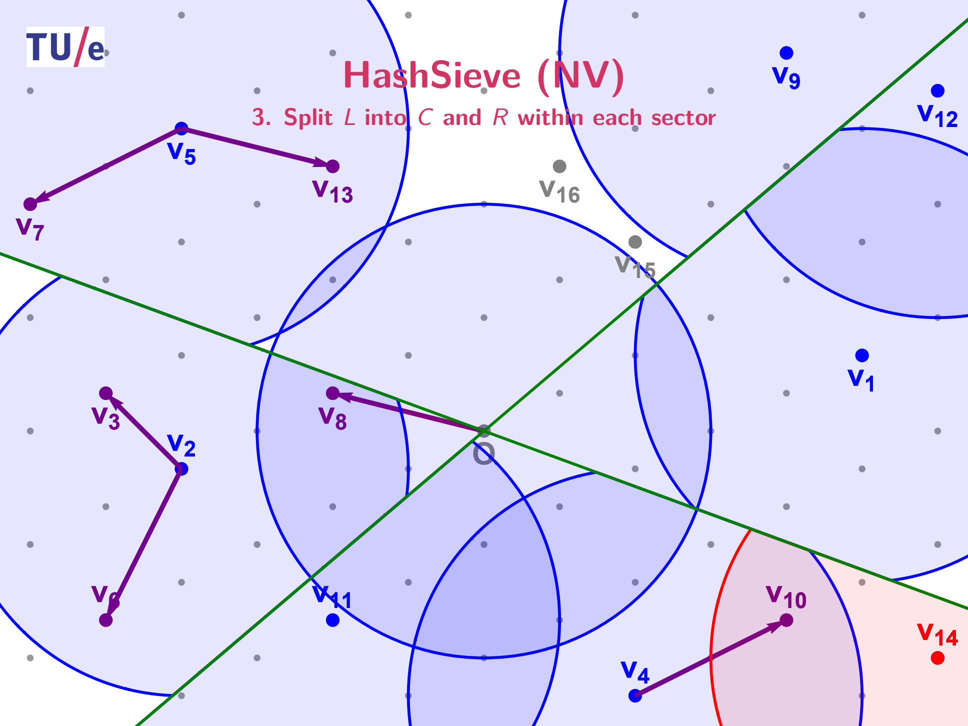

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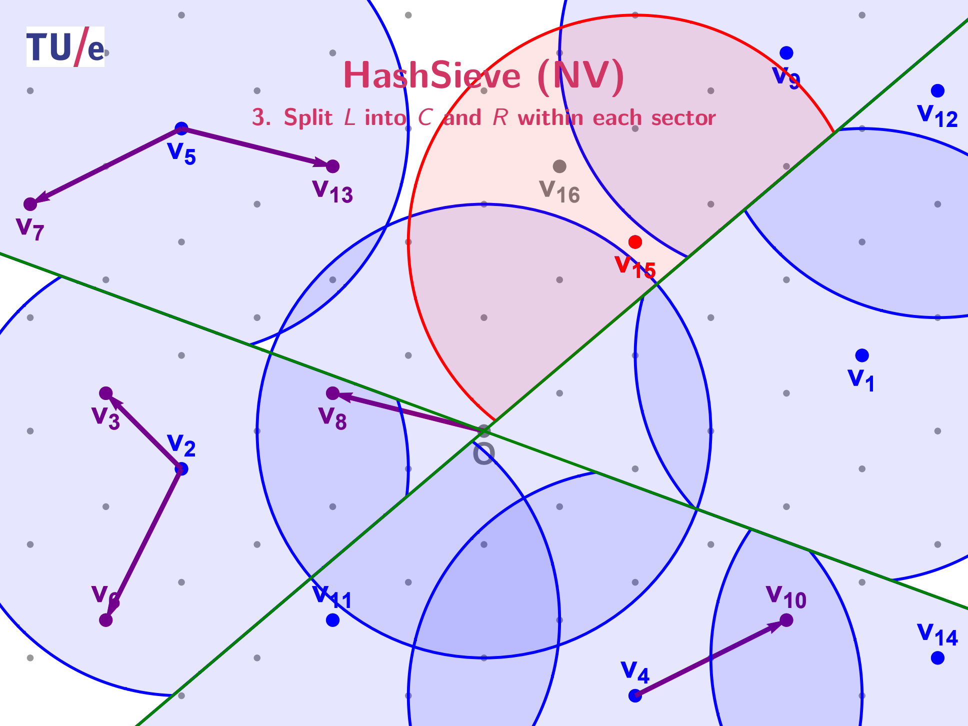

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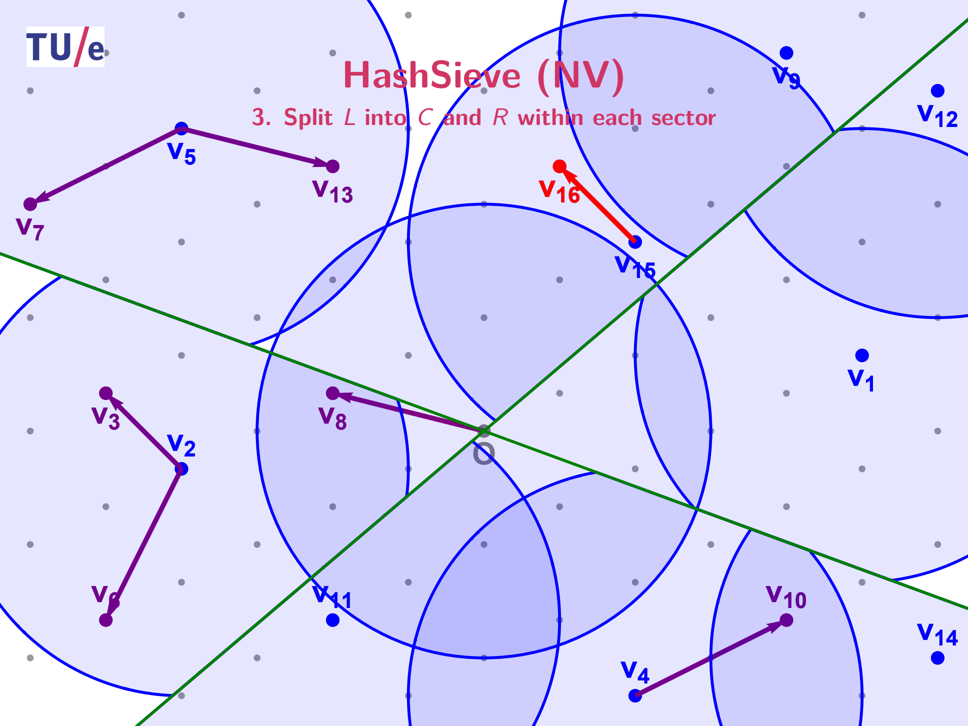

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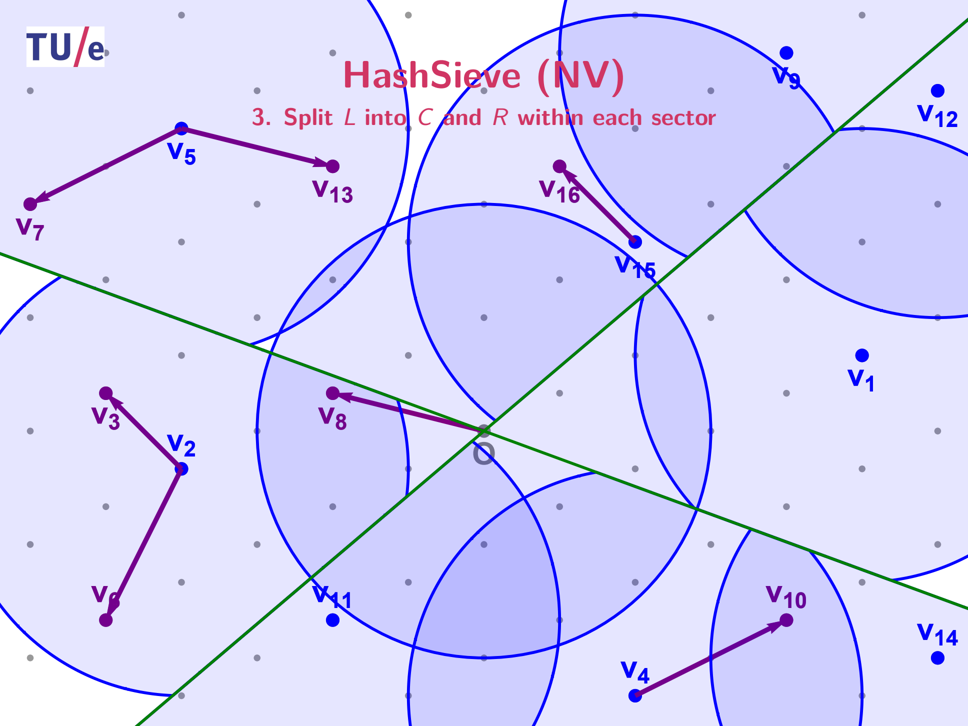

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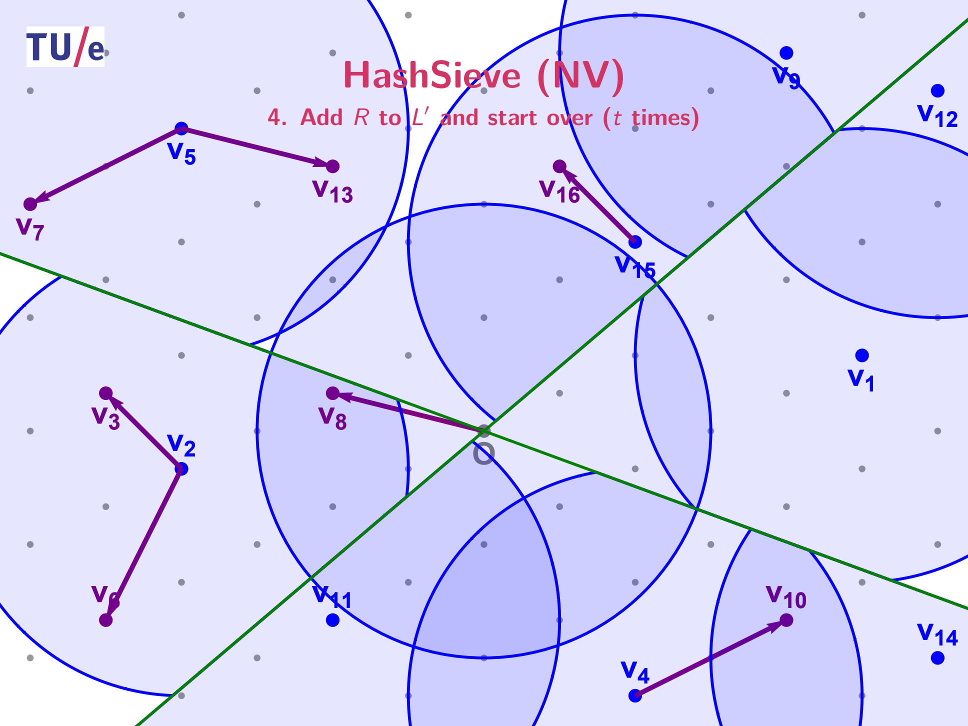

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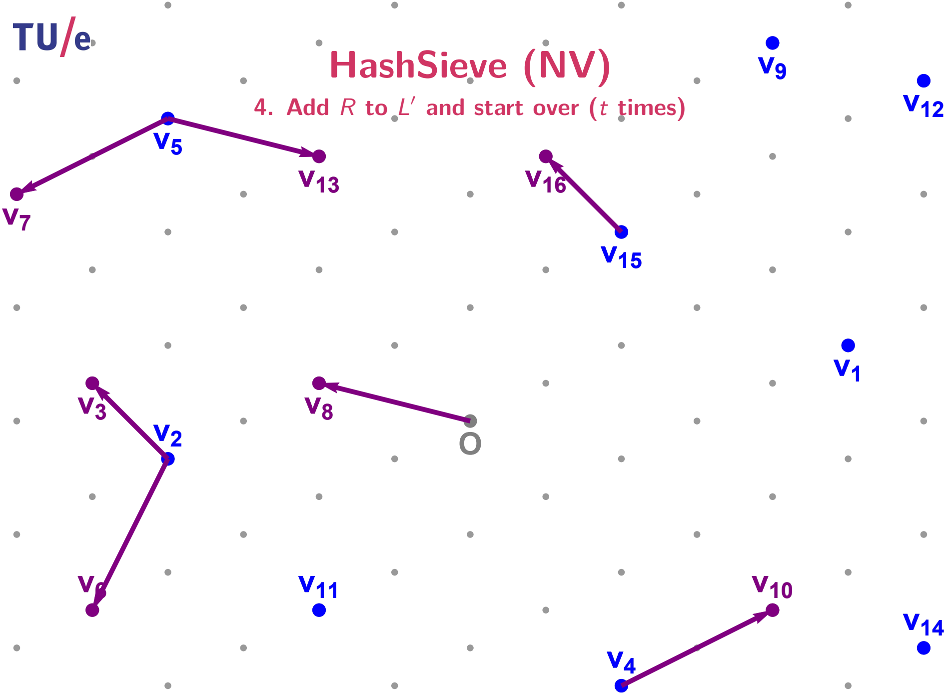

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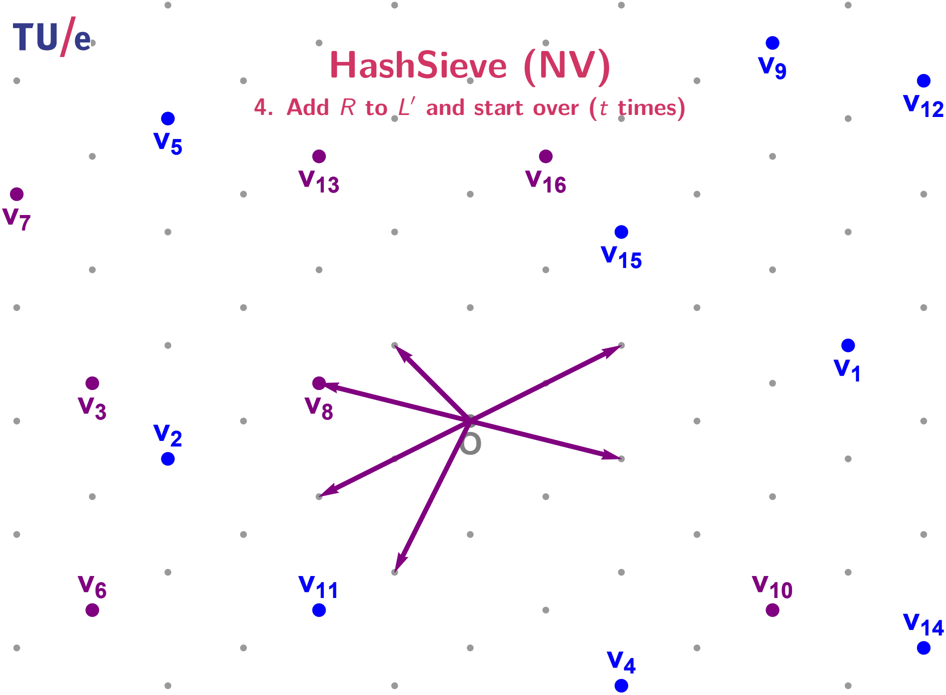

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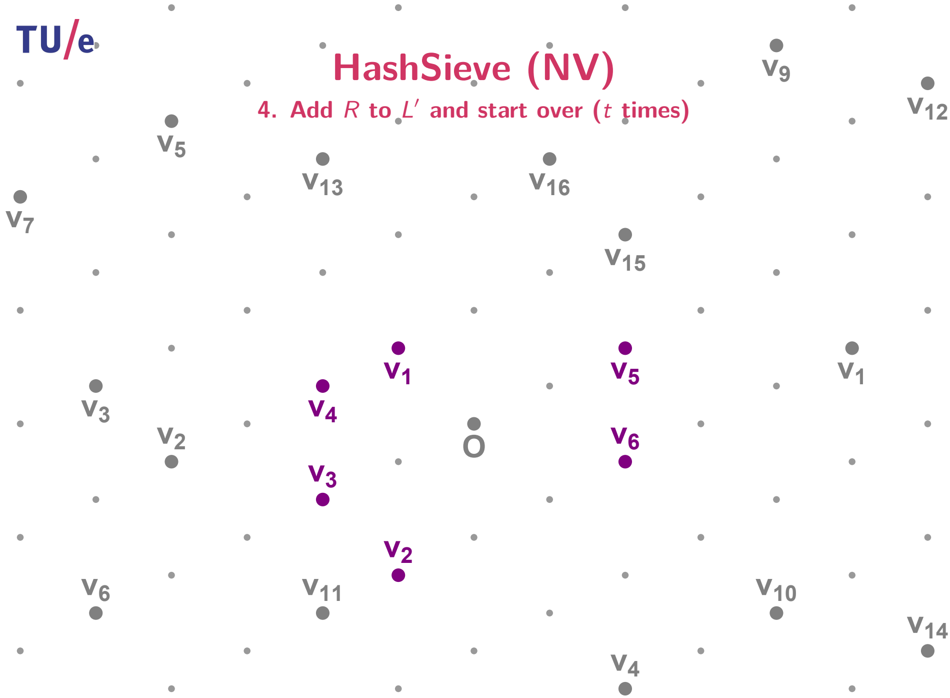

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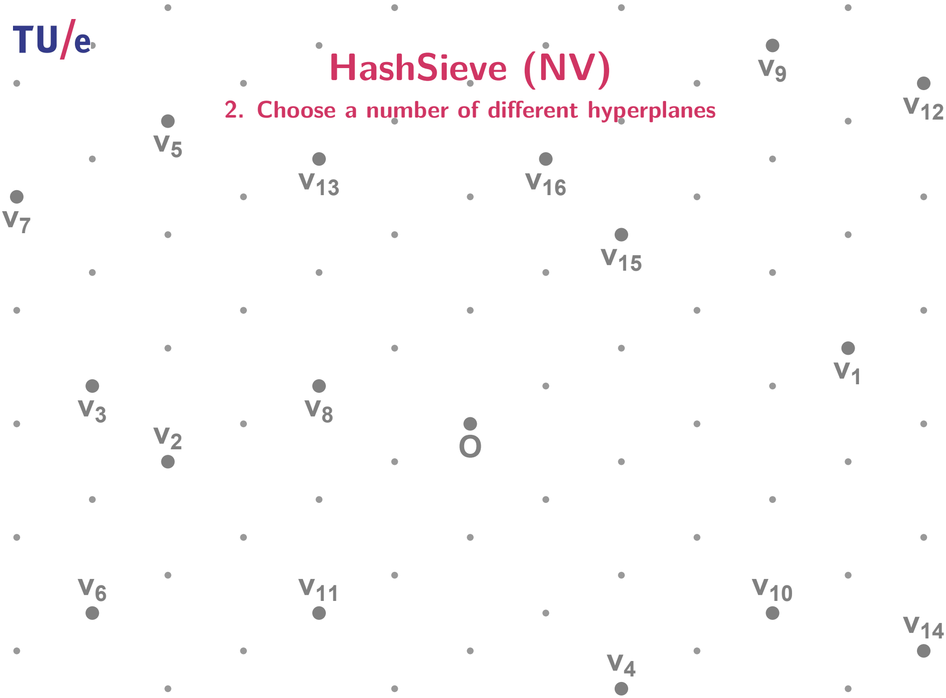

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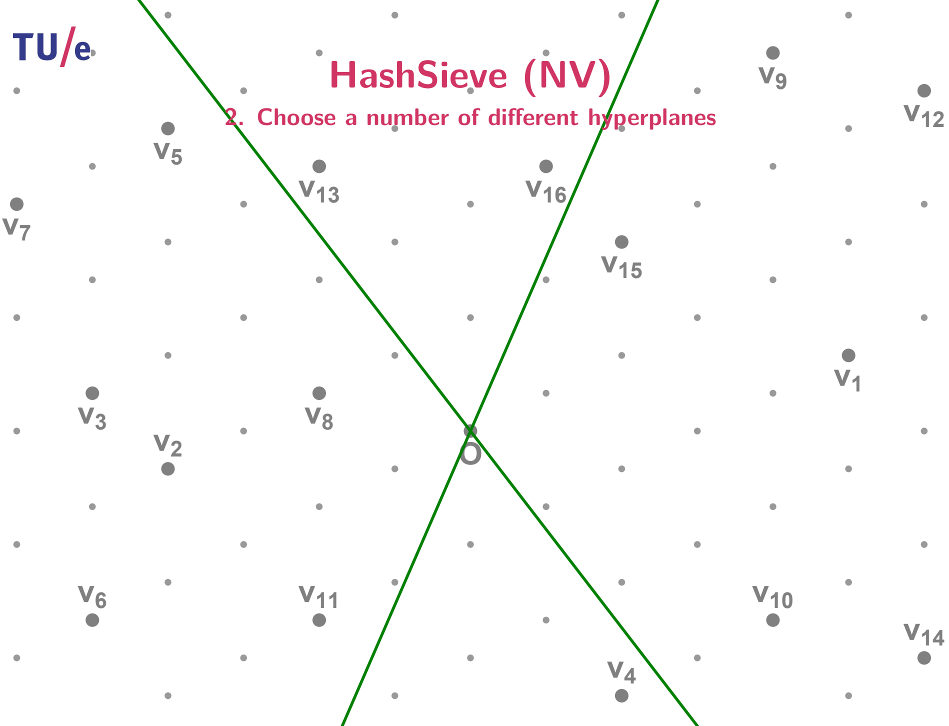

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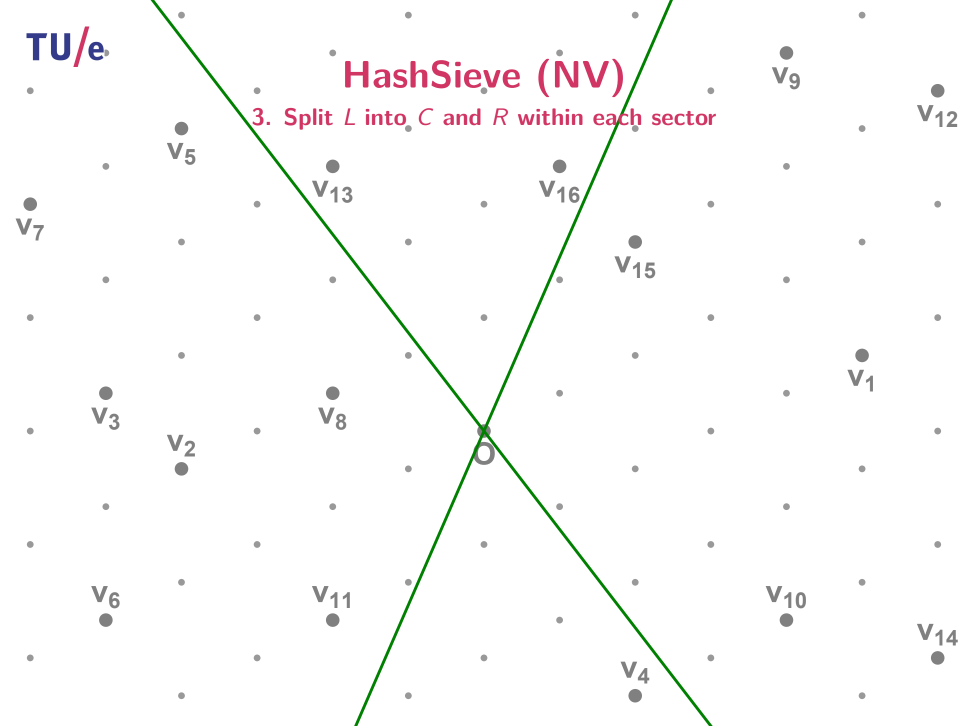

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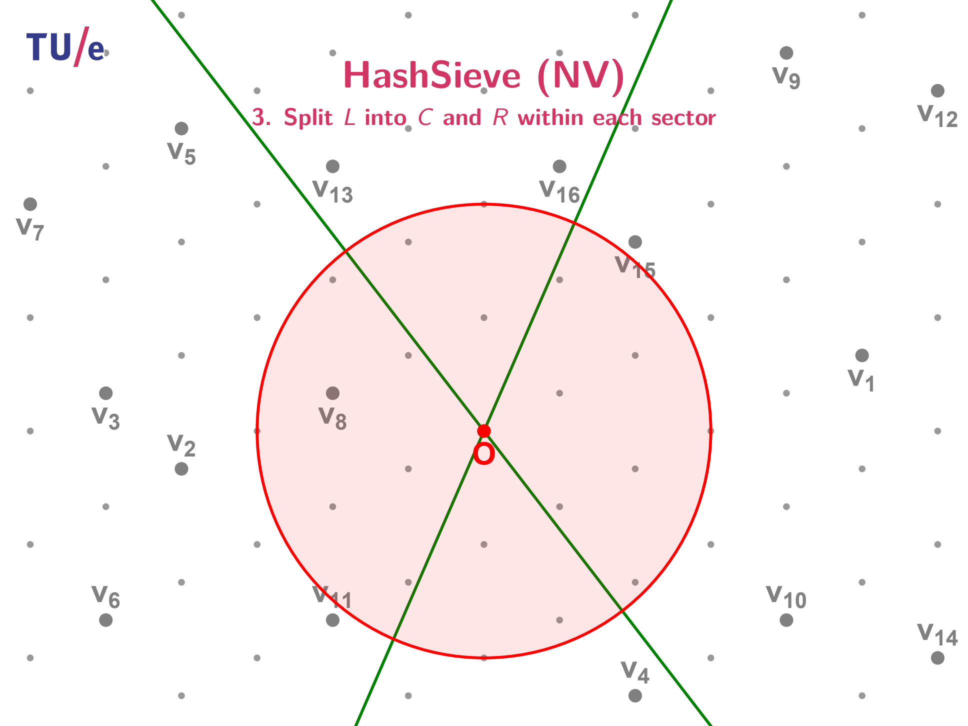

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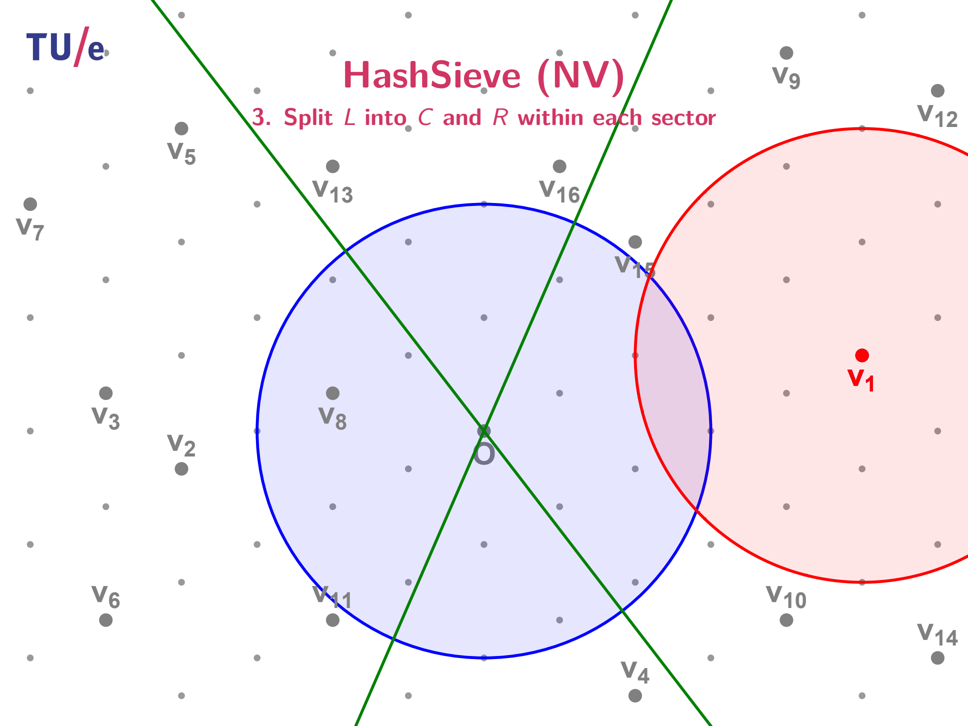

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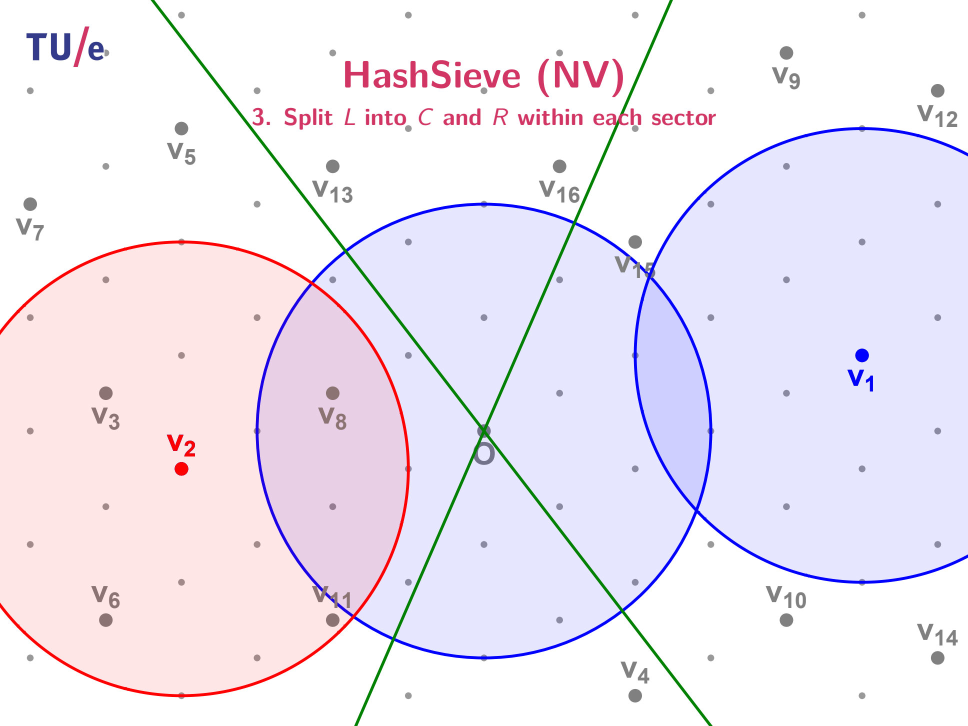

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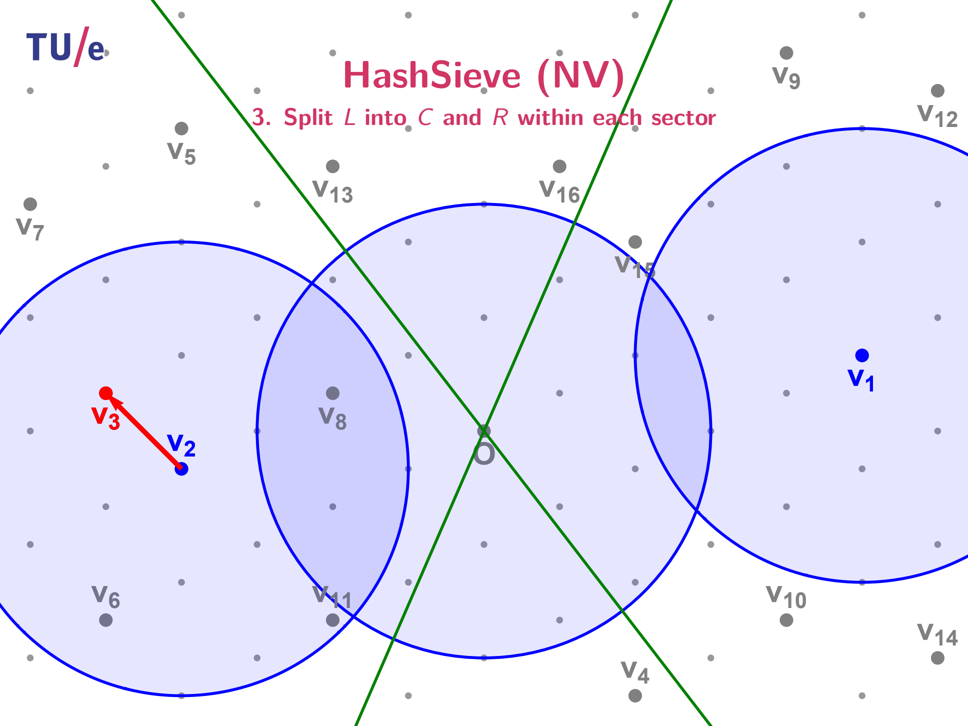

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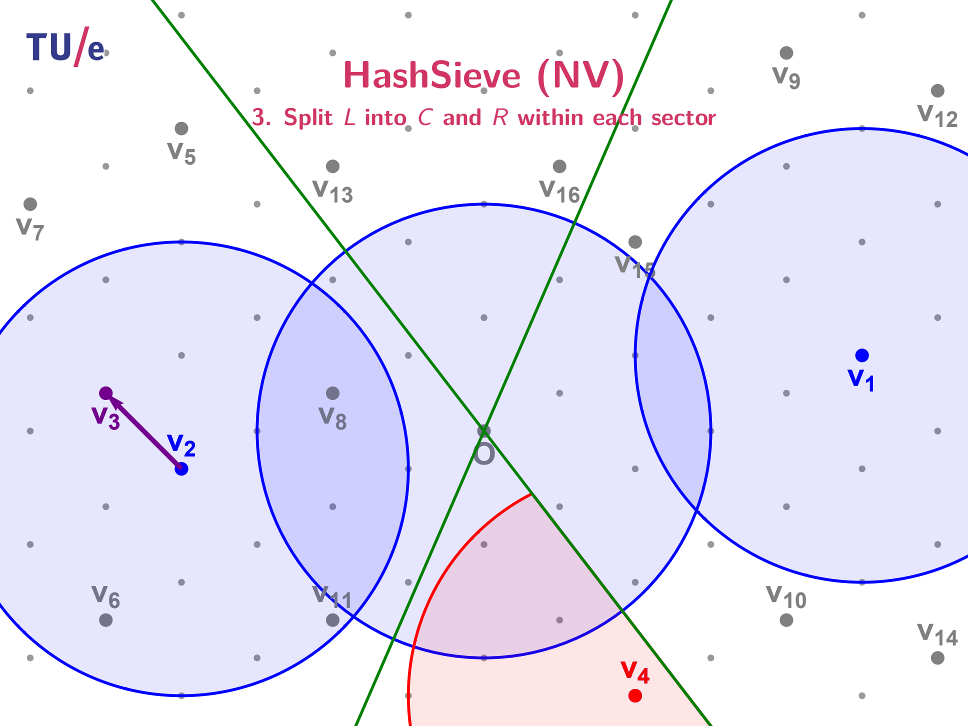

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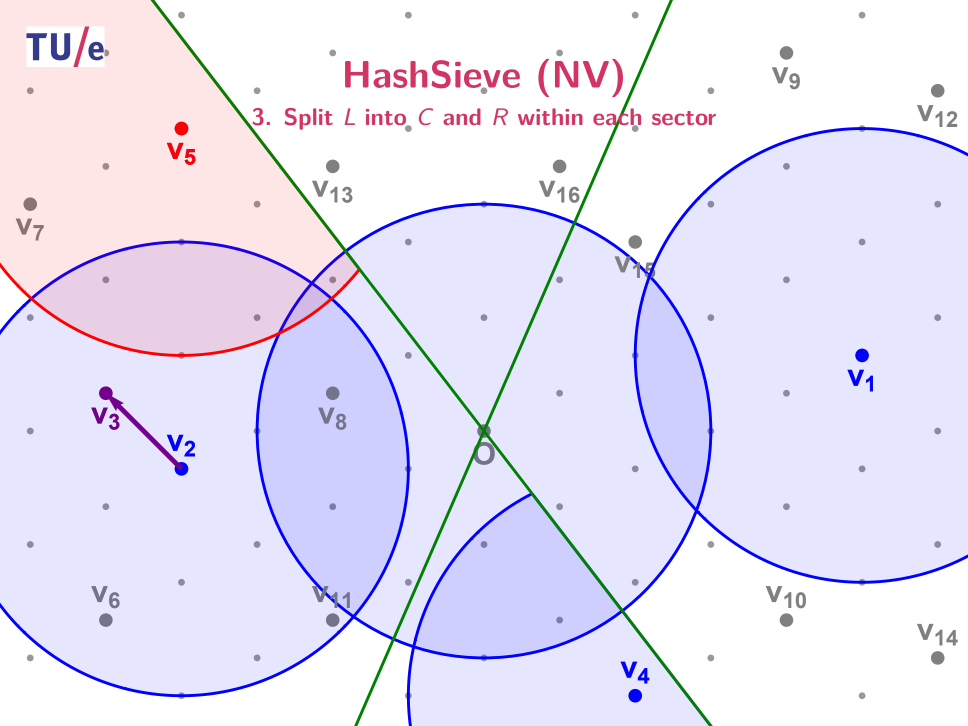

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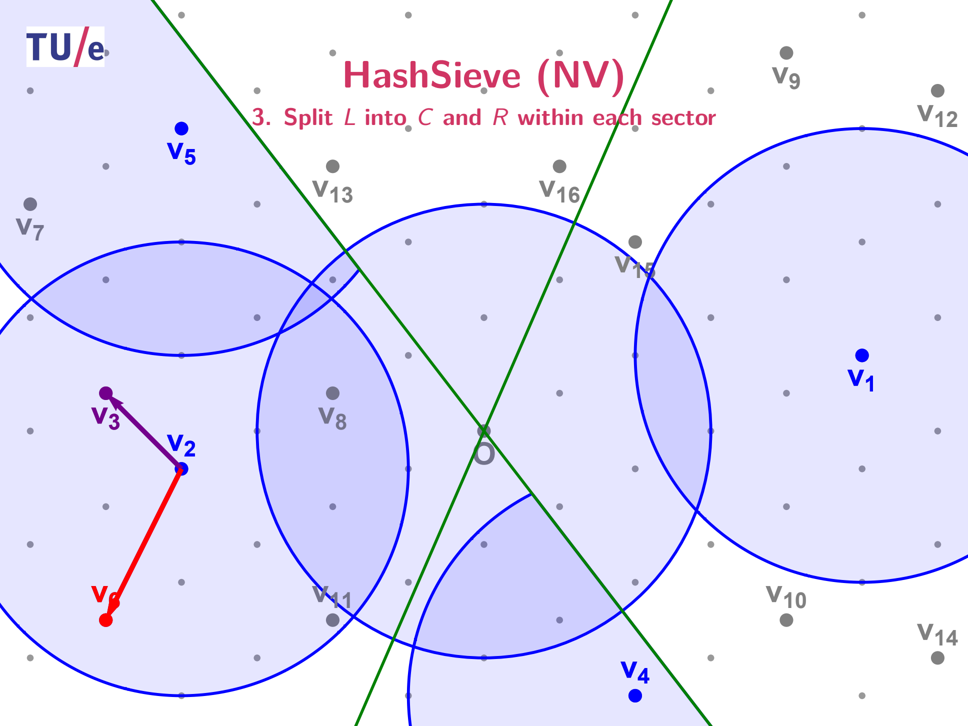

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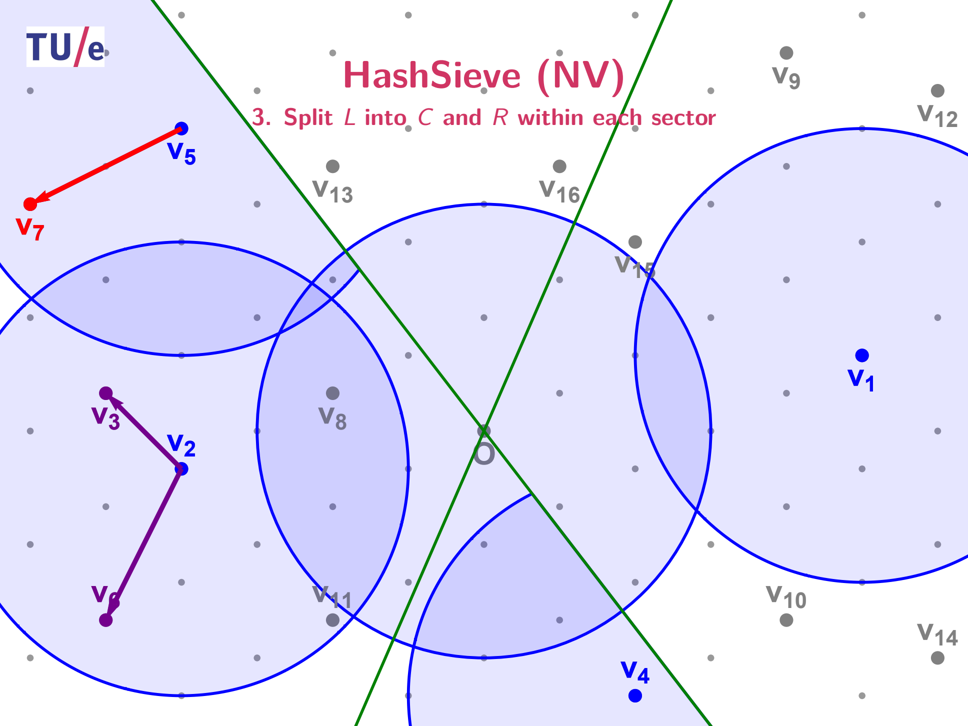

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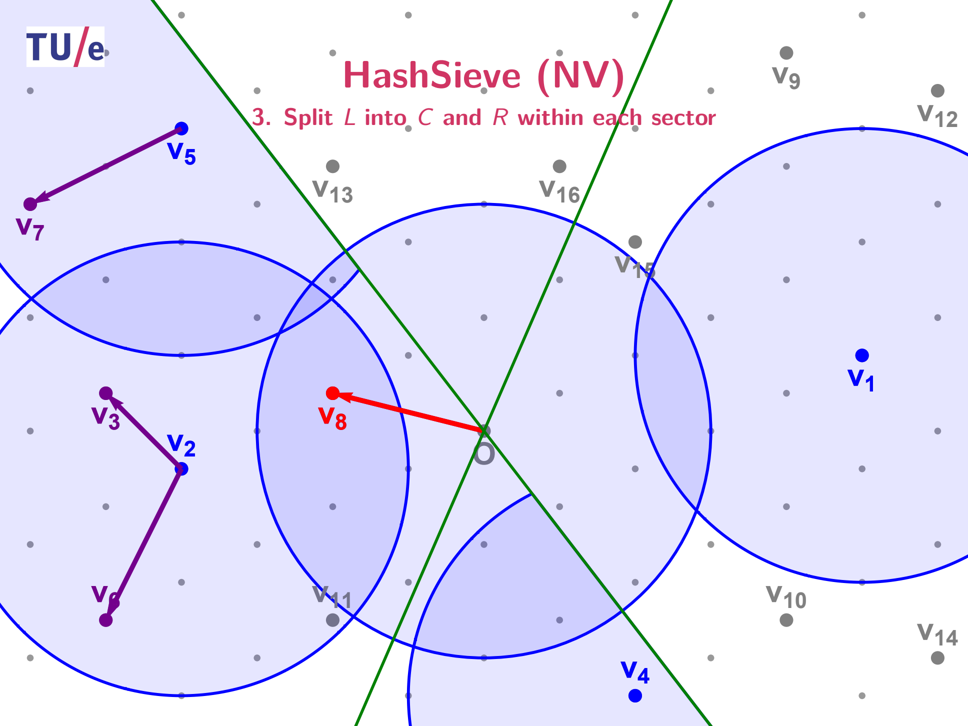

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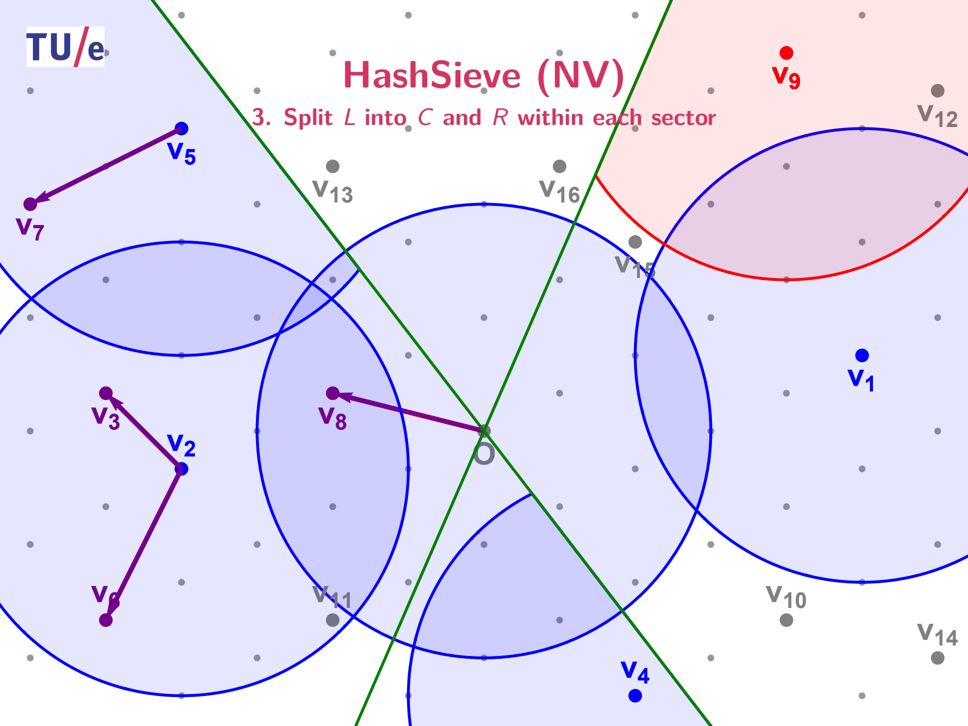

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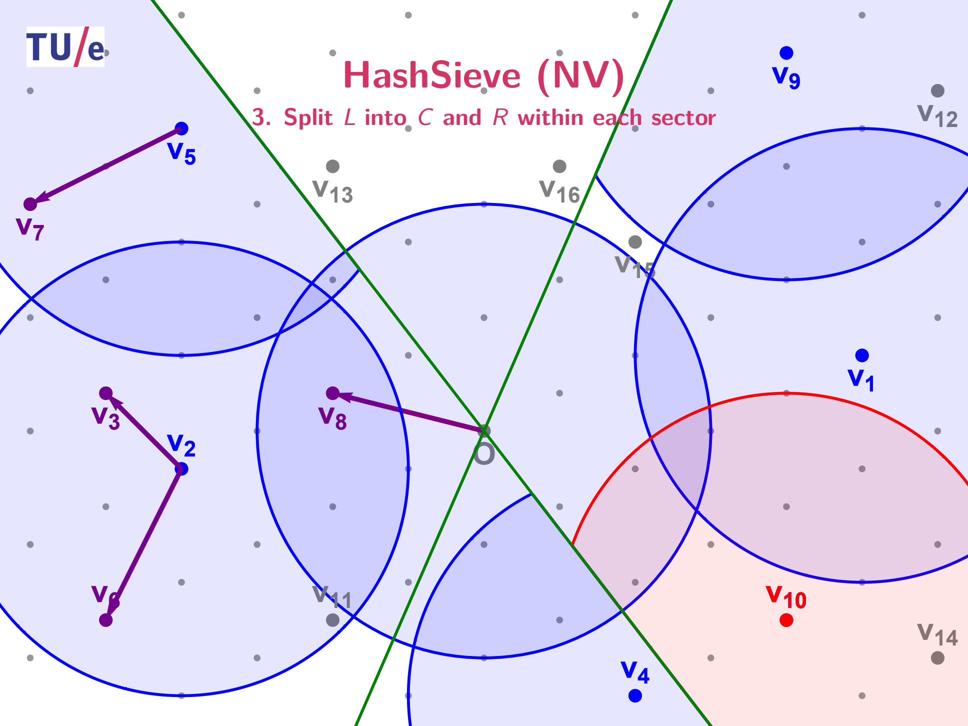

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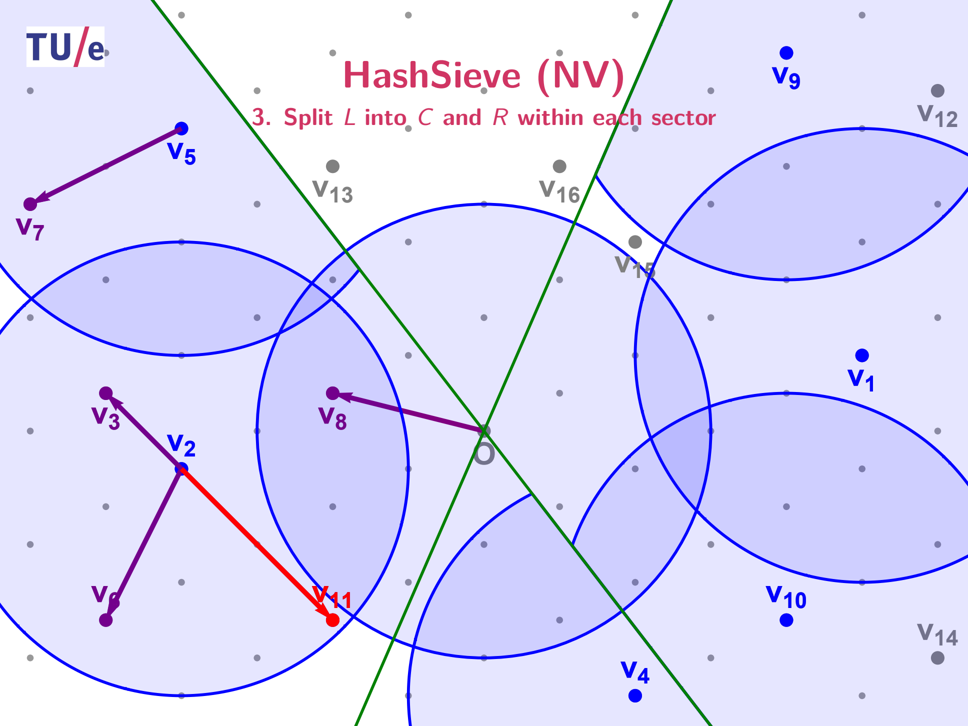

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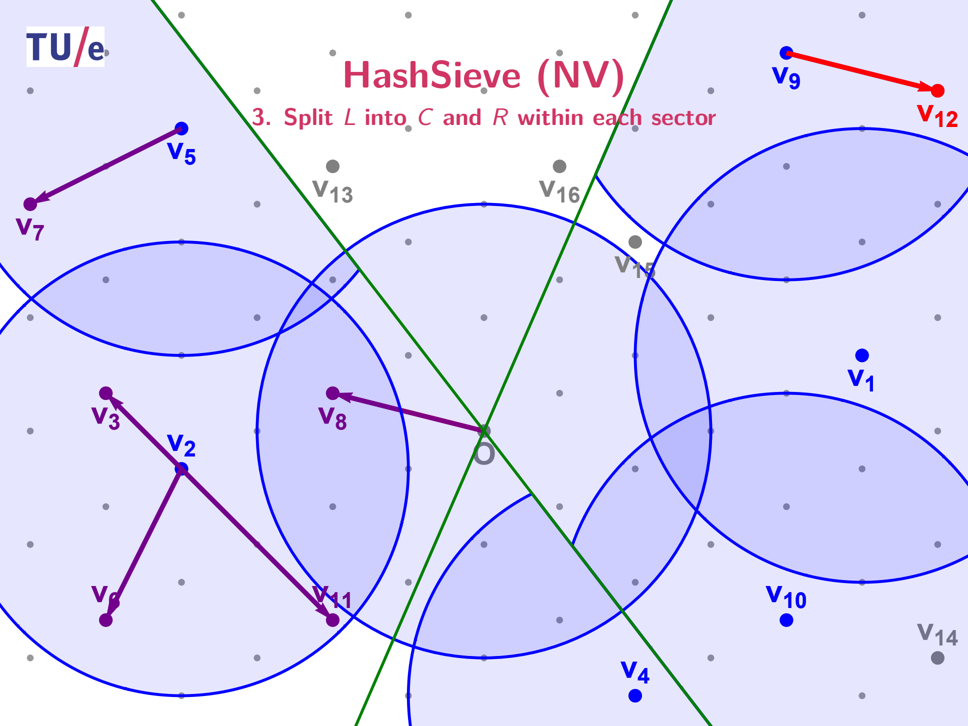

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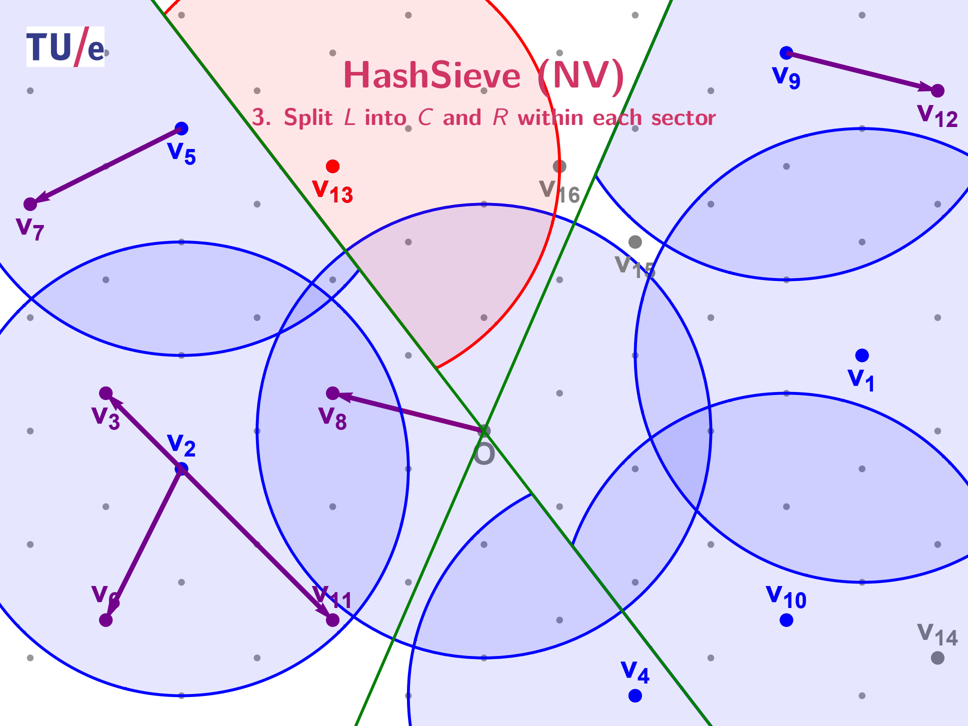

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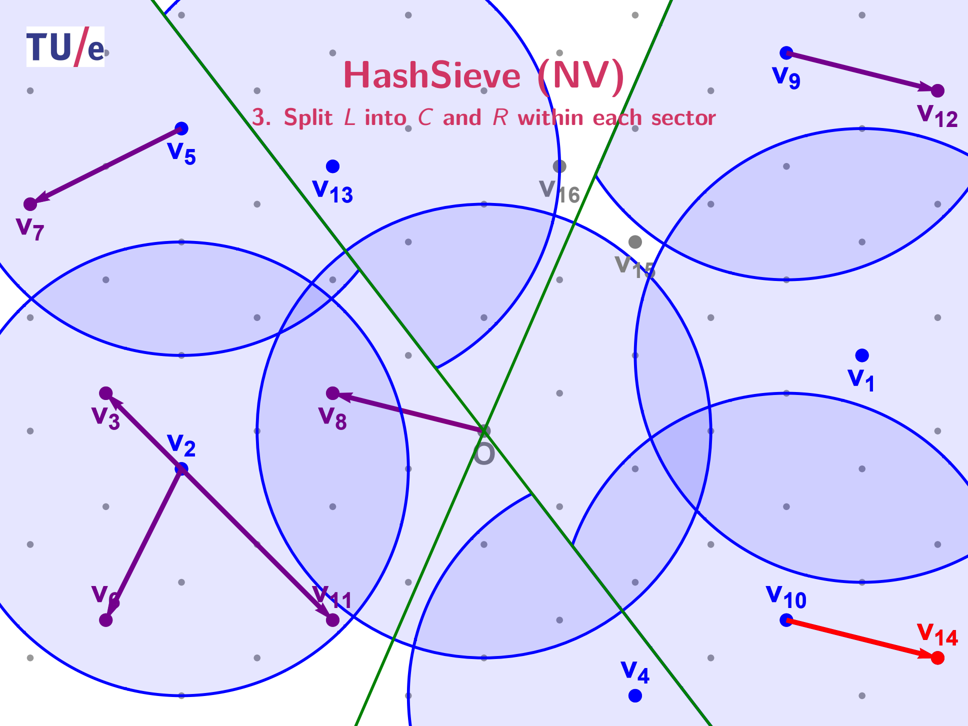

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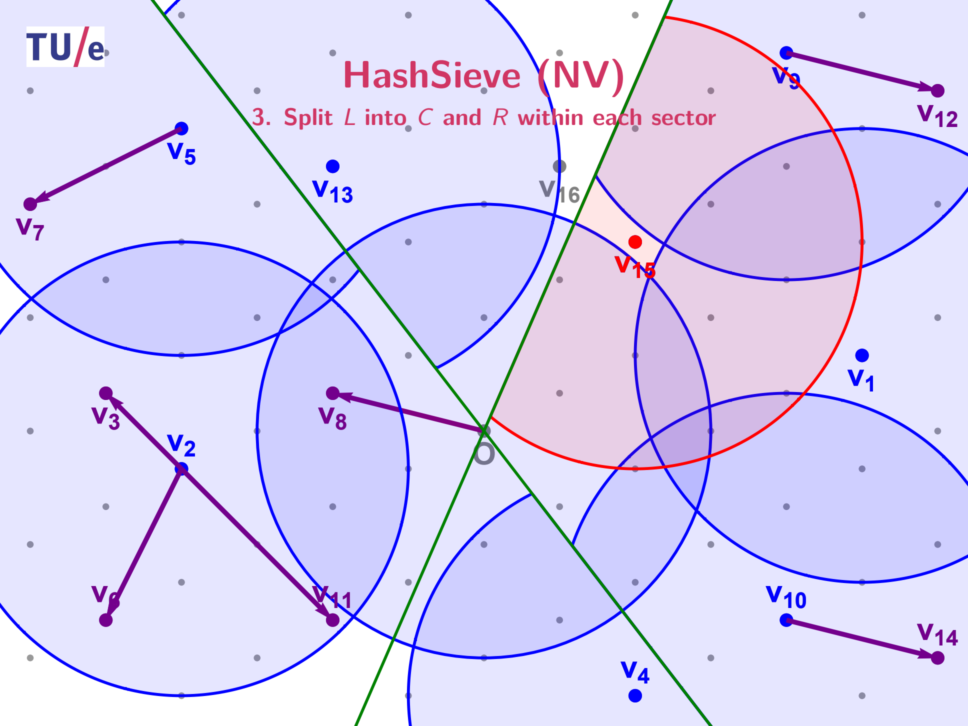

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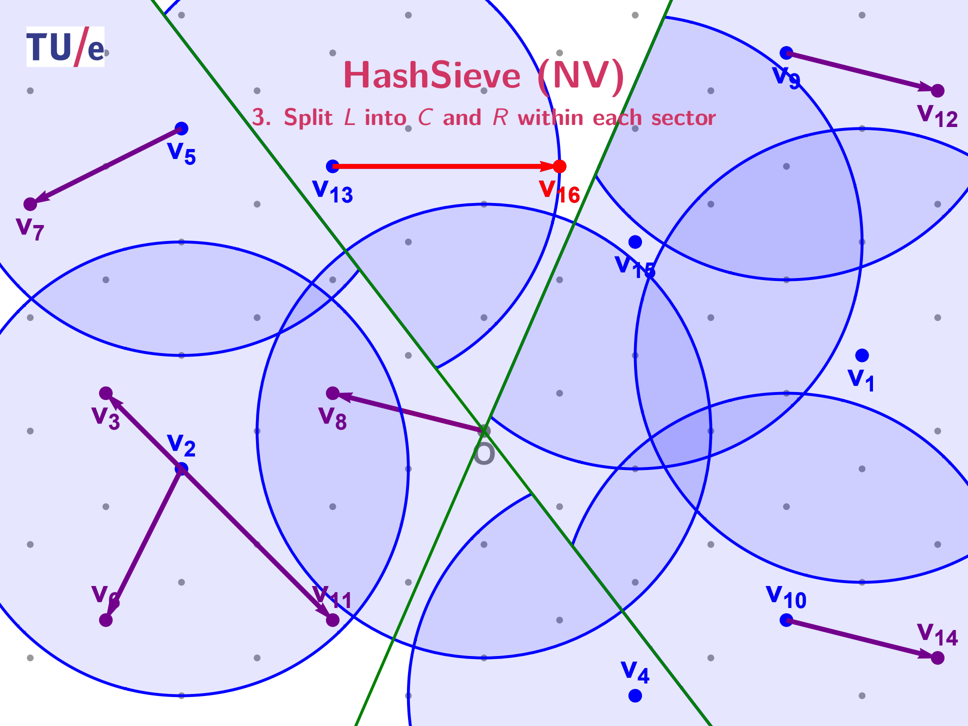

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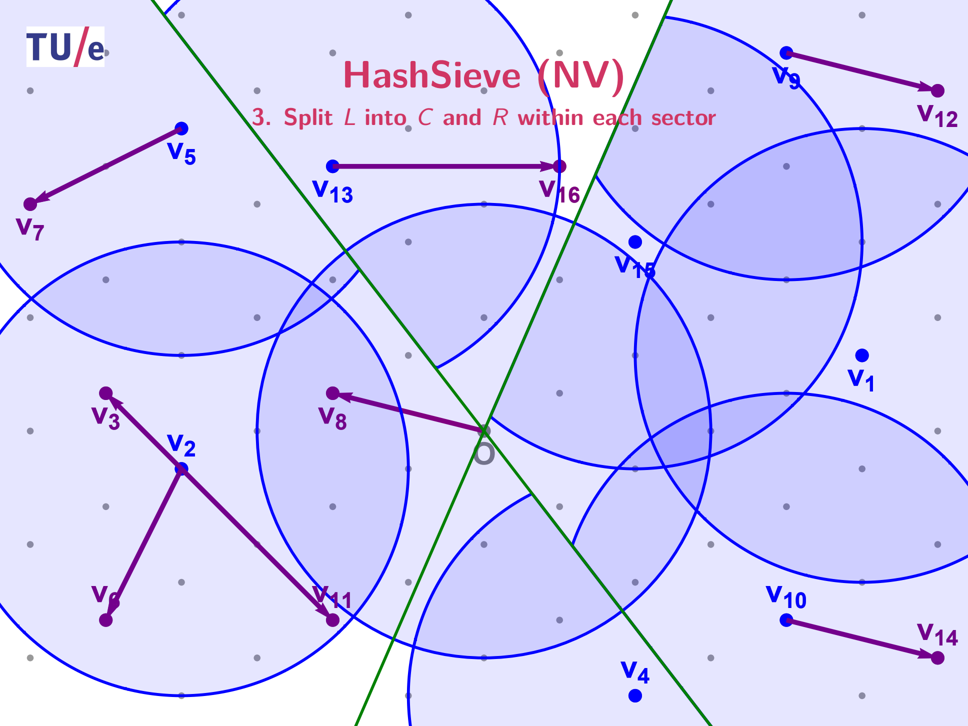

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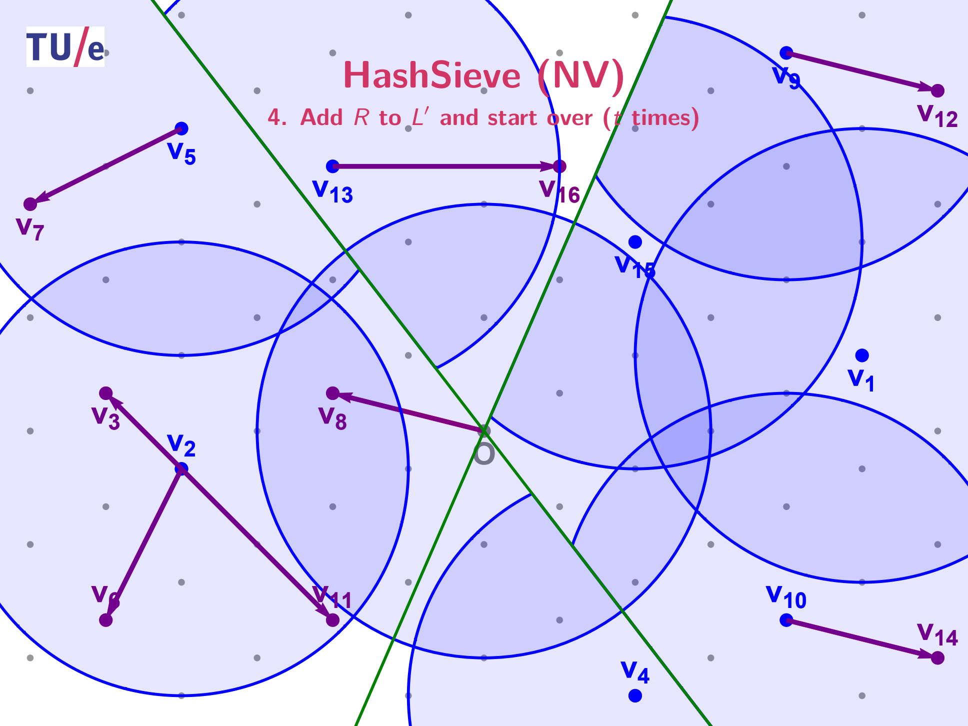

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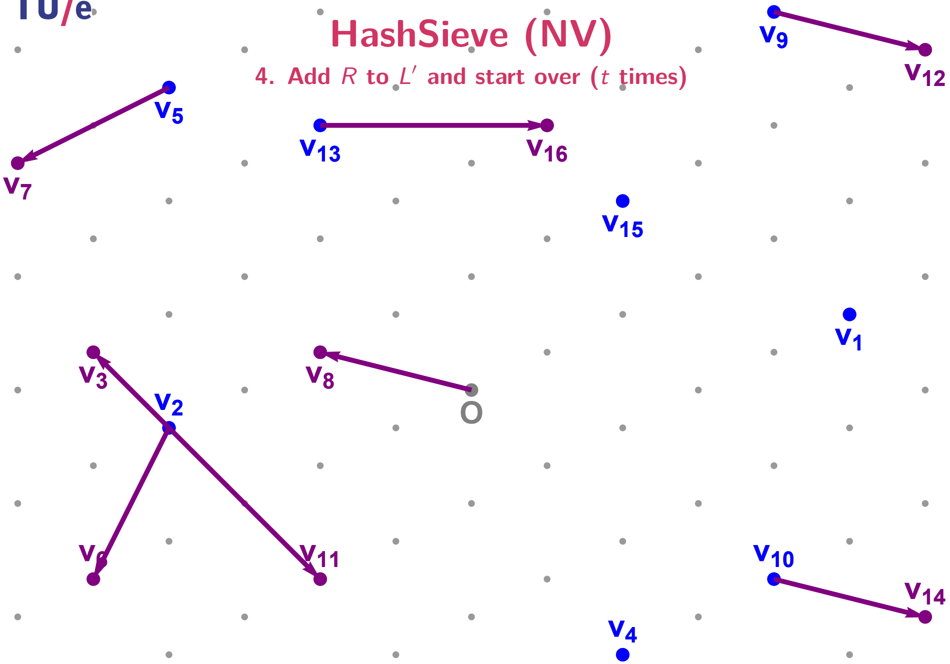

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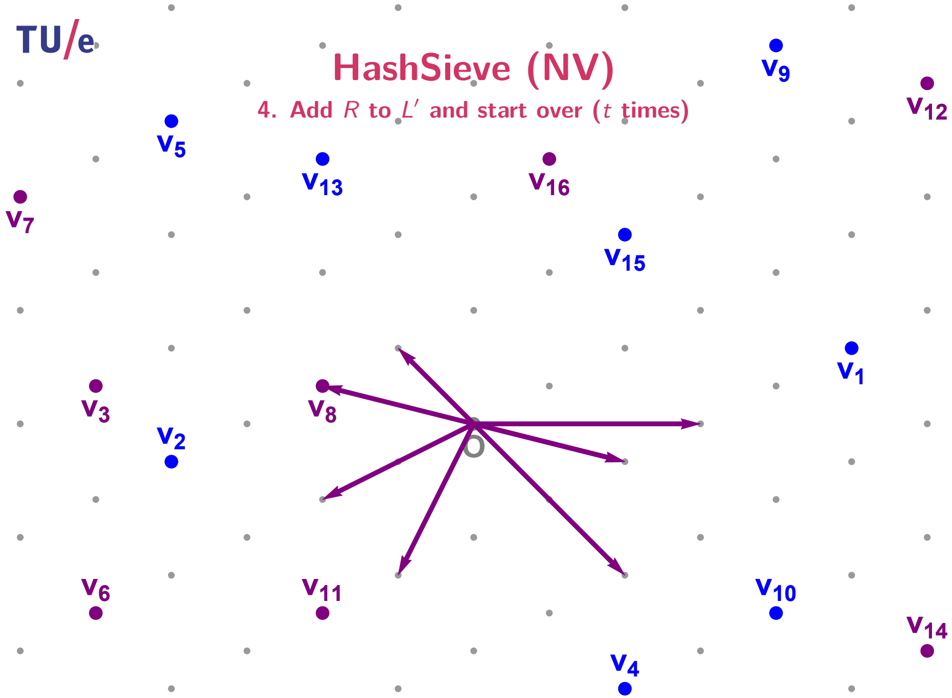

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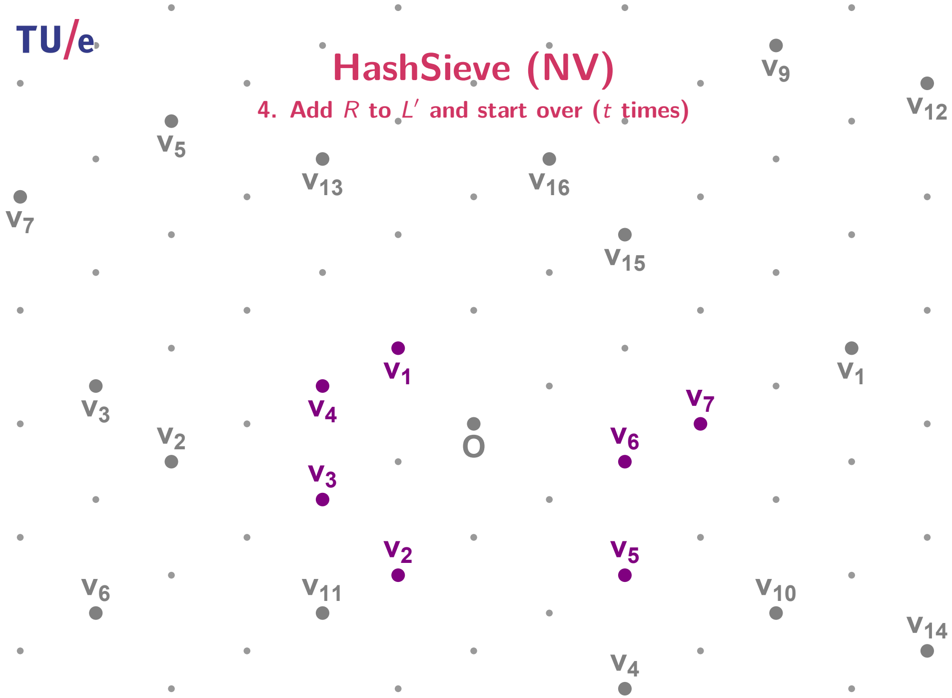

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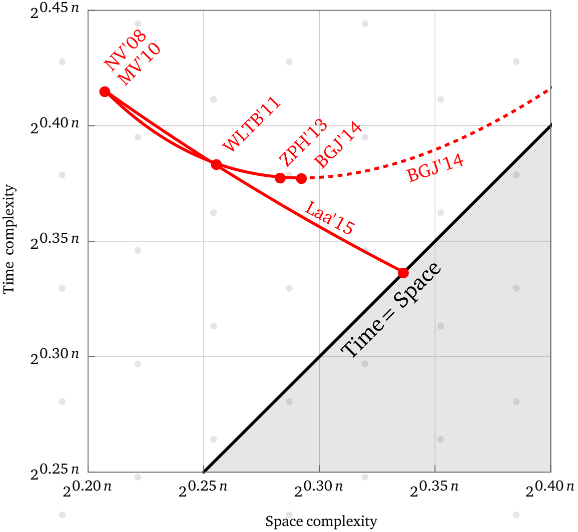
Heuristic

The HashSieve (NV) runs in time $2^{0.34n+o(n)}$ and space $2^{0.21n+o(n)}$.

HashSieve (NV)

Space/time trade-off



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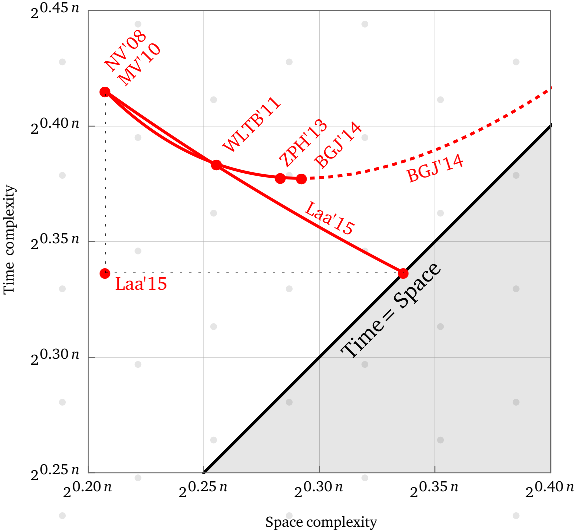
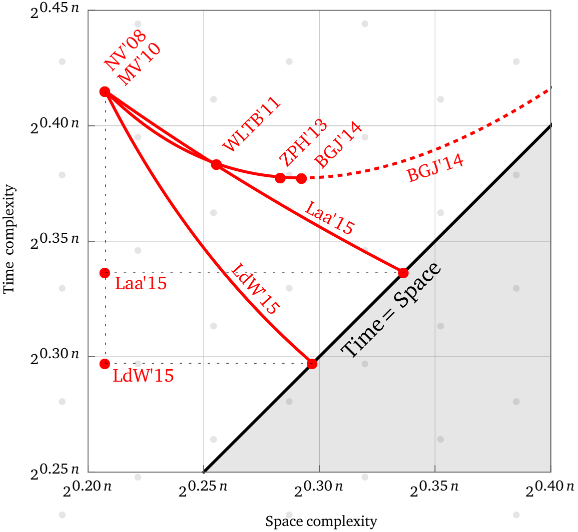


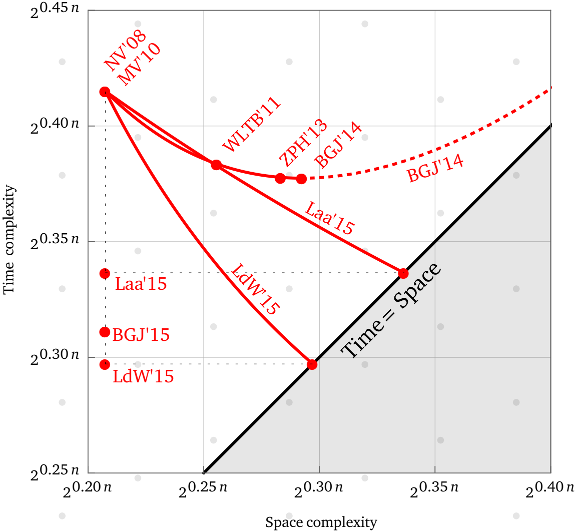
SphereSieve

Space/time trade-off



Another NNS sieve

Space/time trade-off



CrossPolytopeSieve

Main ideas

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TU/e

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- Idea: divide space into regions using regular polytopes
  - Hypercube $\implies$ HashSieve with orthogonal hyperplanes
  - Cross polytope $\implies$ CrossPolytopeSieve
  - Simplex $\implies$ similar to CrossPolytopeSieve

CrossPolytopeSieve

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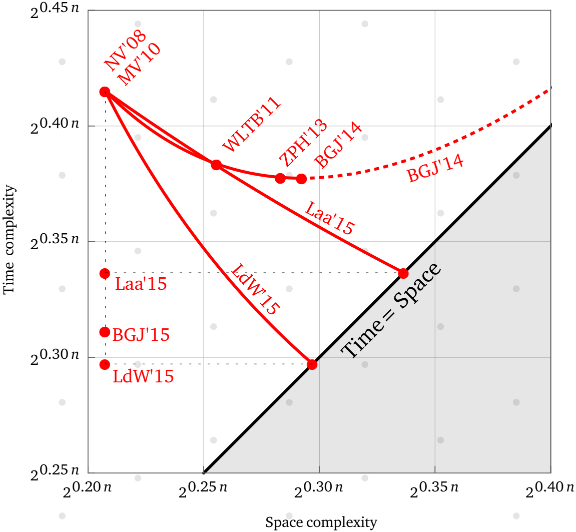

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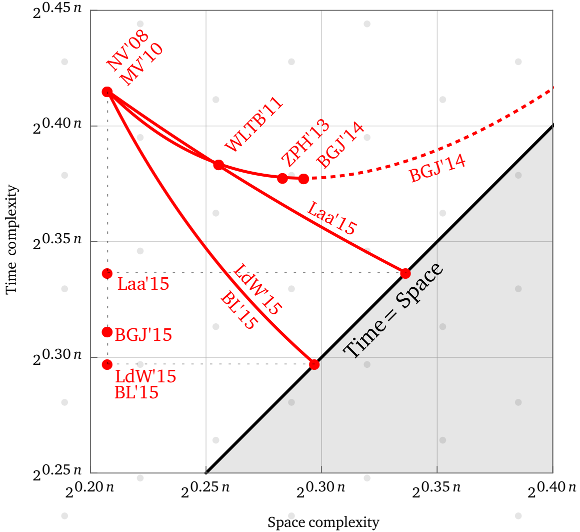

Open problems

- Heuristic sieving
  - Other LSH methods? (other symmetric objects?)
  - SVP challenge records?
  - Crossover point with enumeration?
  - Ideal lattice sieving speedups?
- Provable sieving
  - Better exponent with LSH?
  - Use idea of partitioning the space?
- Other SVP algorithms
  - Voronoi cell with LSH?
  - Solve CVPP faster with LSH?
  - Combine sieving with enumeration?

Questions