Sieving for shortest vectors in lattices using angular locality-sensitive hashing

Thijs Laarhoven

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Outline

Lattices

Enumeration algorithms
- Fincke-Pohst enumeration
- Kannan enumeration
- Pruning the enumeration tree

The Voronoi cell algorithm

Sieving algorithms
- Nguyen-Vidick sieve
- Multiple levels
- GaussSieve

Sieving using angular locality-sensitive hashing
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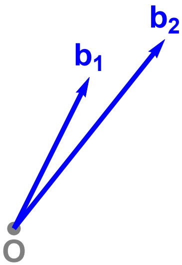

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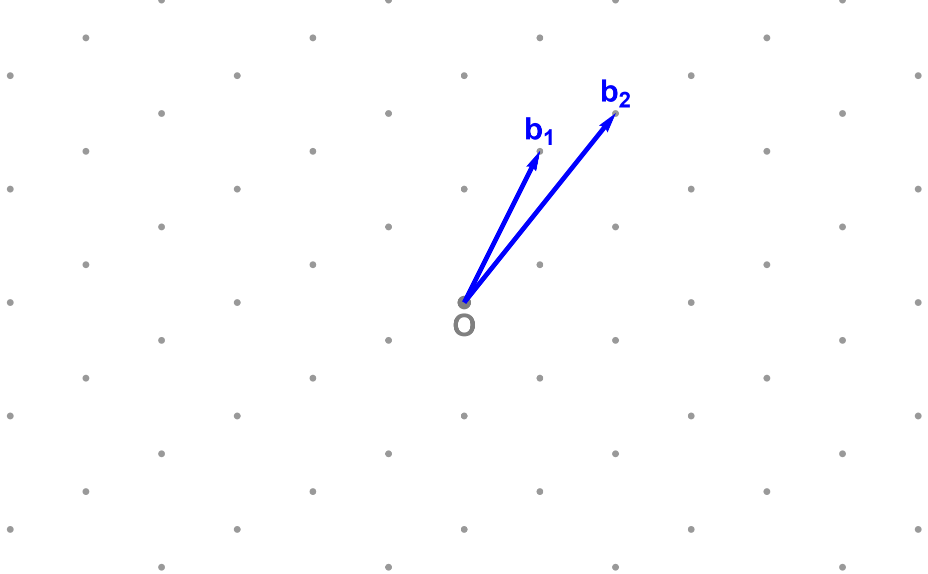

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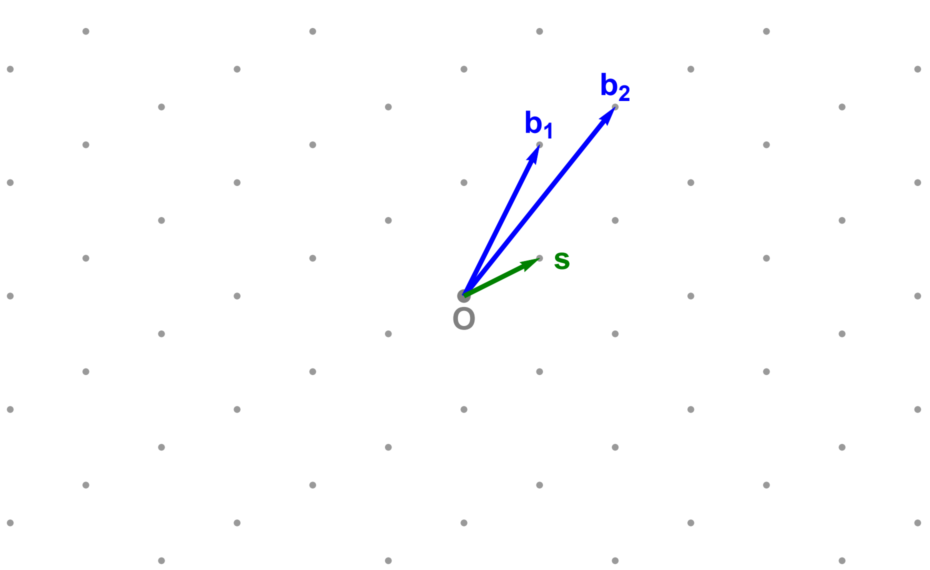
Shortest Vector Problem (SVP)



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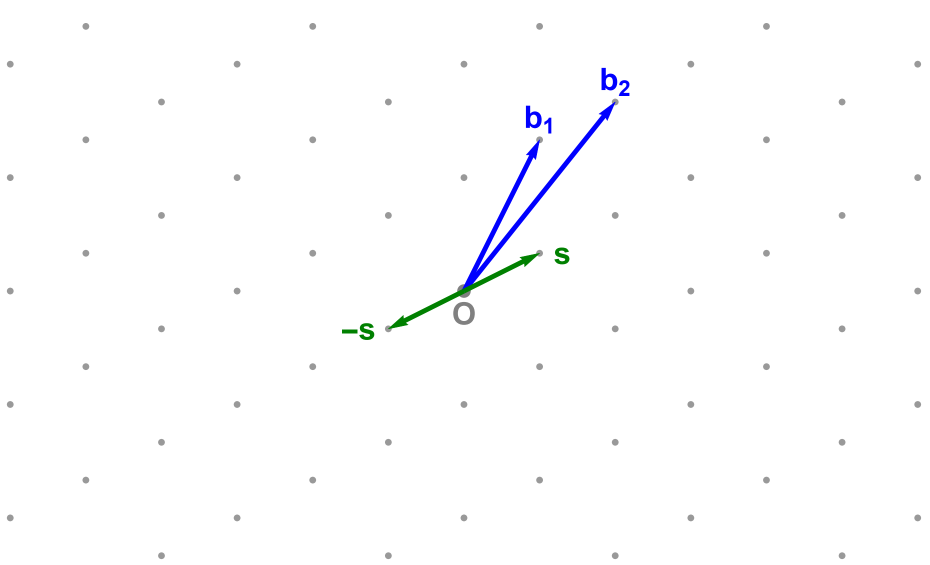

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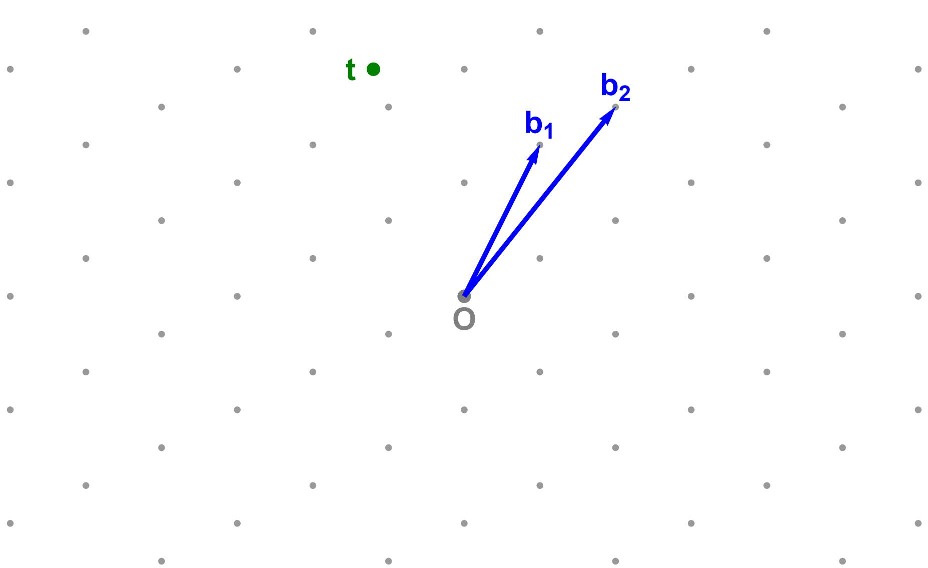

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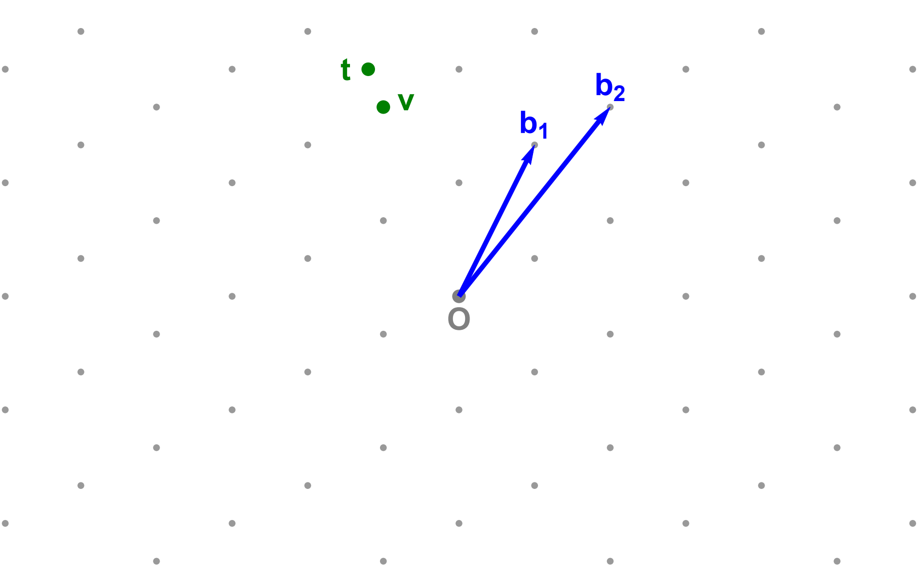

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  - Based on hard lattice problems (SVP, CVP)
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How hard are hard lattice problems such as SVP?

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Studied since the '80s [Poh81, Kan83, FP85, ..., GNR10, MW14]

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Recursive: Reduces SVP_n (CVP_n) to several instances of CVP_{n-1}

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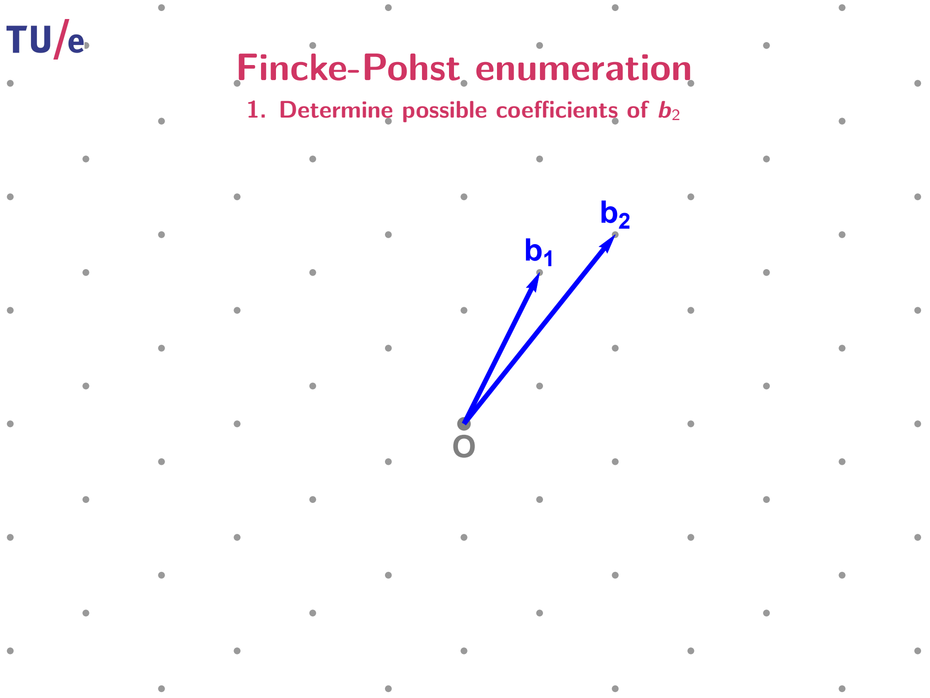

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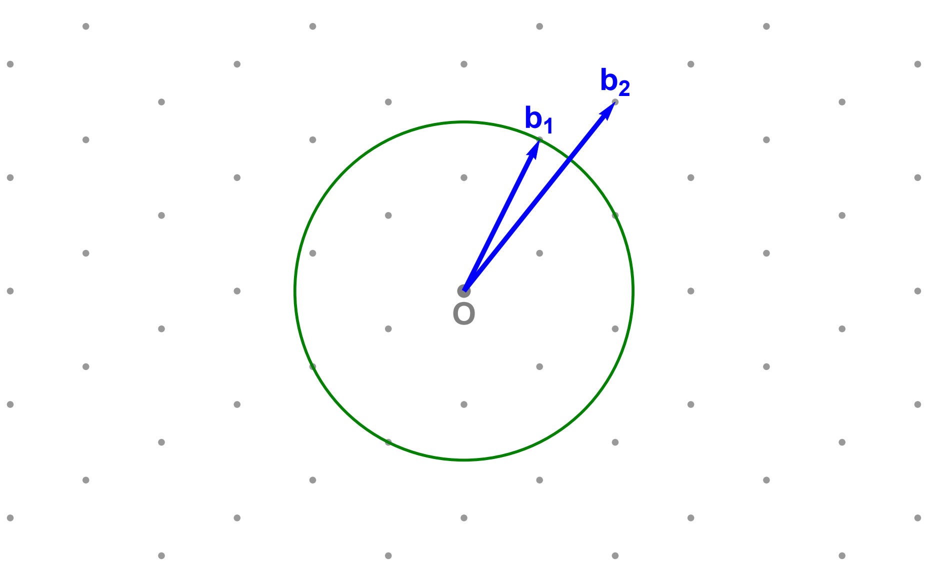

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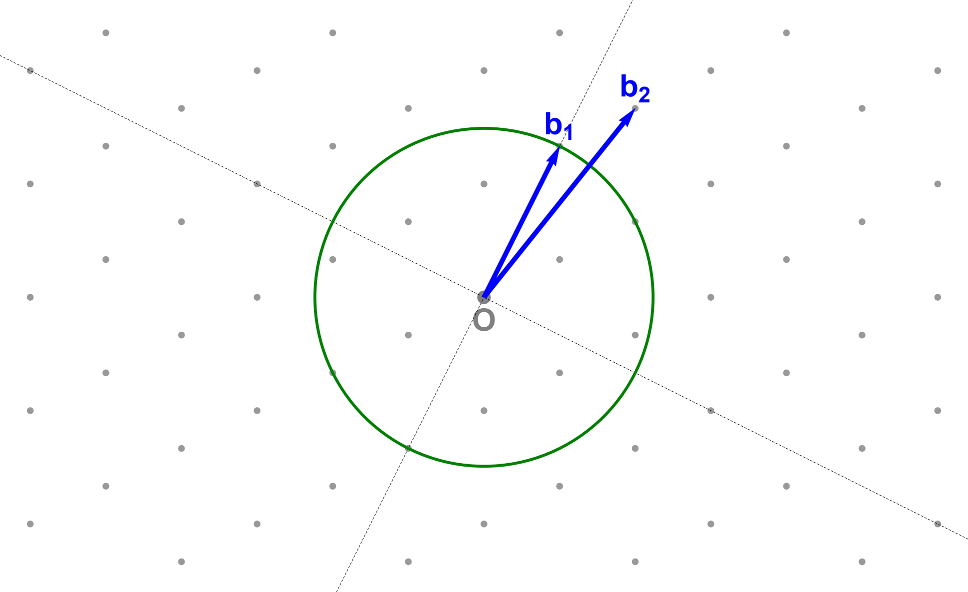

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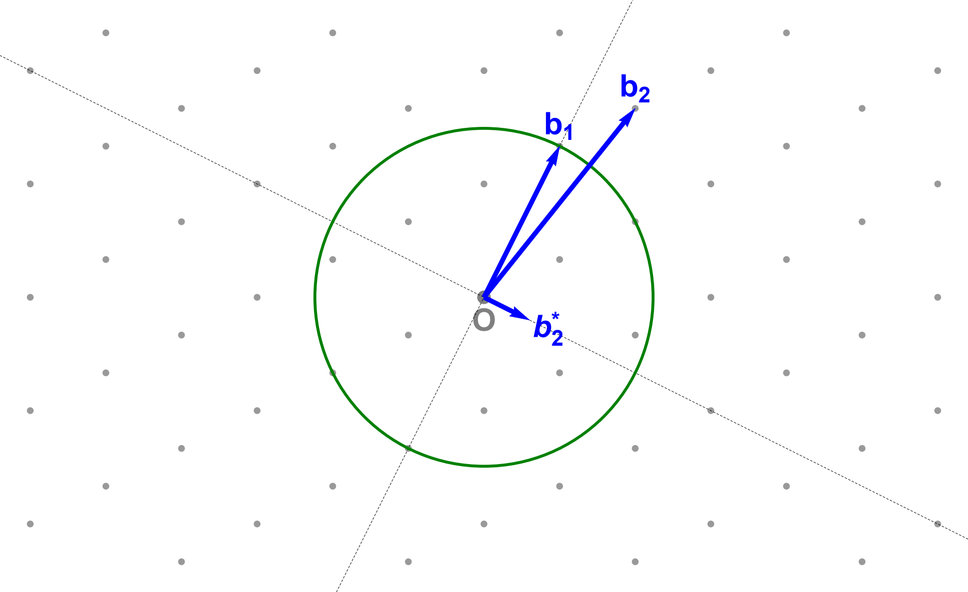

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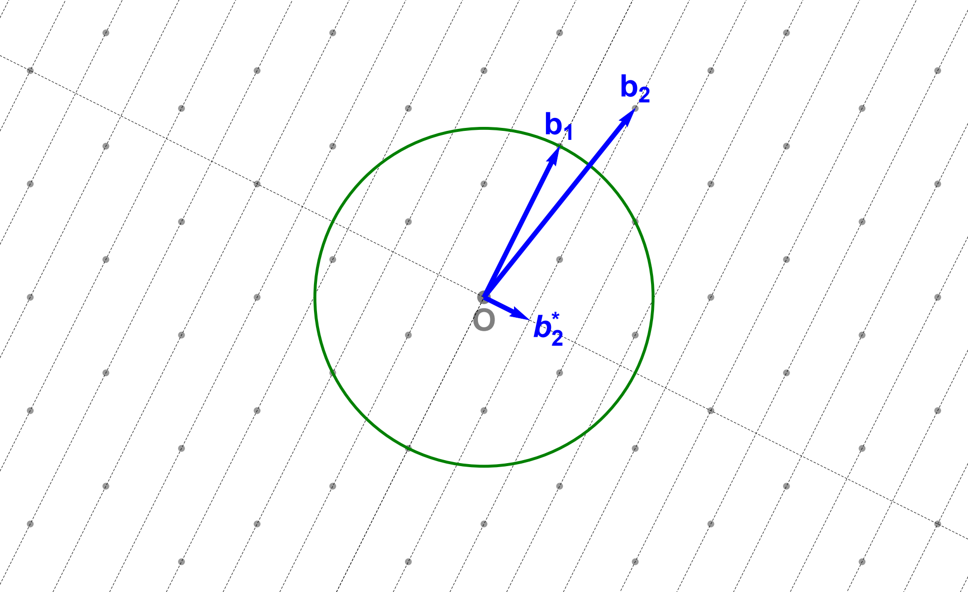

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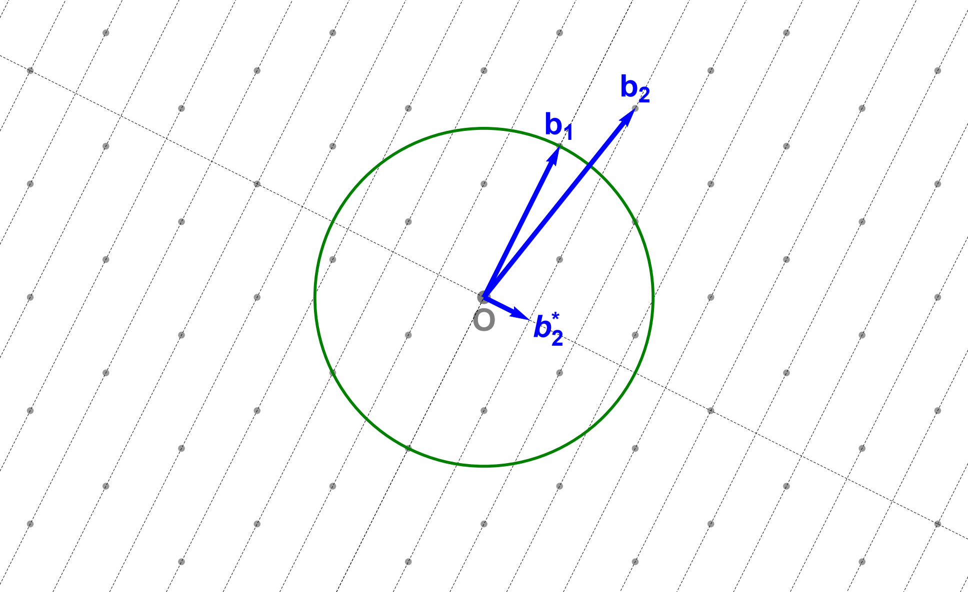

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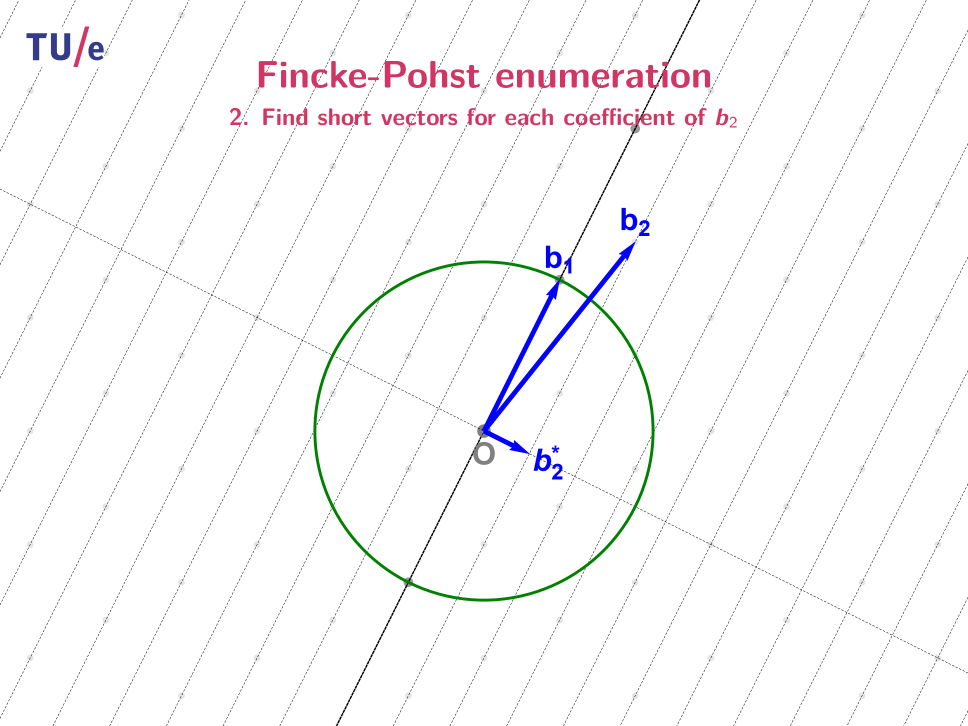

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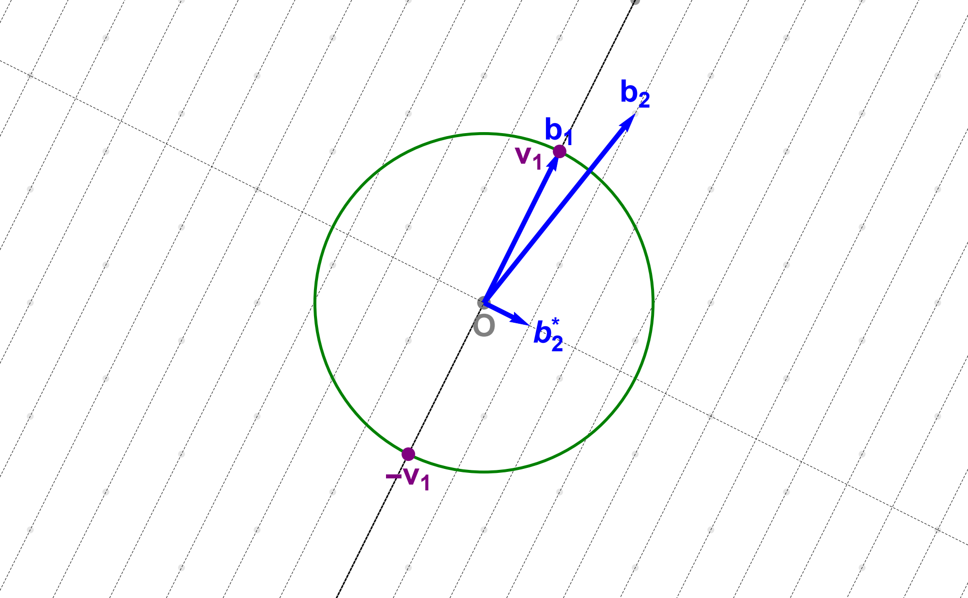

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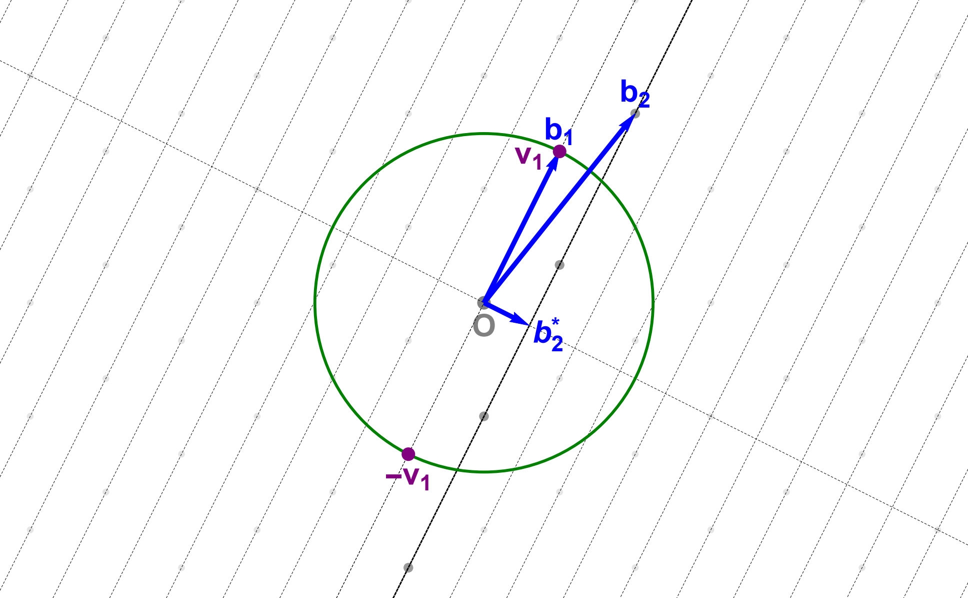

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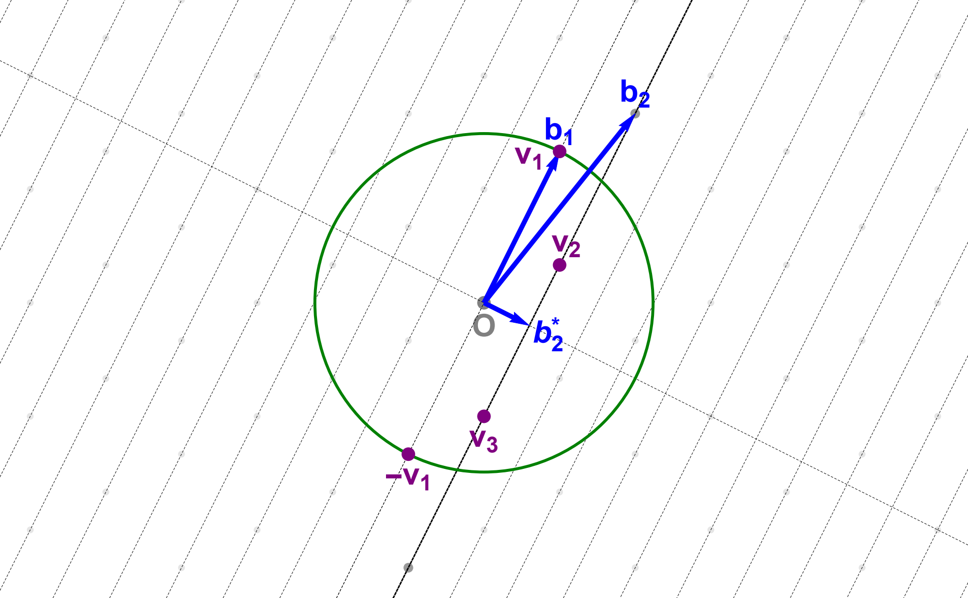

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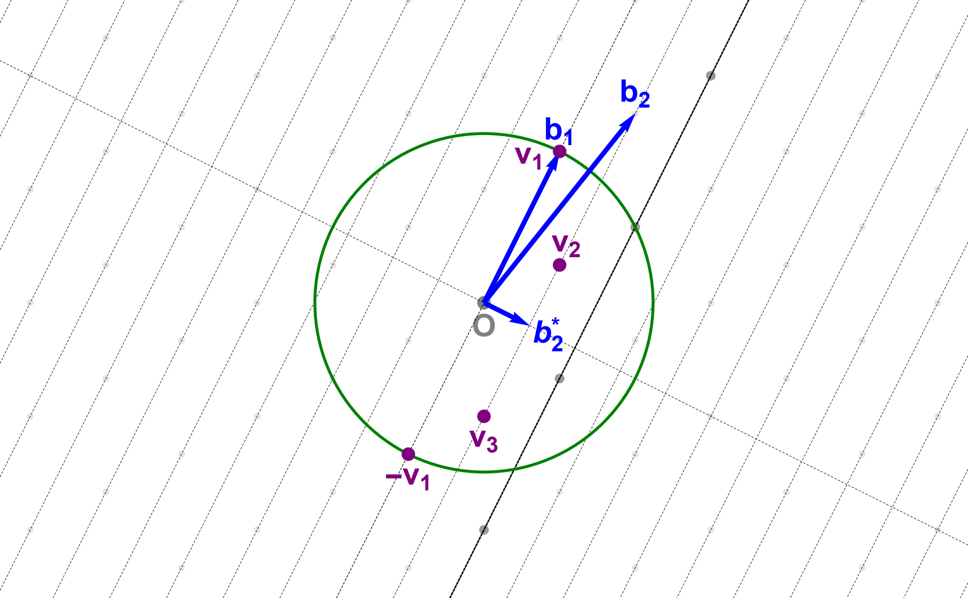

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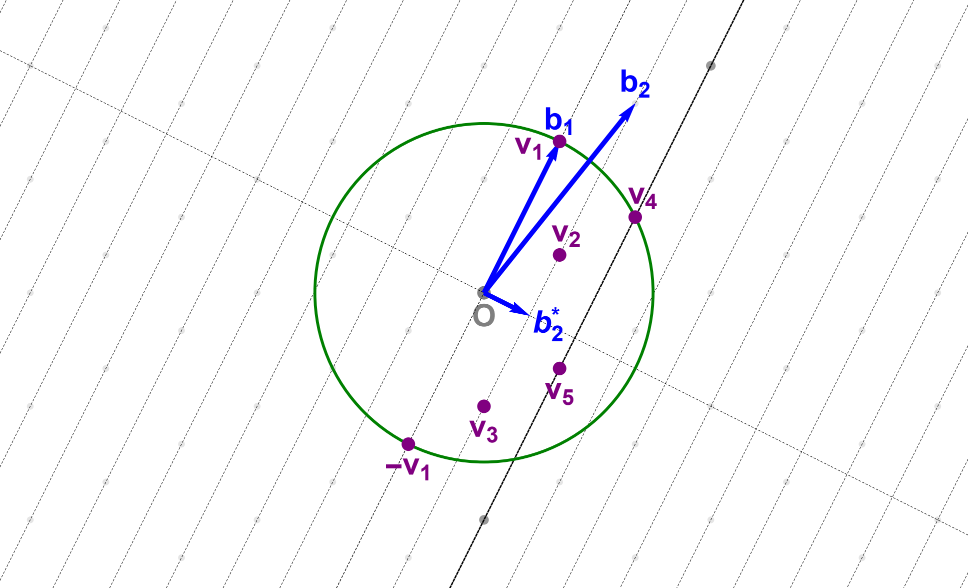

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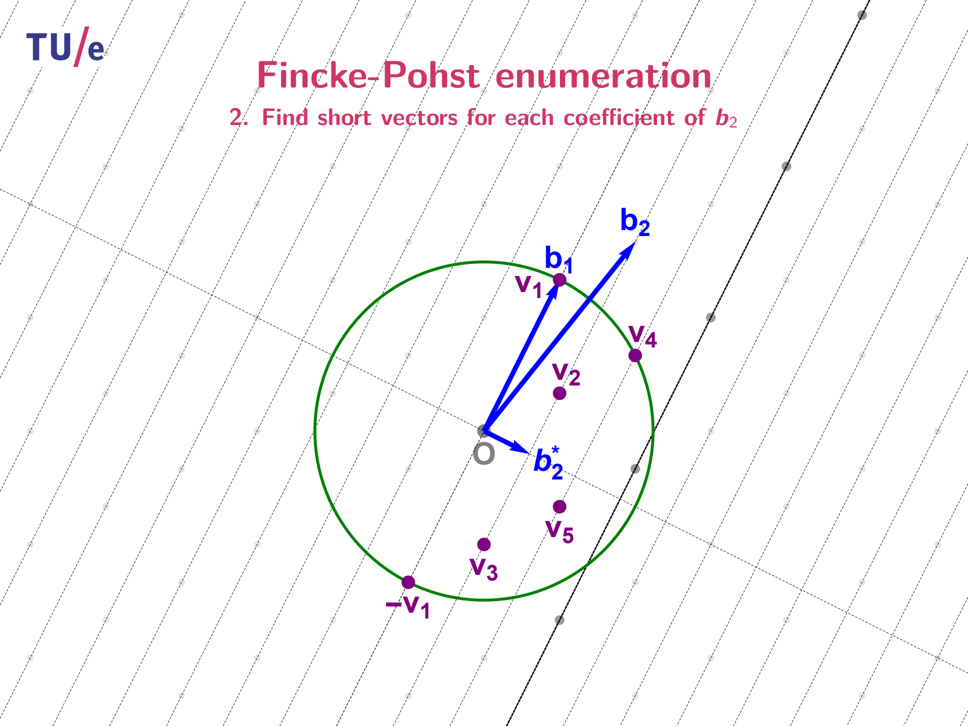

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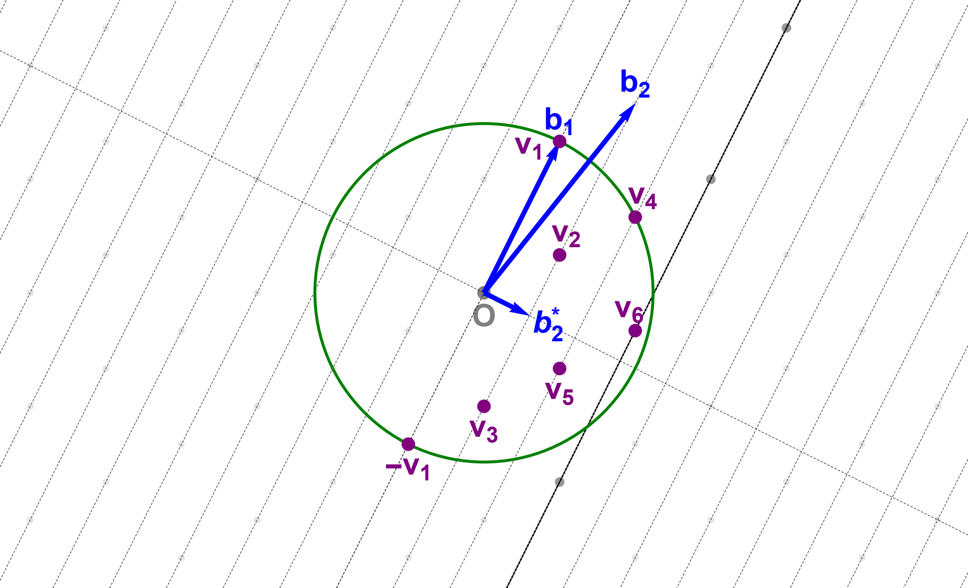

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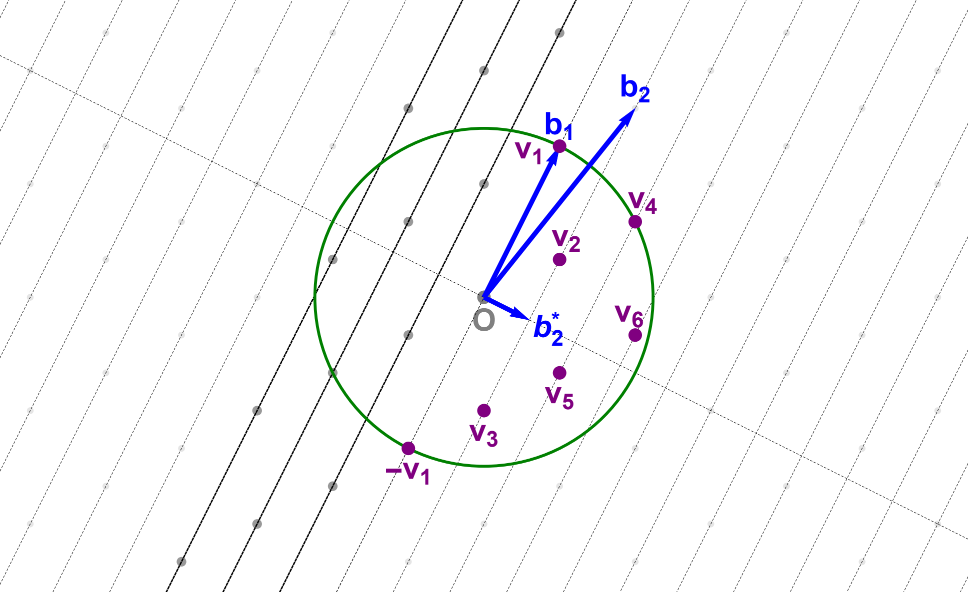

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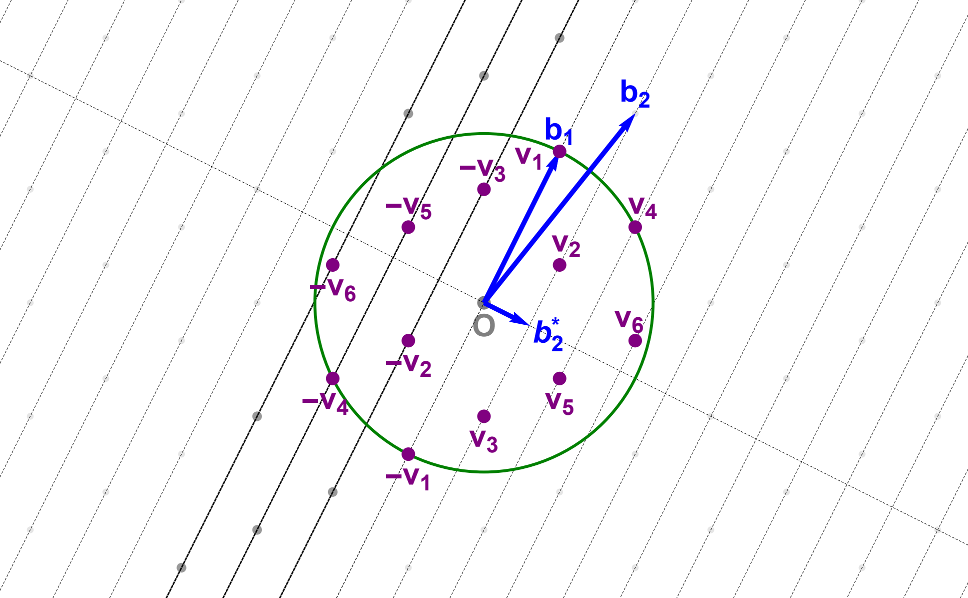

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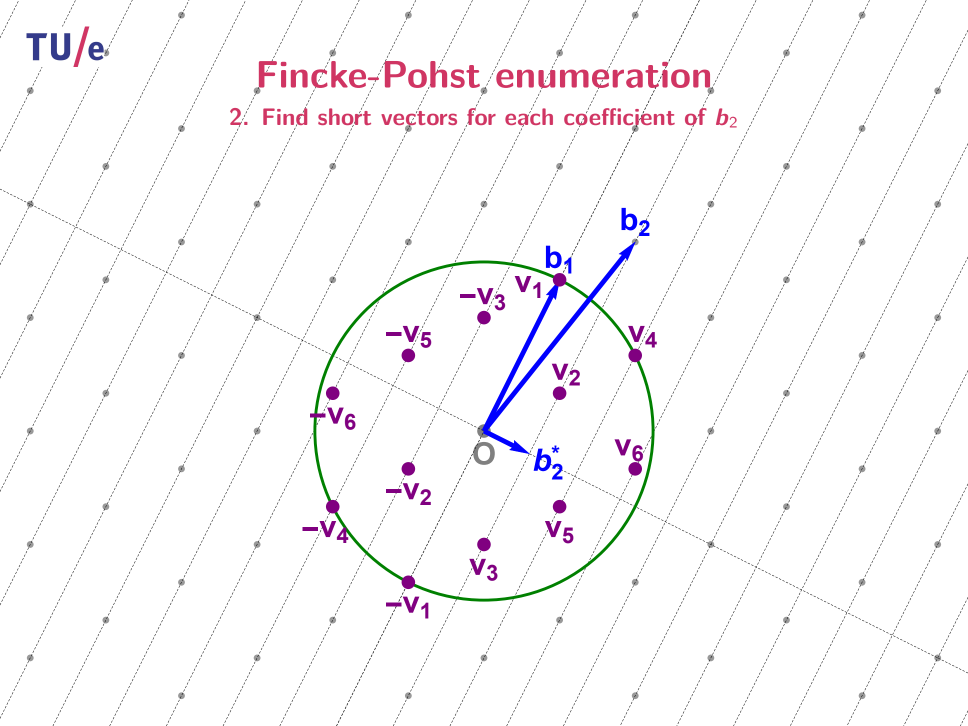

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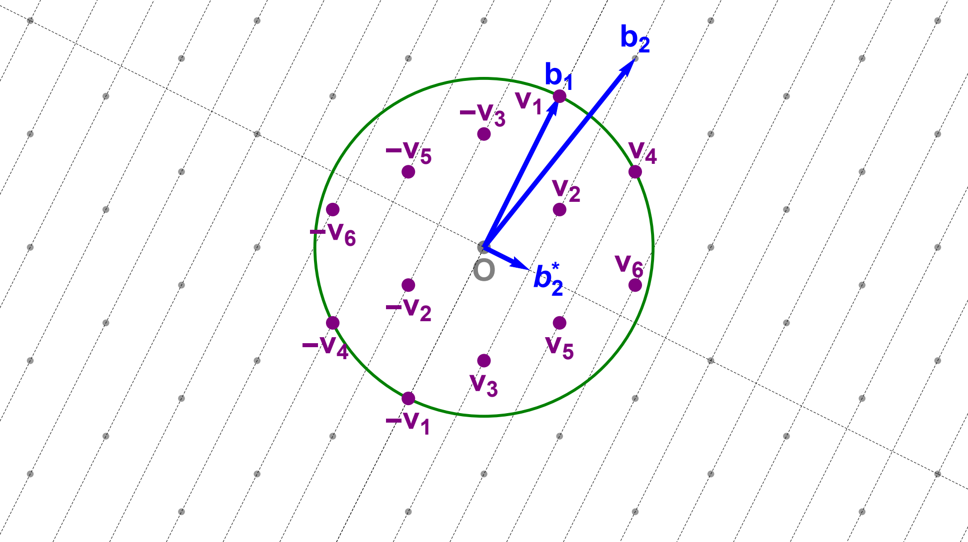

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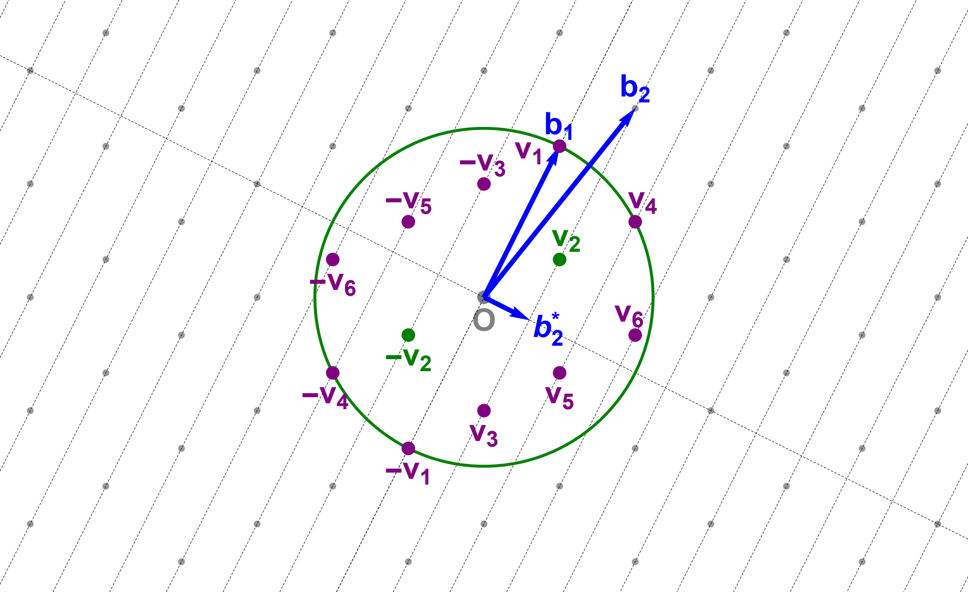

Fincke-Pohst enumeration

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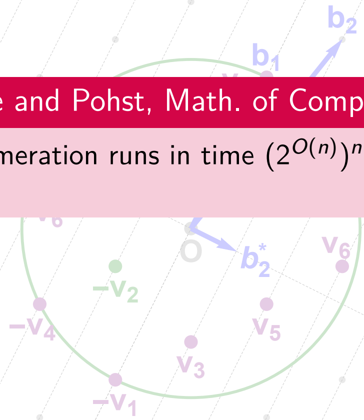

Theorem (Fincke and Pohst, Math. of Comp. '85)

Fincke-Pohst enumeration runs in time $(2^{O(n)})^n = 2^{O(n^2)}$ and space poly(n).

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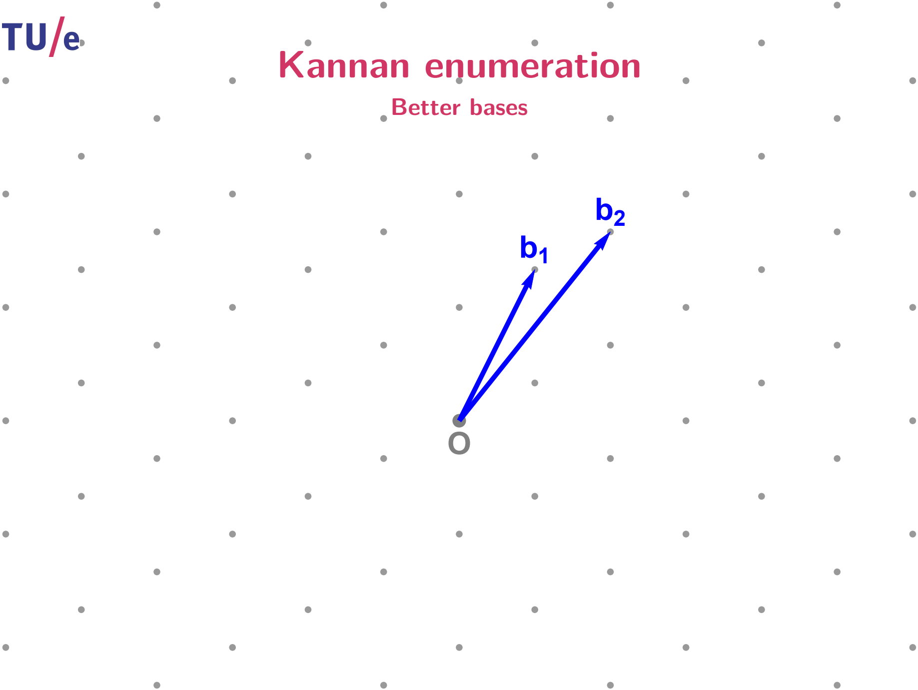
Essentially reduces SVP_n (CVP_n) to $2^{O(n)}$ instances of CVP_{n-1}

Kannan enumeration

Better bases



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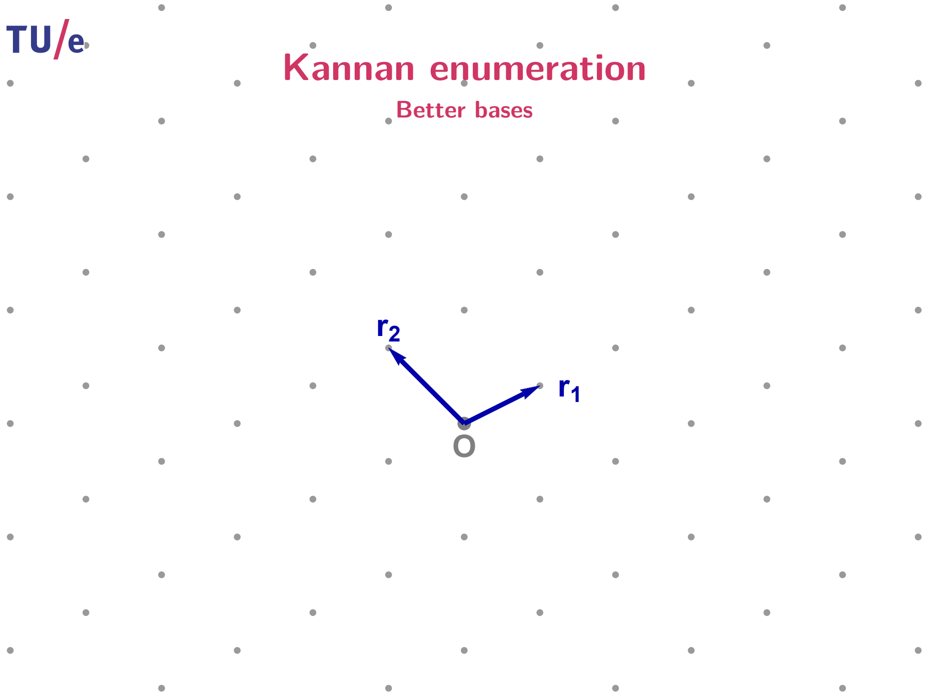

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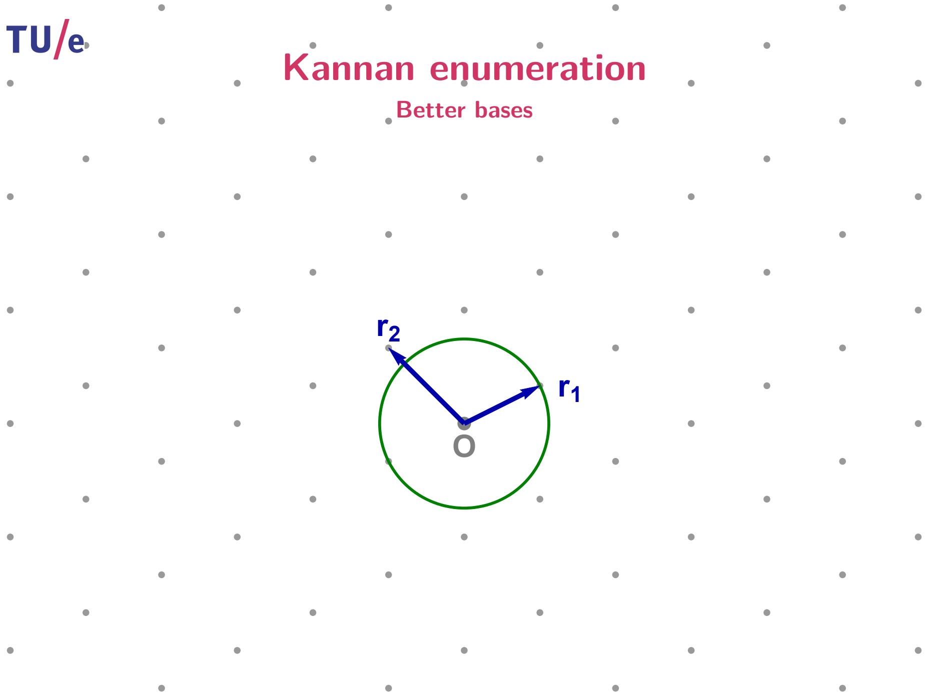

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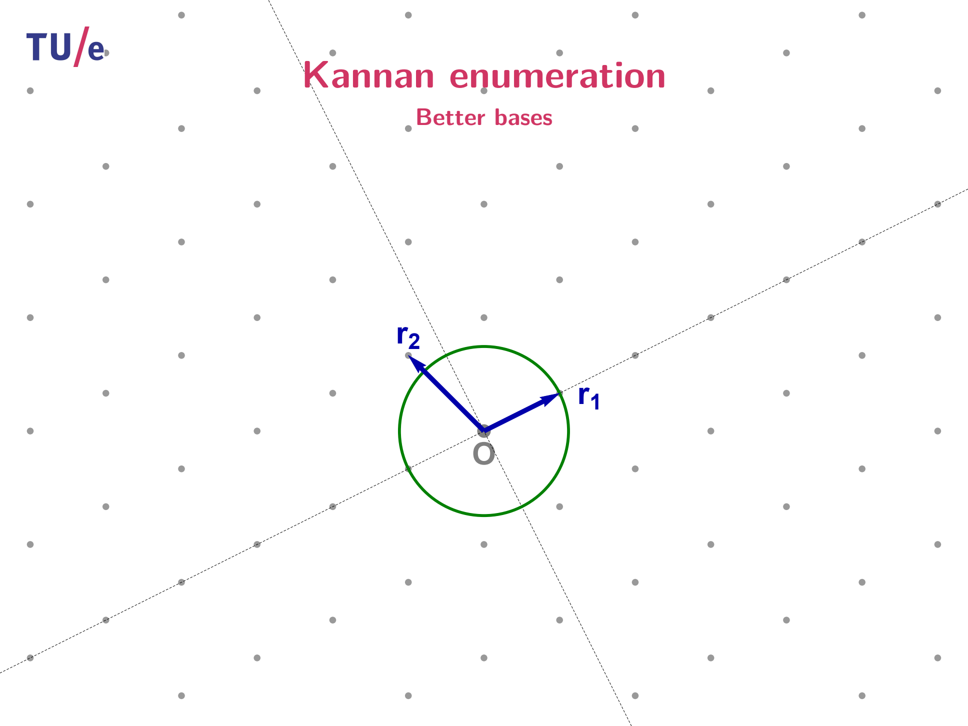

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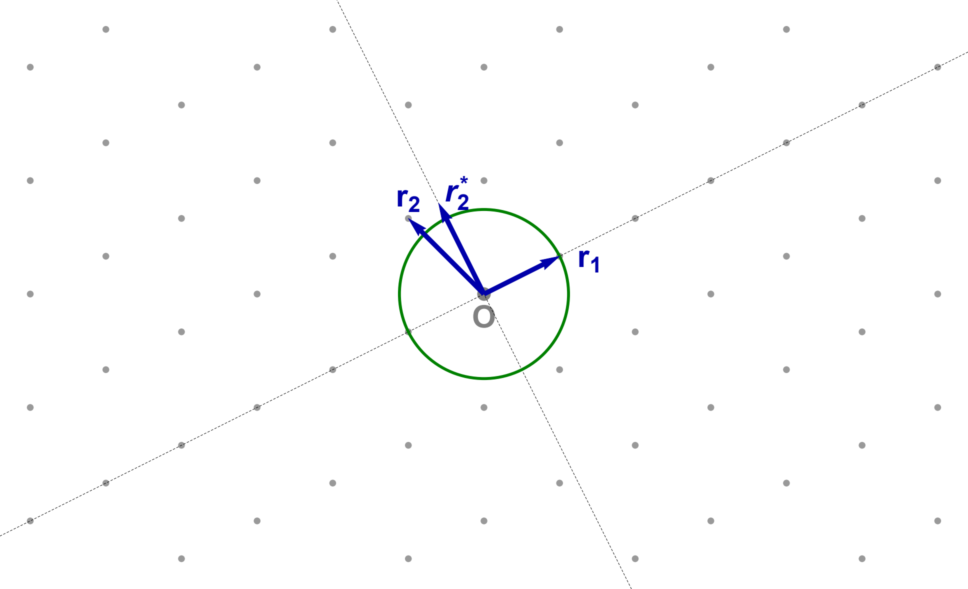

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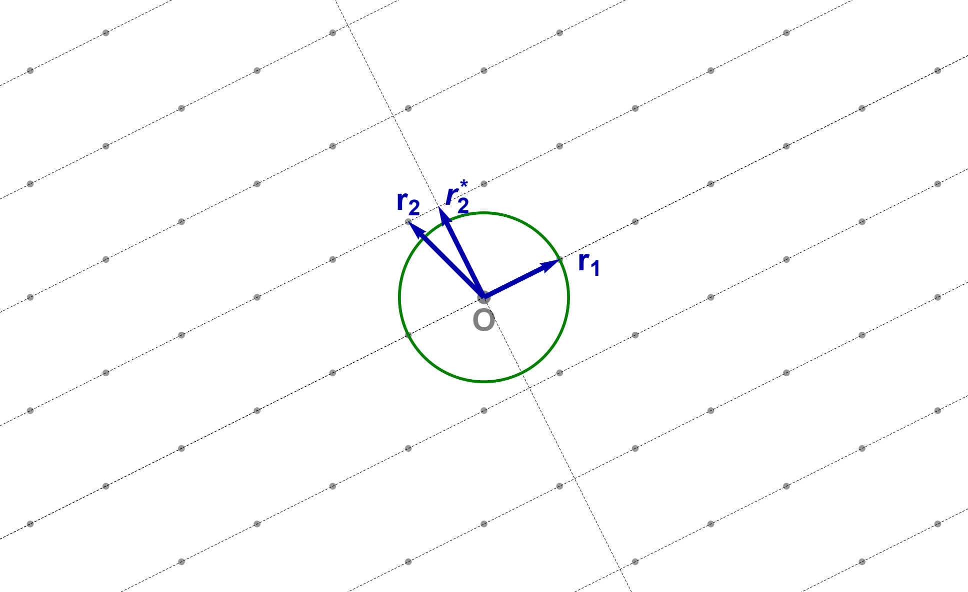

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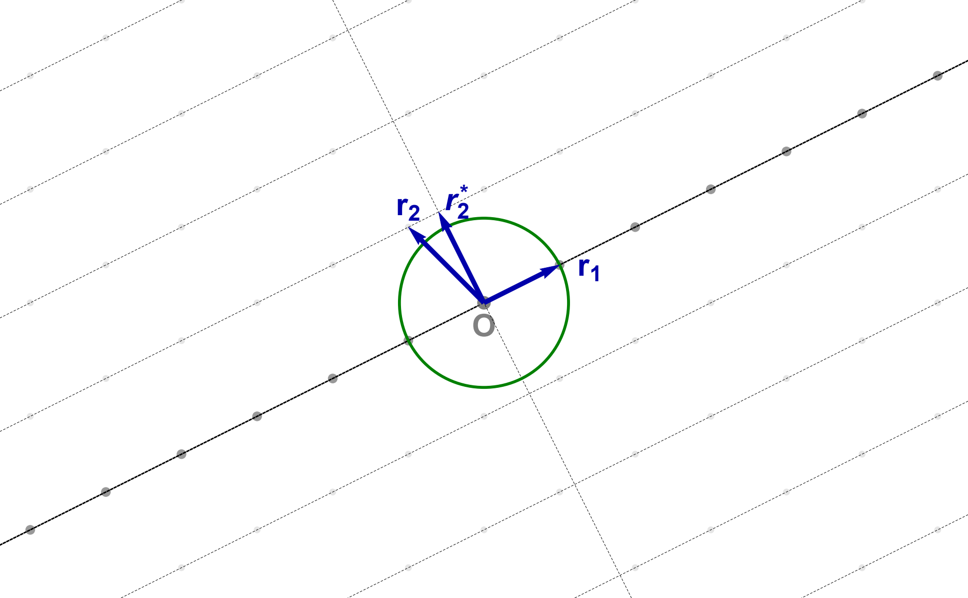

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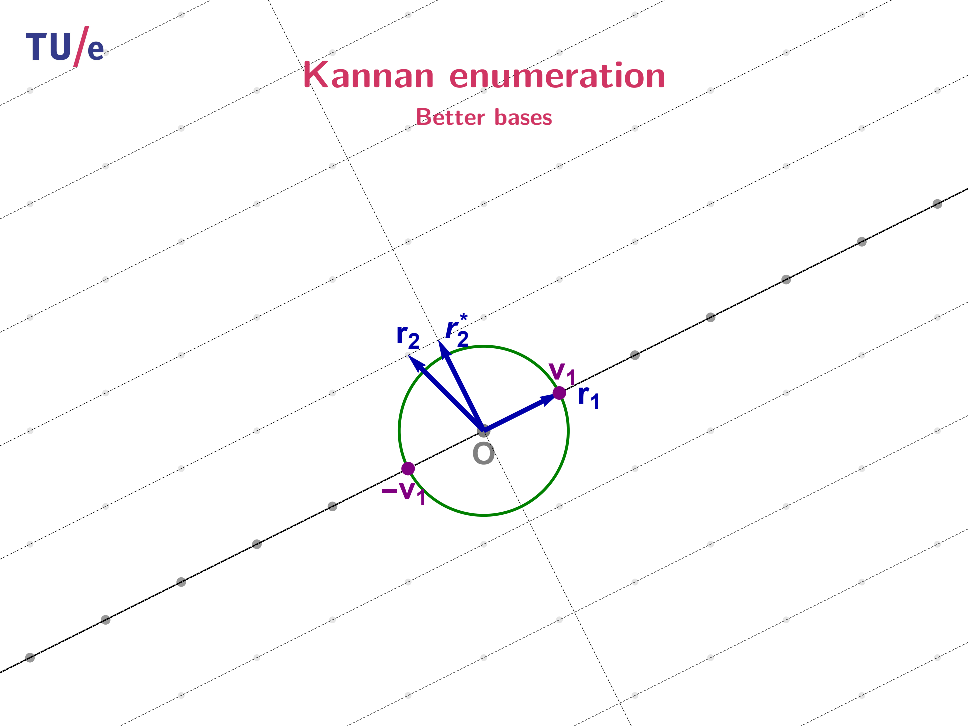

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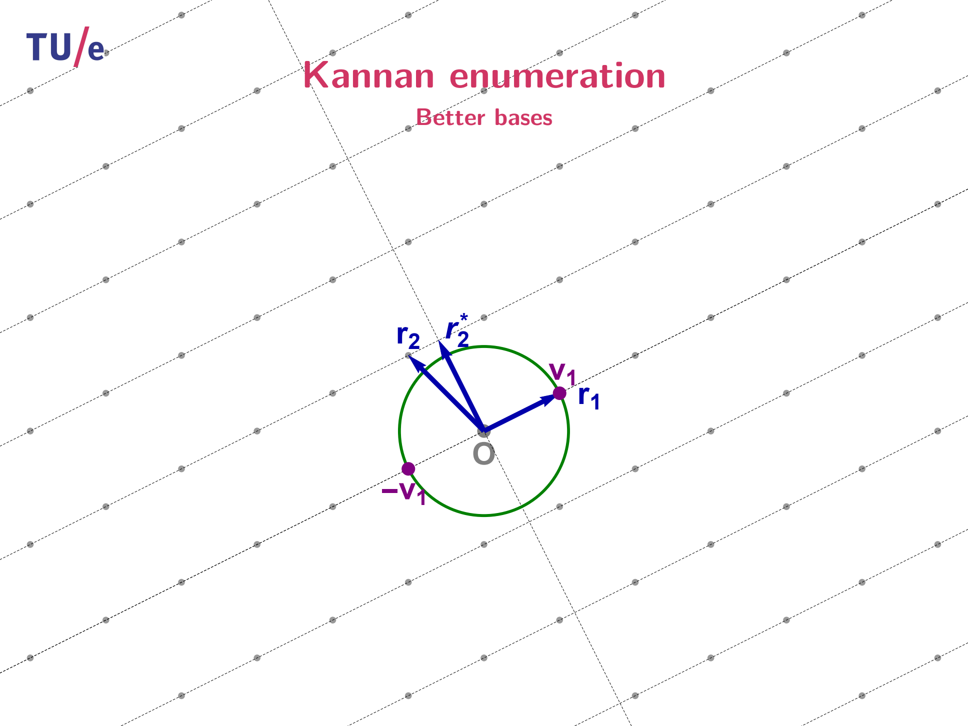

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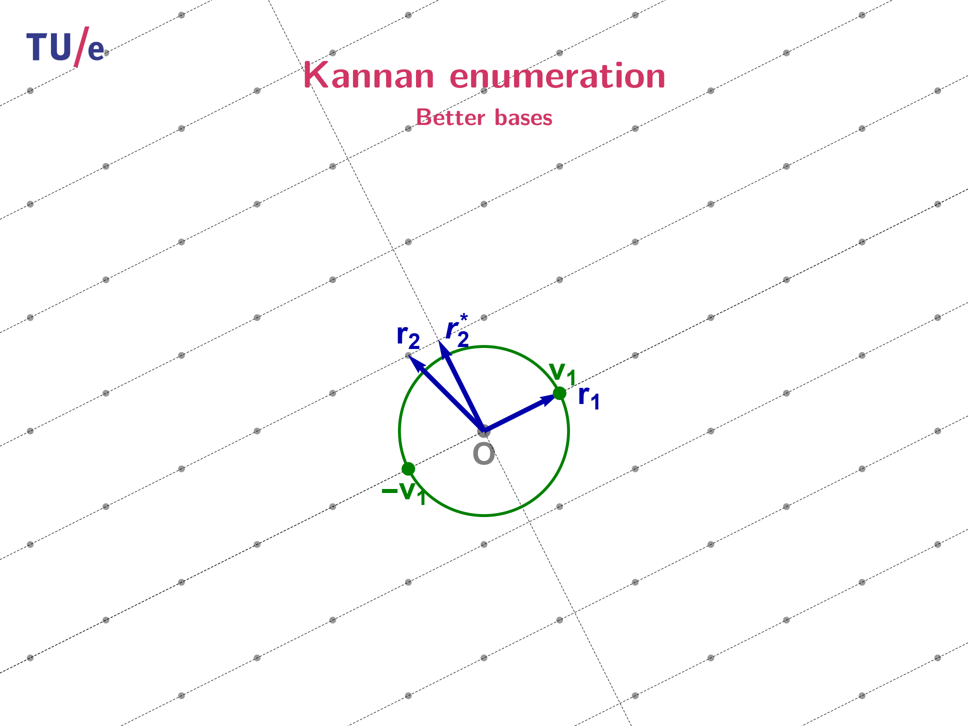

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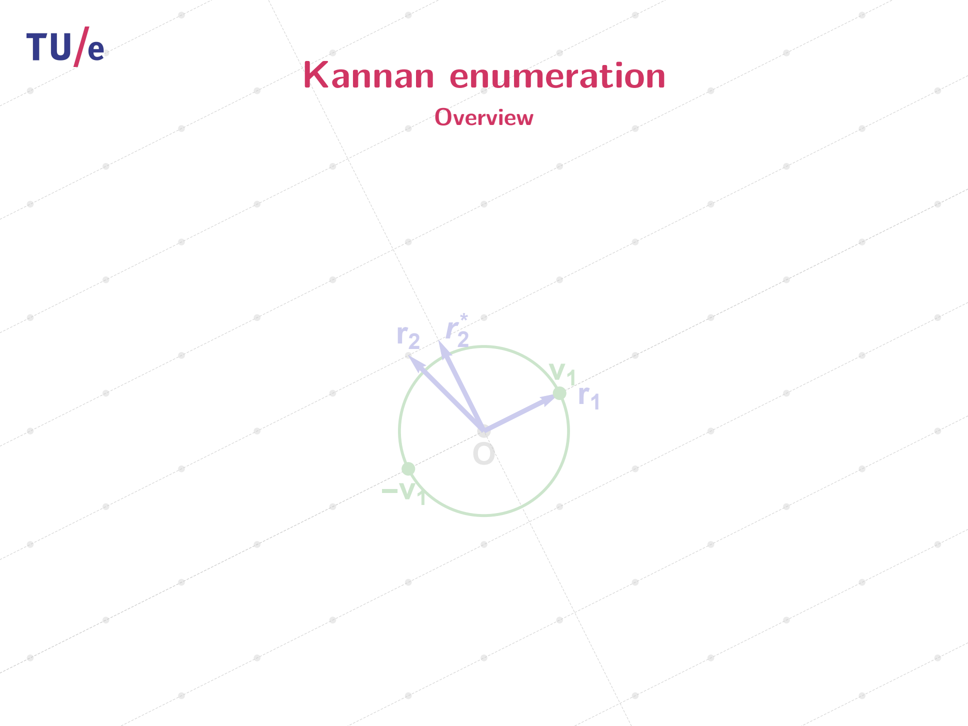

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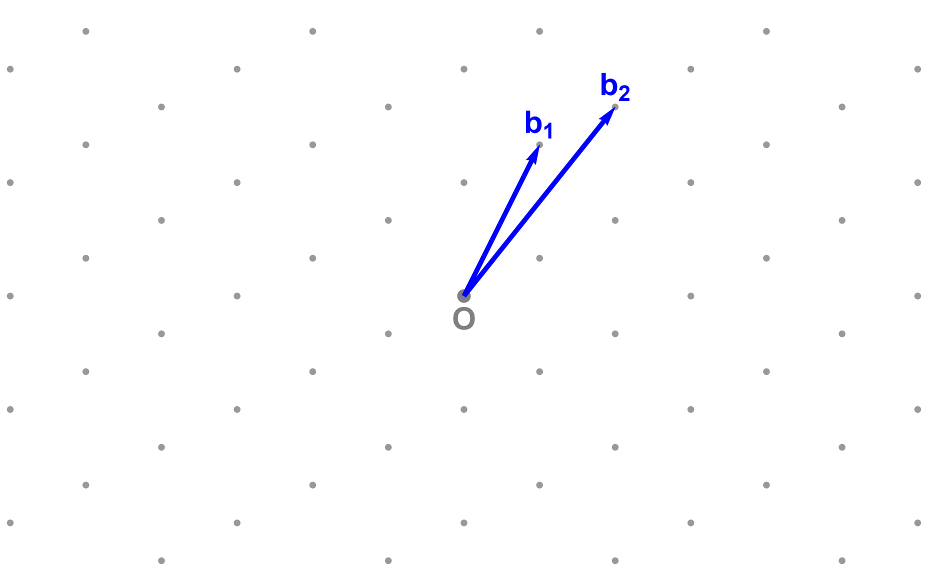
> *"Our algorithm reduces an n-dimensional problem to polynomially many (instead of $2^{O(n)}$) $(n-1)$-dimensional problems. [...] The algorithm we propose, first finds a more orthogonal basis for a lattice in time $2^{O(n \log n)}$."*
>
> – Kannan, *STOC'83*

Pruned enumeration

Reducing the search space



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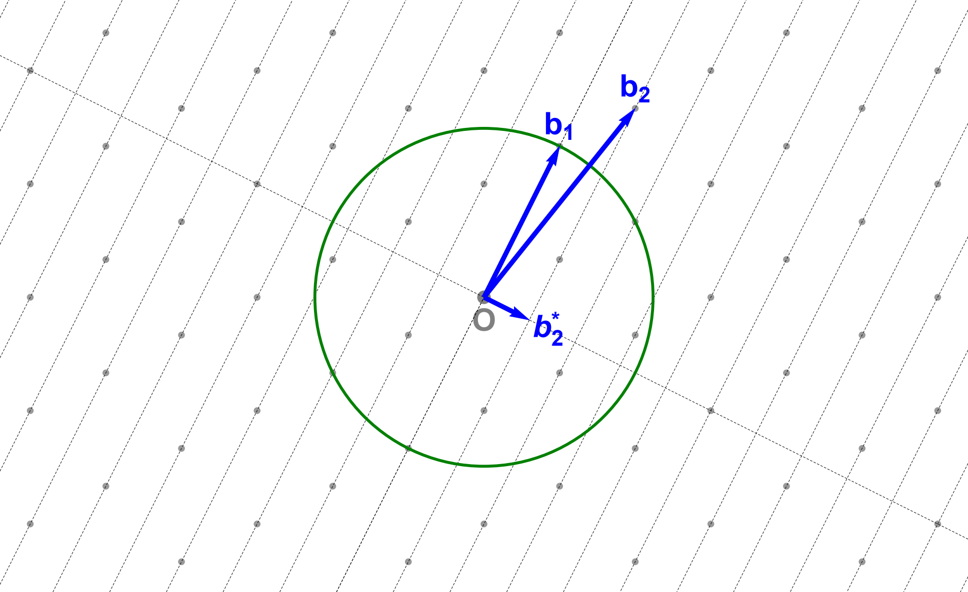

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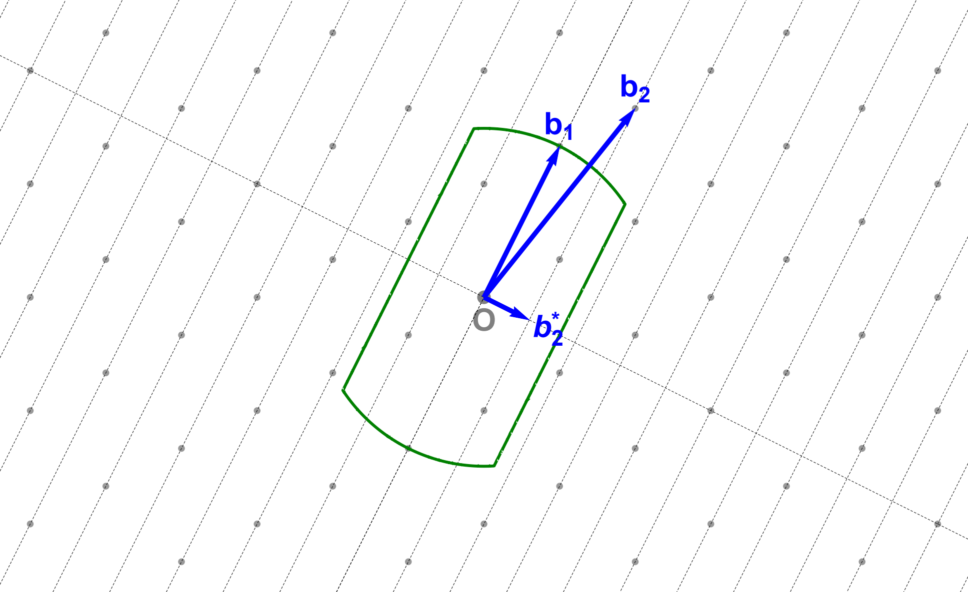

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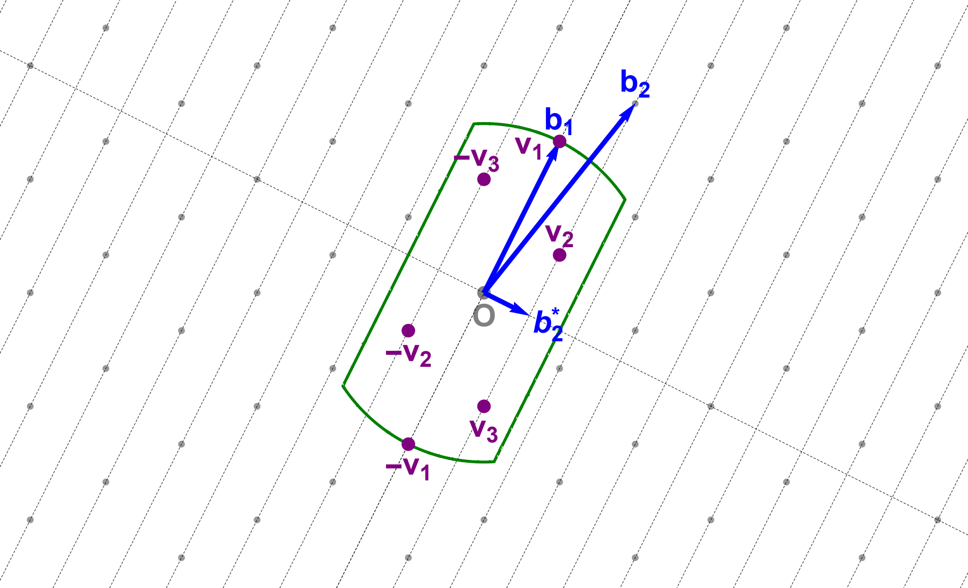

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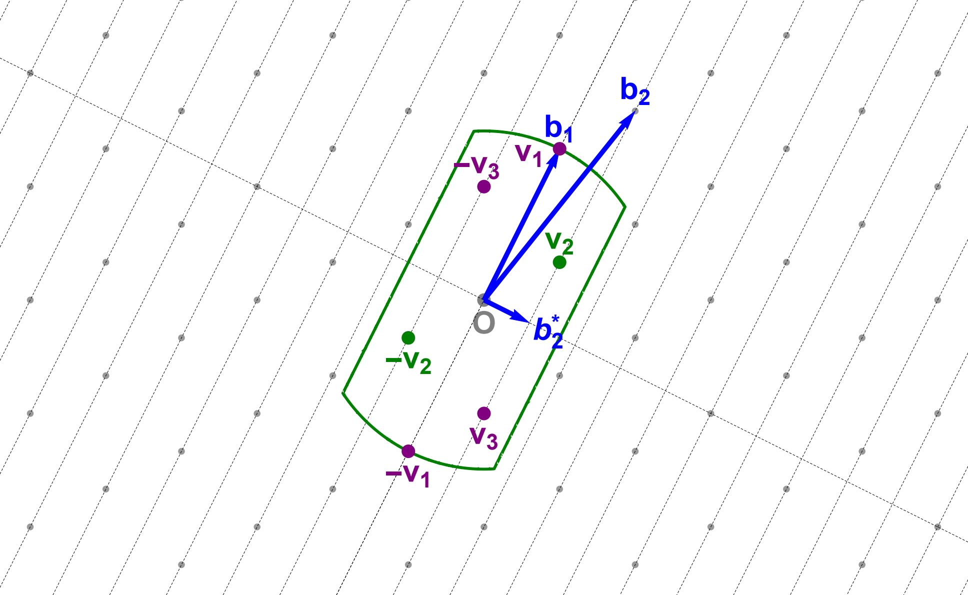

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"Well-chosen bounding functions lead asymptotically to an exponential speedup of about $2^{n/4}$ over basic enumeration, maintaining a success probability $\geq$ 95%."

– Gama et al., *EUROCRYPT'10*

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"With extreme pruning, the probability of finding the desired vector is actually rather low (say, 0.1%*), but surprisingly, the running time of the enumeration is reduced by a much more significant factor (say, much more than* 1000*)."*

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Very recent algorithm [MV10]

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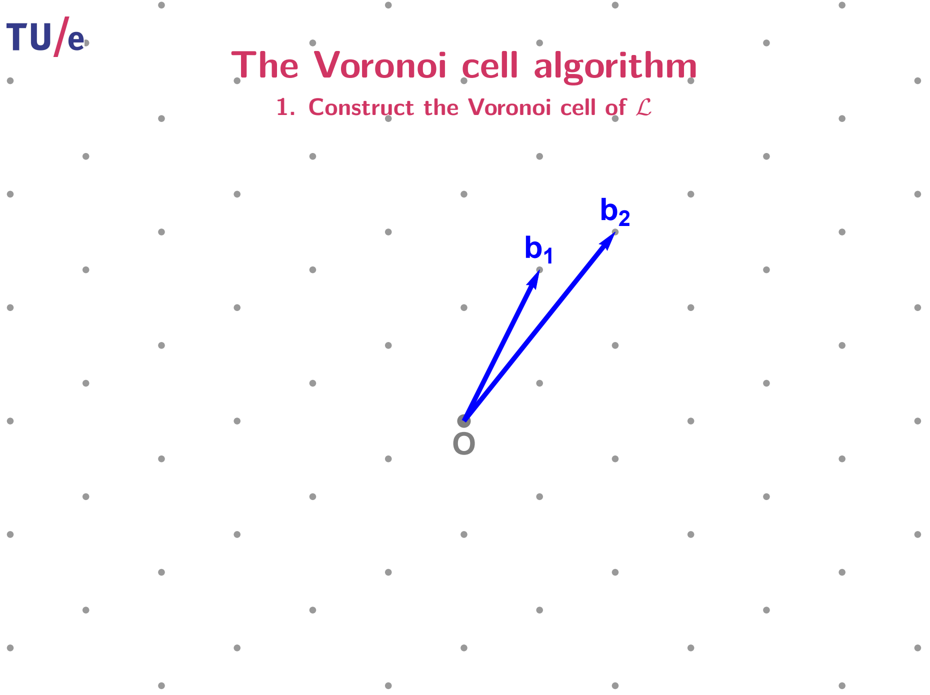

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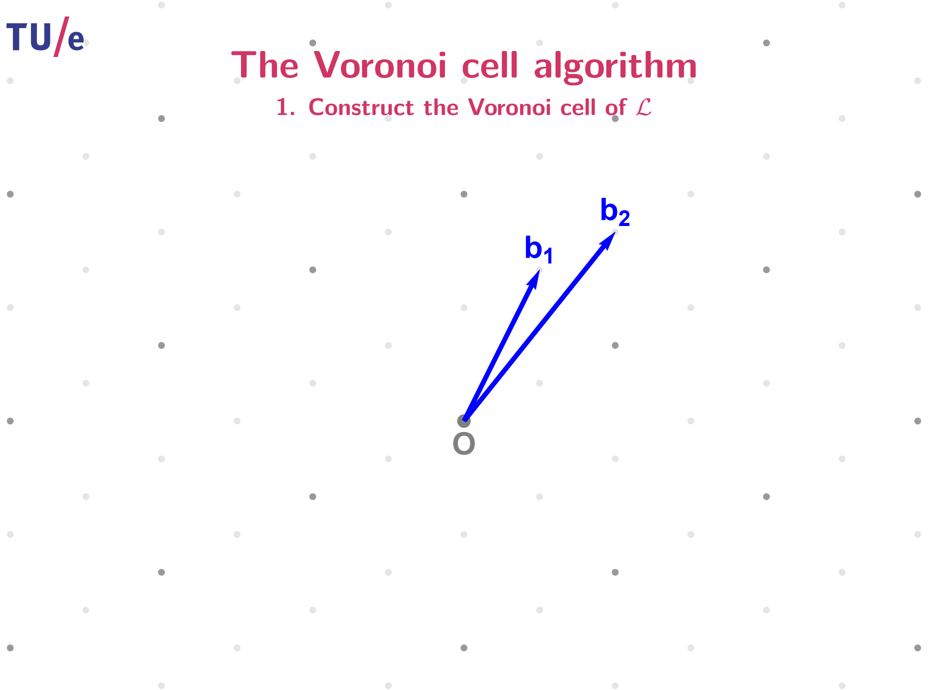

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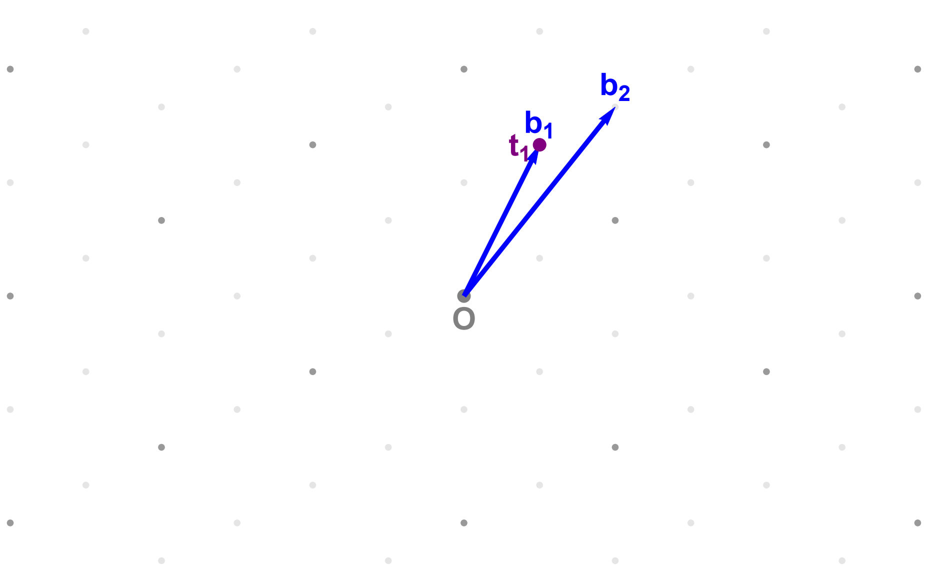

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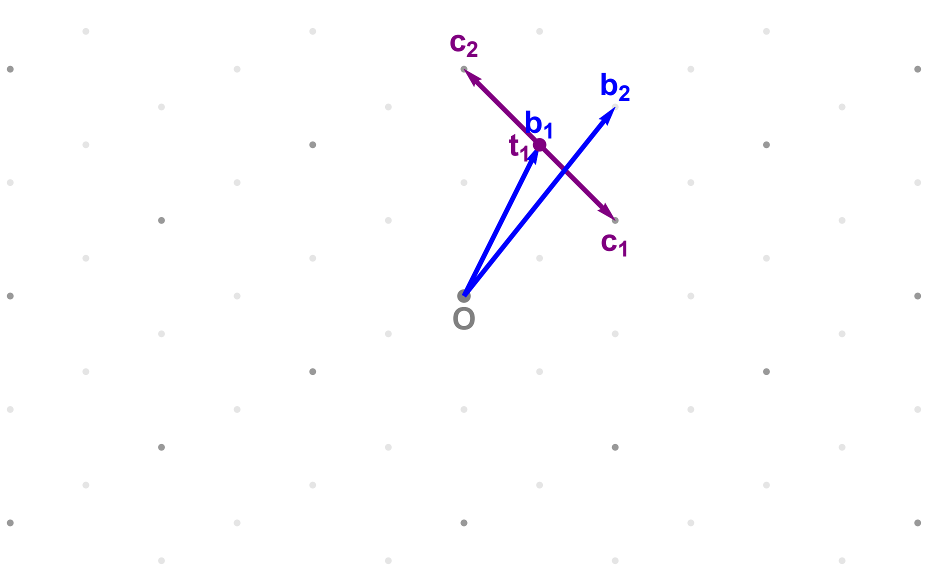

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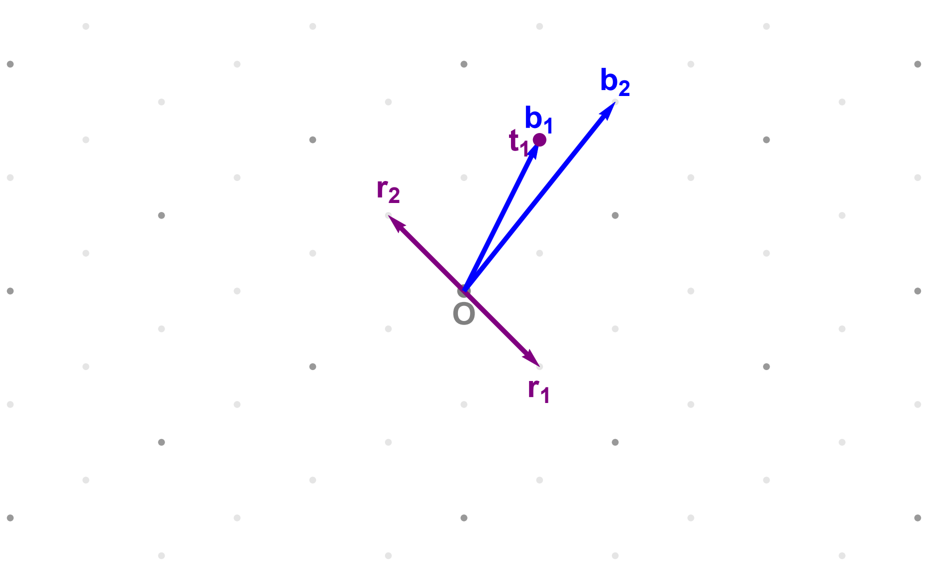

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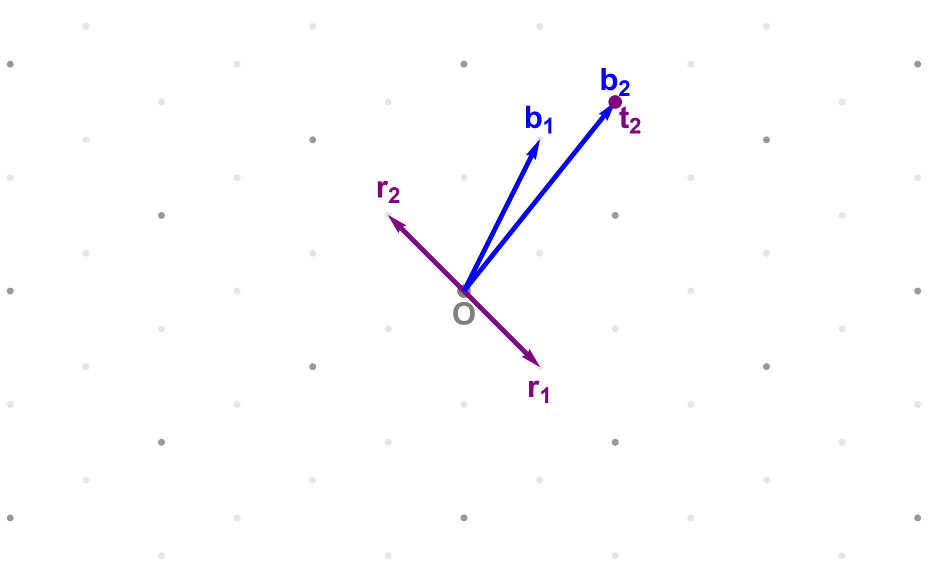

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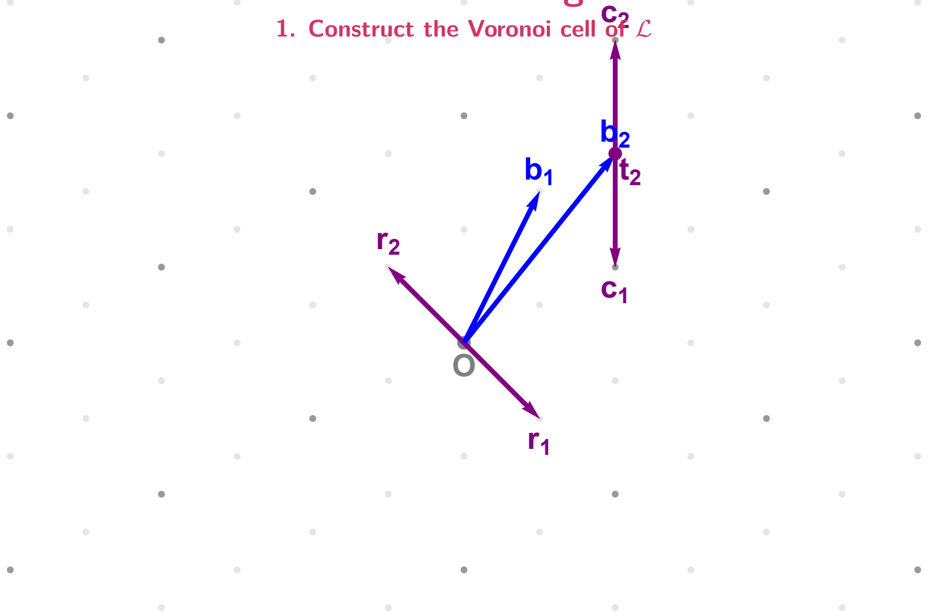
1. Construct the Voronoi cell of $\mathcal{L}$



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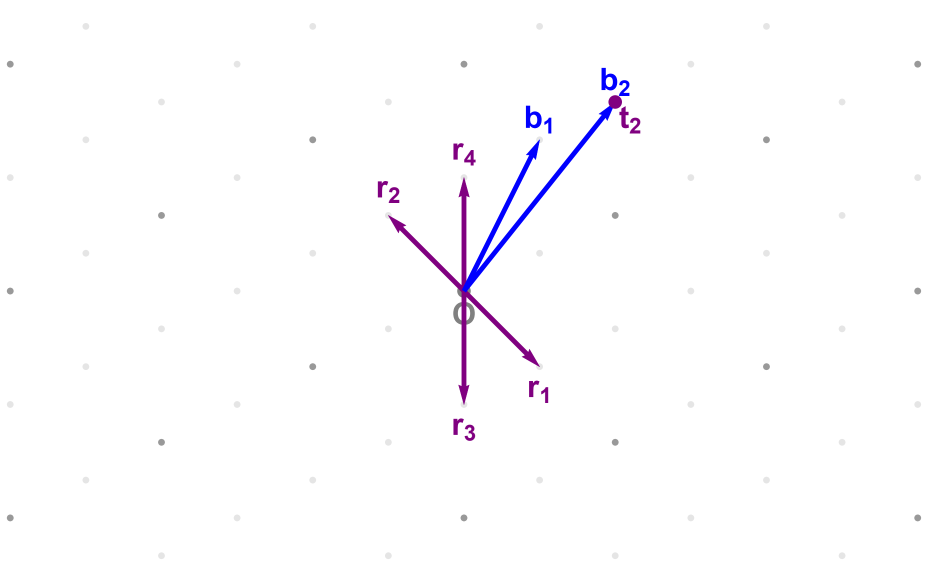

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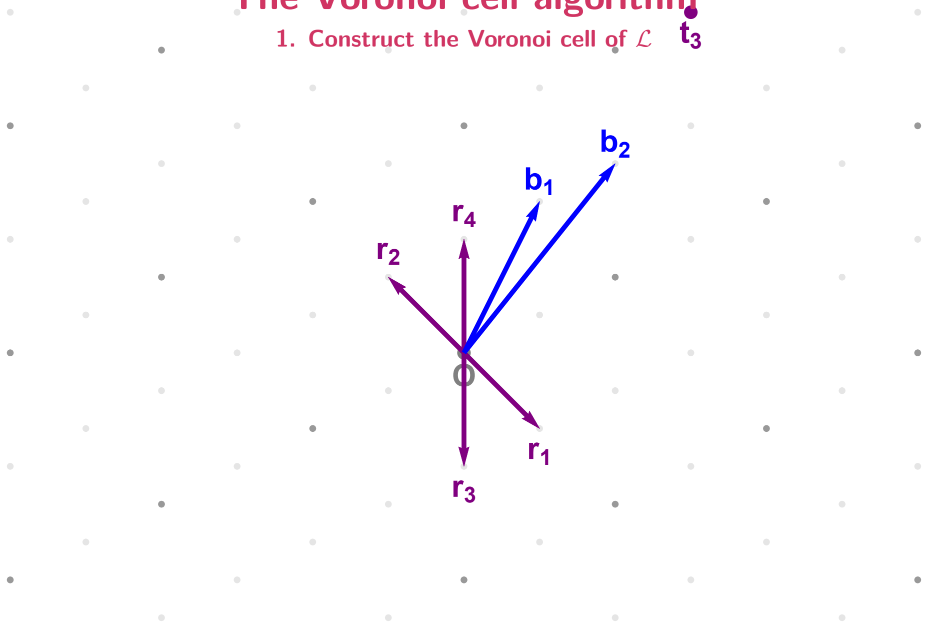

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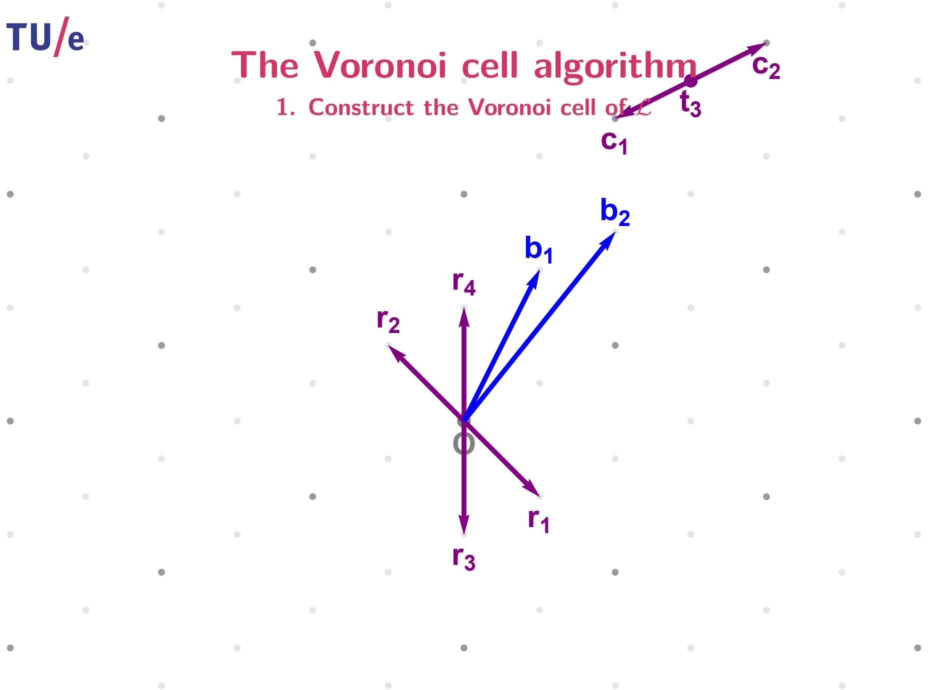

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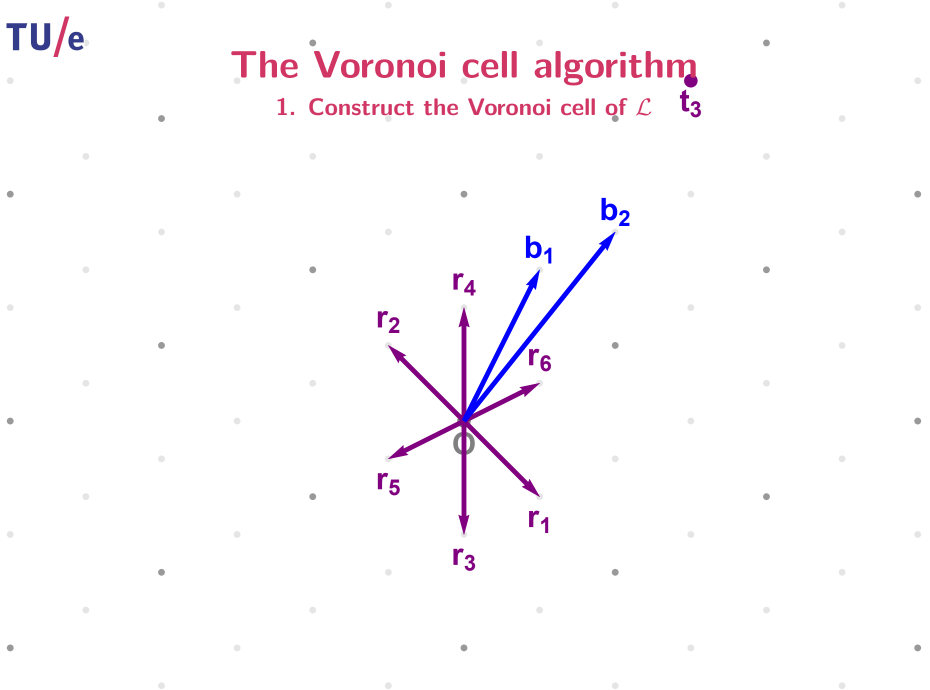

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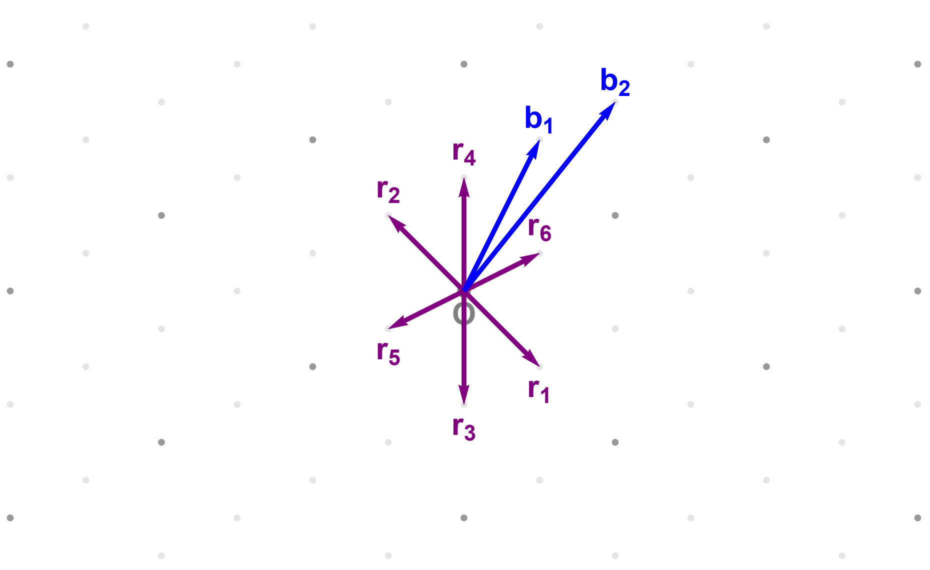

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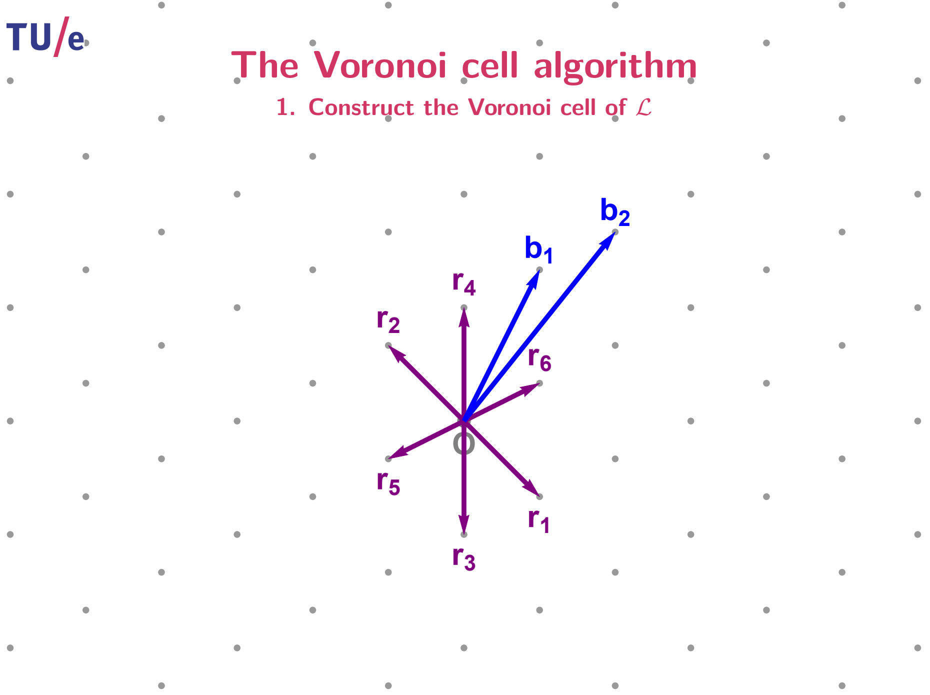

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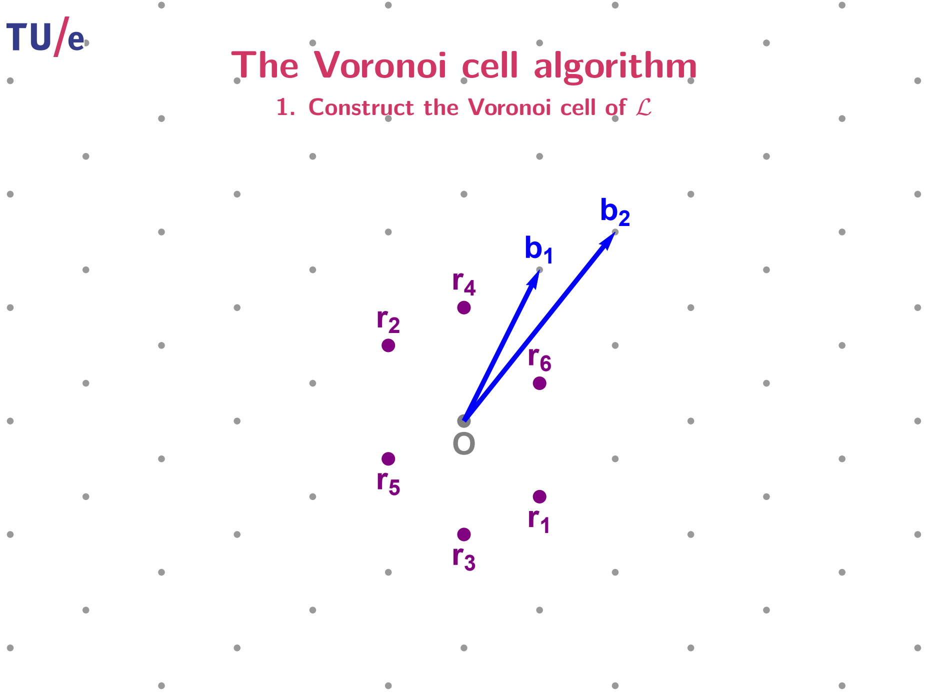

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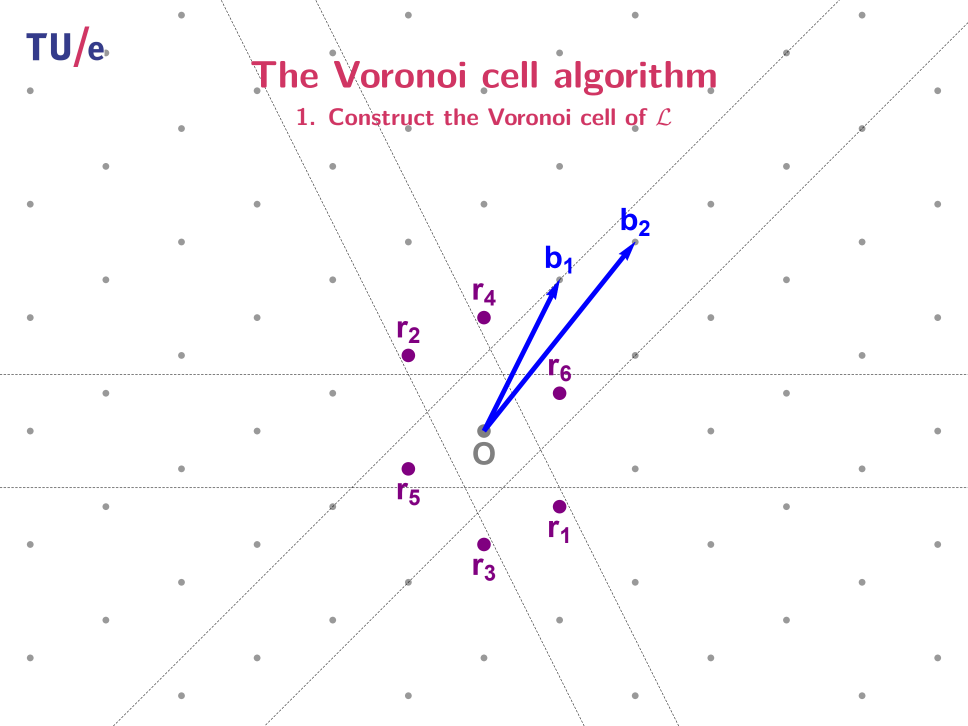

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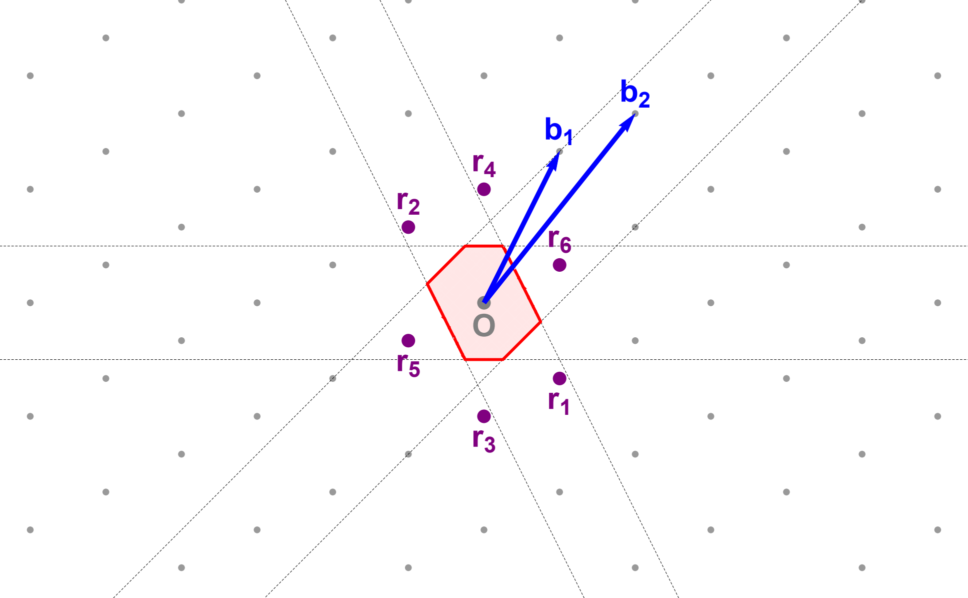

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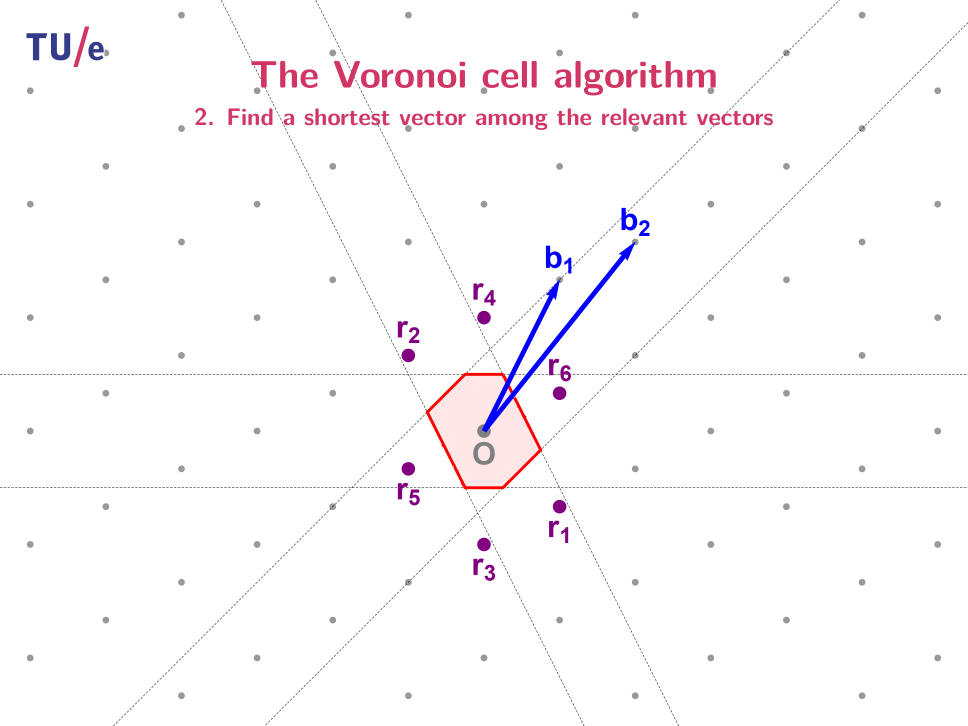
2. Find a shortest vector among the relevant vectors



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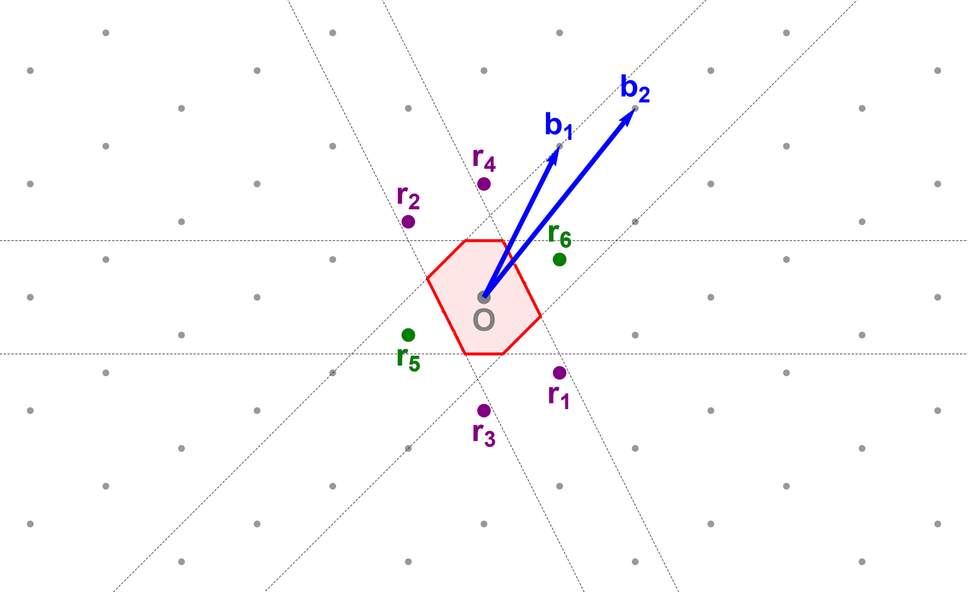


The Voronoi cell algorithm

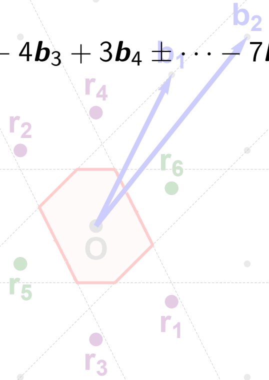
Analysis

The Voronoi cell algorithm

Analysis

Suppose the shortest vector $\boldsymbol{s}$ satisfies:

$$\boldsymbol{s} = 5\boldsymbol{b}_1 + 2\boldsymbol{b}_2 - 4\boldsymbol{b}_3 + 3\boldsymbol{b}_4 \pm \cdots - 7\boldsymbol{b}_n$$



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Choose the vector $\boldsymbol{t} \in \{0,1\}^n \cdot B$ such that $\boldsymbol{s} + \boldsymbol{t} \in 2\mathcal{L}$:

$$\boldsymbol{t} = 1\boldsymbol{b}_1 + 0\boldsymbol{b}_2 + 0\boldsymbol{b}_3 + 1\boldsymbol{b}_4 \pm \cdots + 1\boldsymbol{b}_n$$

$$\boldsymbol{s} + \boldsymbol{t} = 6\boldsymbol{b}_1 + 2\boldsymbol{b}_2 - 4\boldsymbol{b}_3 + 4\boldsymbol{b}_4 \pm \cdots - 6\boldsymbol{b}_n \qquad (\in 2\mathcal{L})$$

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The vectors closest to $\boldsymbol{t}$ in the lattice $2\mathcal{L}$ are $\boldsymbol{t} \pm \boldsymbol{s}$.

The Voronoi cell algorithm

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Theorem (Micciancio and Voulgaris, SODA'10)

The Voronoi cell algorithm runs in time $2^{2n+o(n)}$ and space $2^{n+o(n)}$.

Outline

Lattices

Enumeration algorithms
- Fincke-Pohst enumeration
- Kannan enumeration
- Pruning the enumeration tree

The Voronoi cell algorithm

Sieving algorithms
- Nguyen-Vidick sieve
- Multiple levels
- GaussSieve

Sieving using angular locality-sensitive hashing
- Nguyen-Vidick sieve
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Sieving

Studied since 2001 [AKS01, Reg04, NV08, . . ., ZPH13, BGJ14]

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   - Set $C = \emptyset$ and $R = \emptyset$
   - For each $\boldsymbol{v} \in L$, find the closest $\boldsymbol{c} \in C$
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3. Set $L \leftarrow R$ and repeat until L contains a shortest vector

Nguyen-Vidick sieve

1. Sample a list L of random lattice vectors

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2. Split L into centers C and short vectors R

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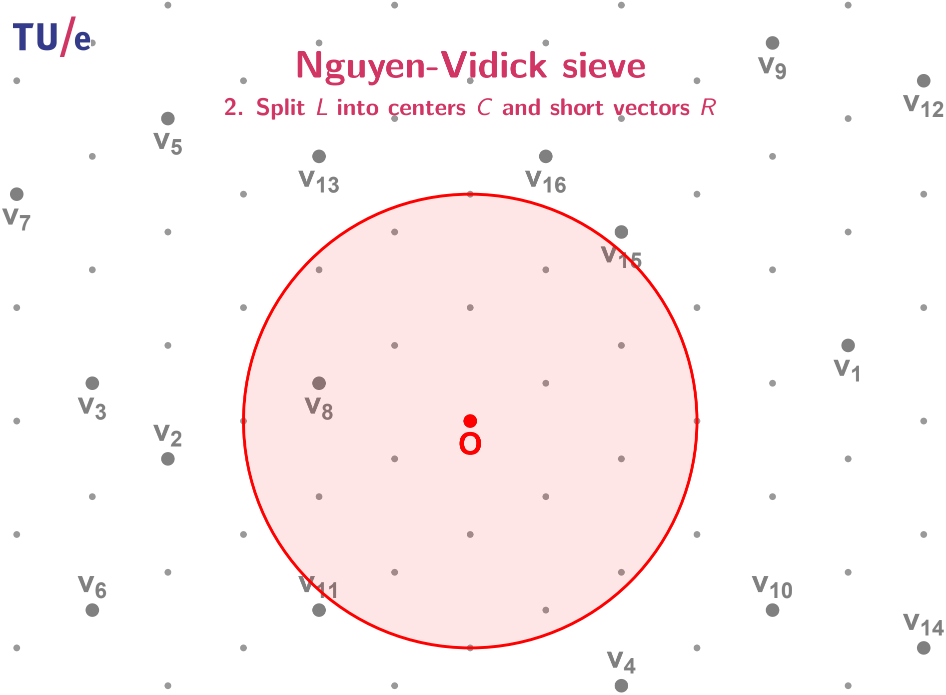

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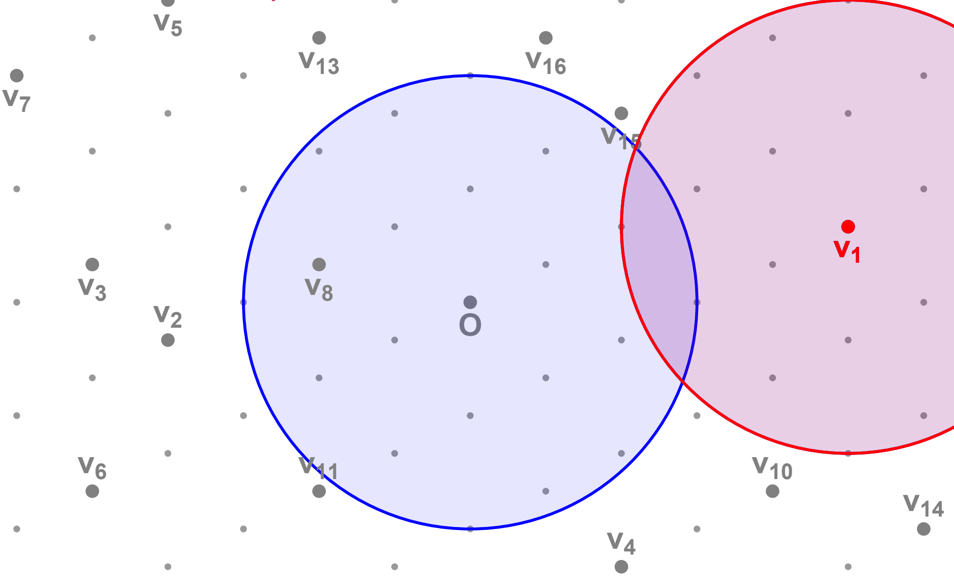

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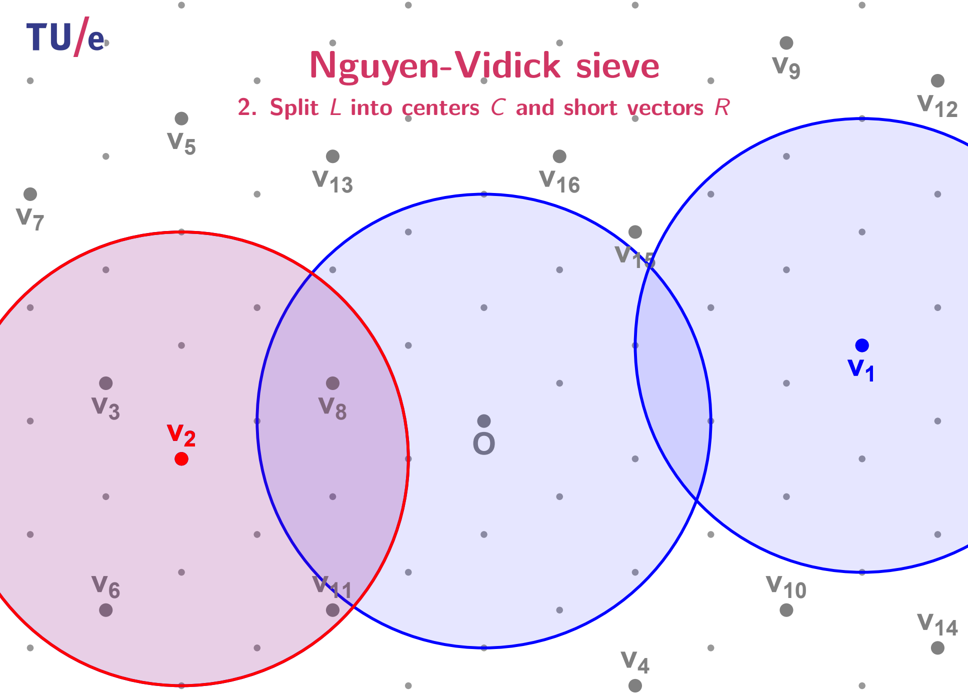

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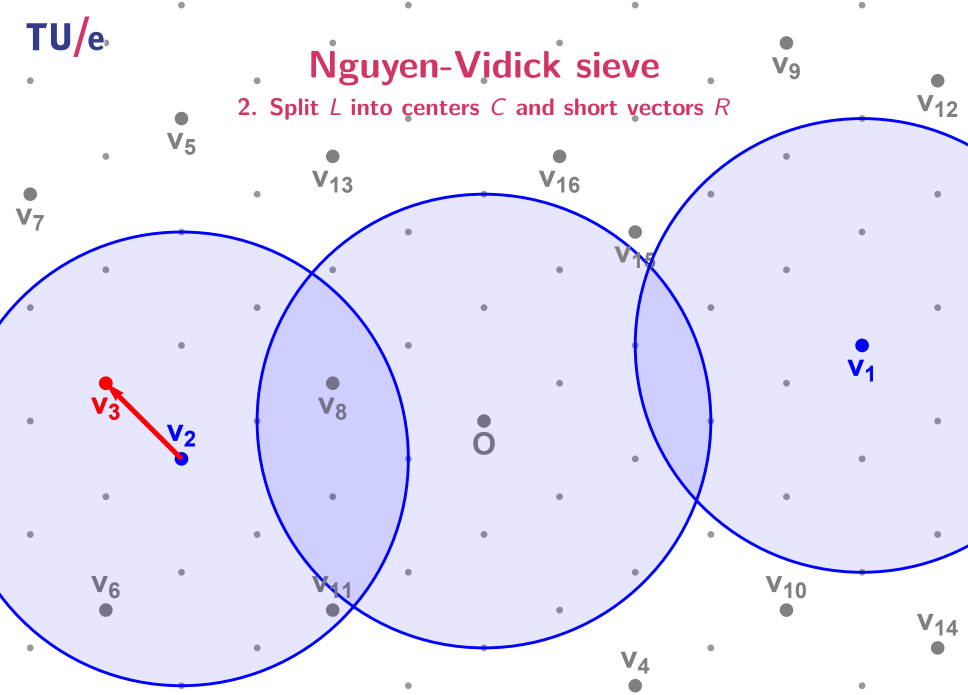

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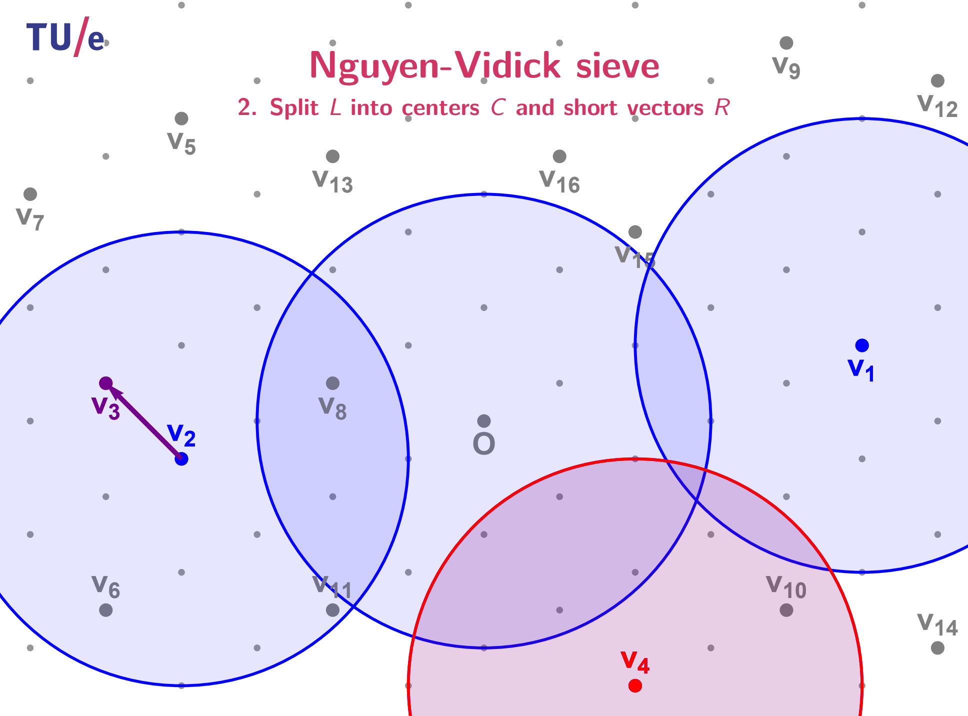

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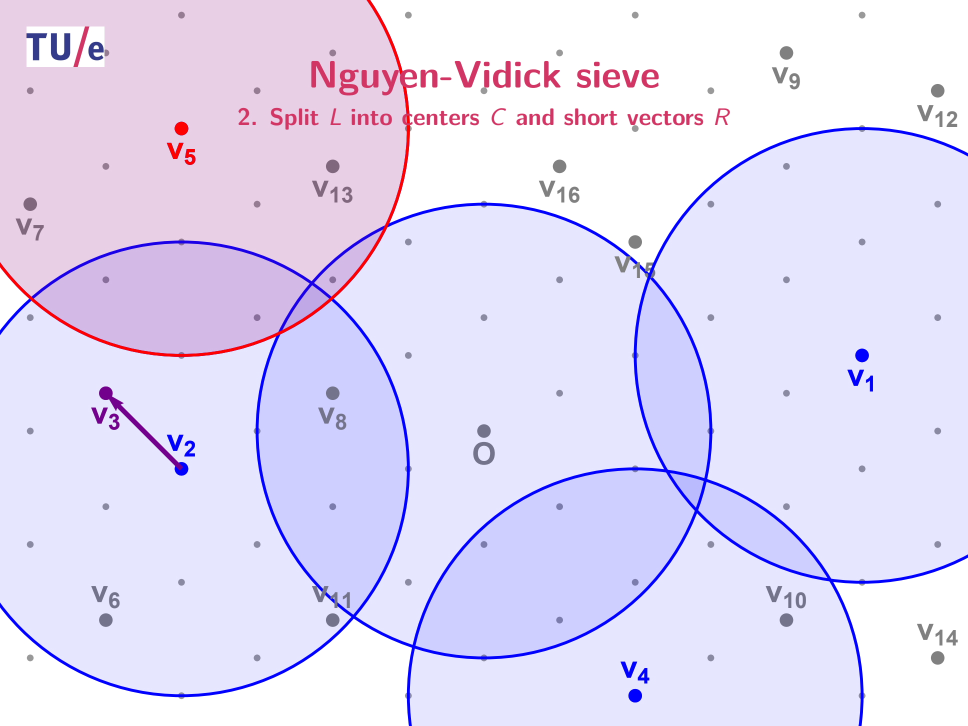

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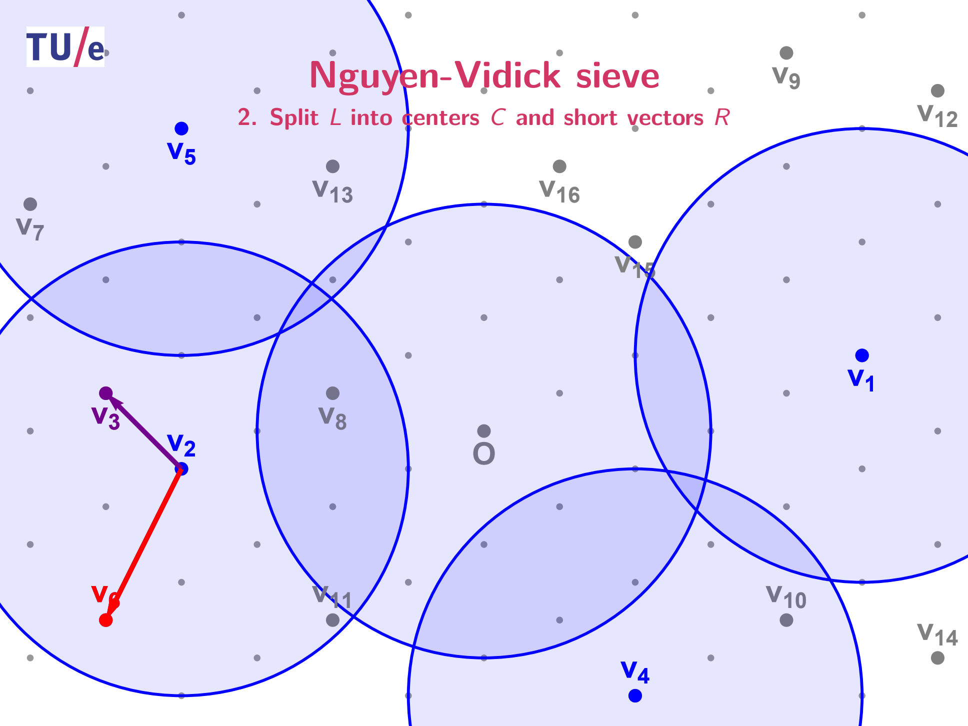

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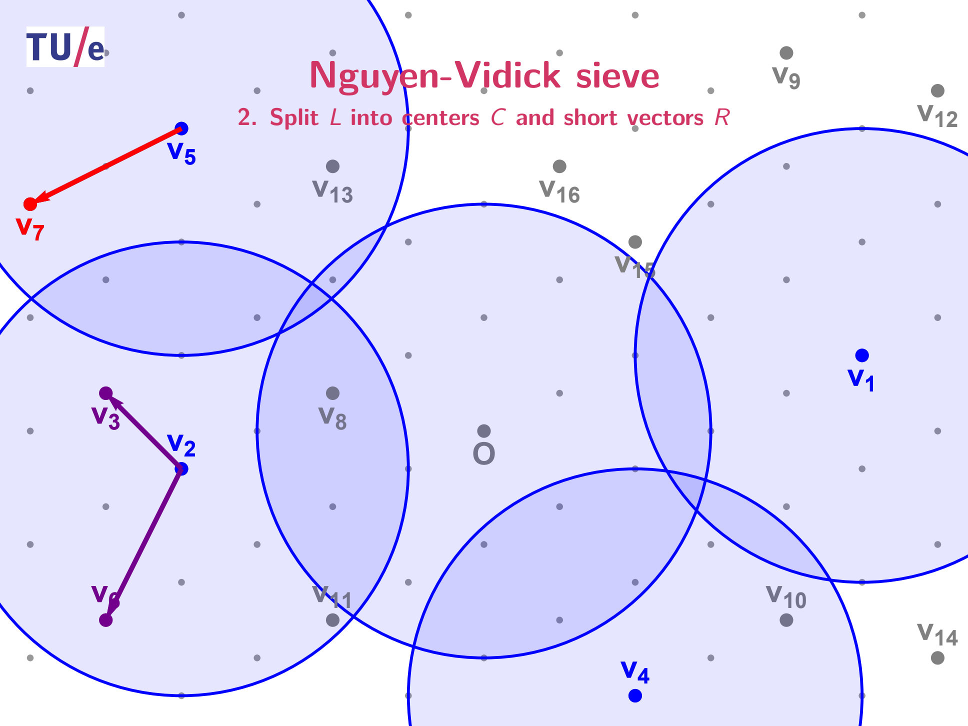

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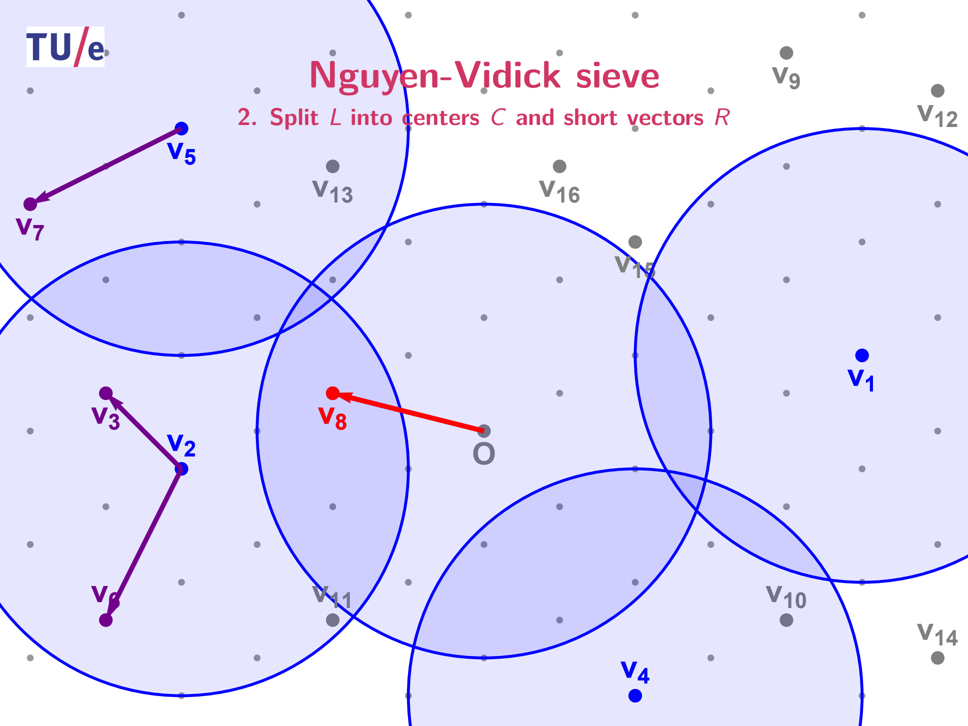

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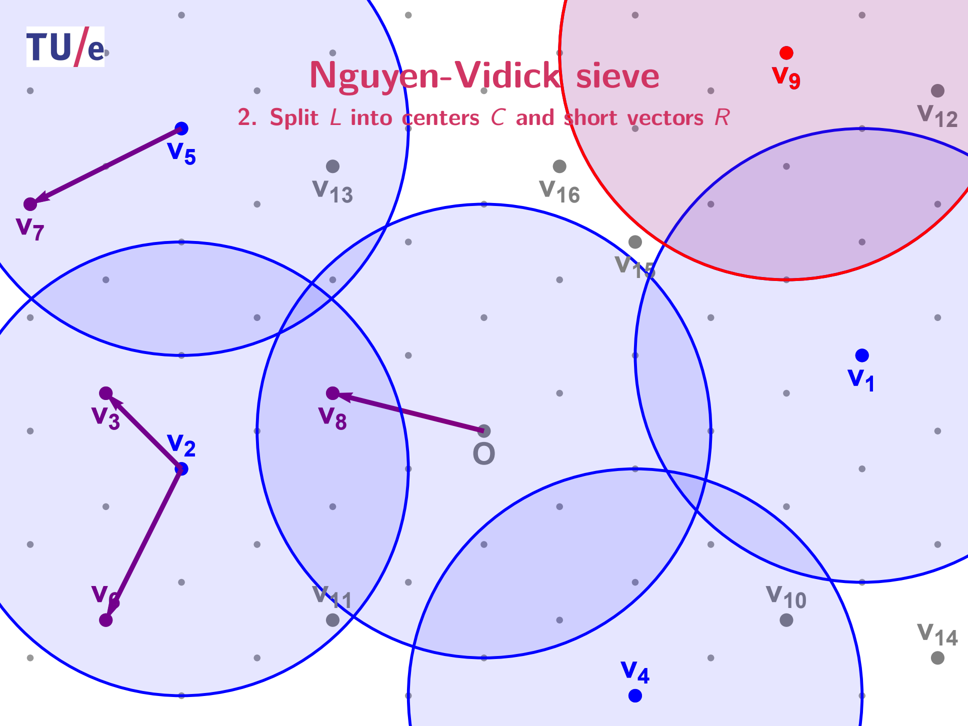

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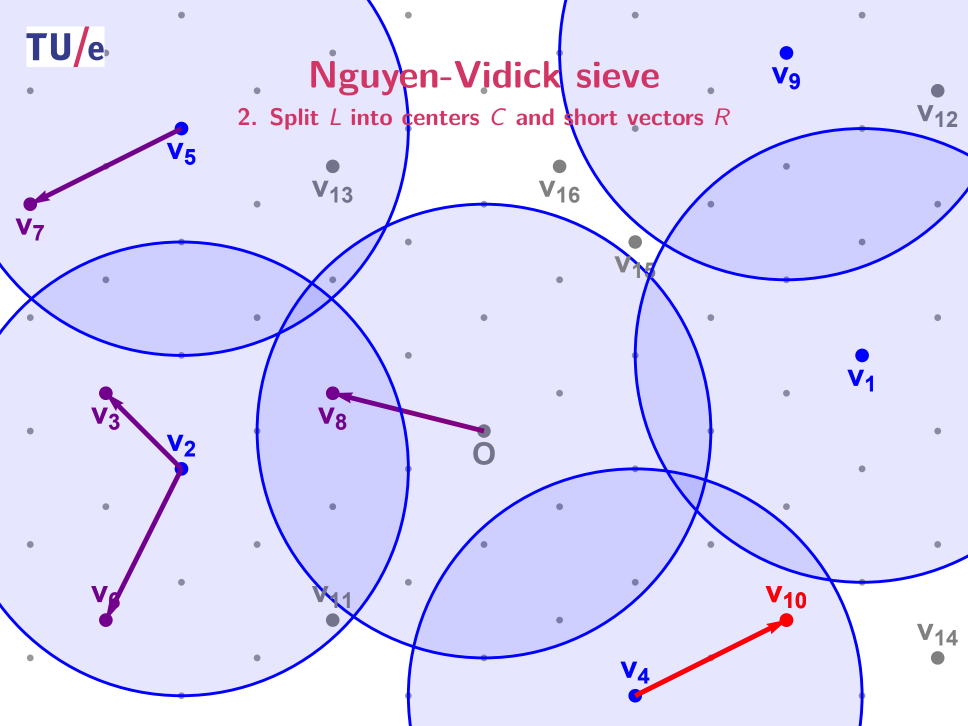

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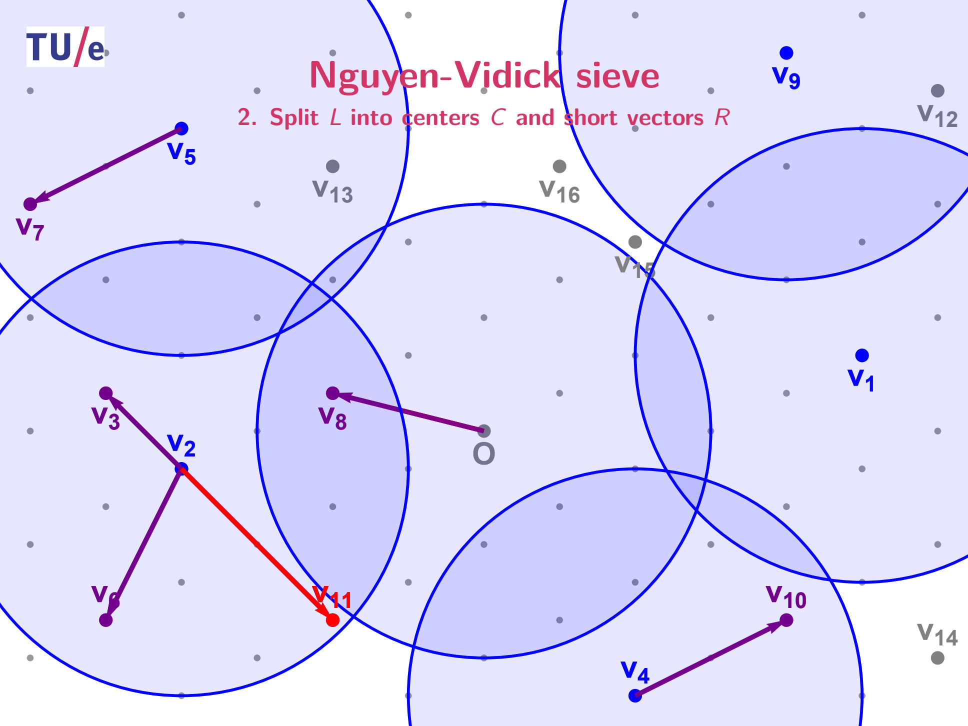

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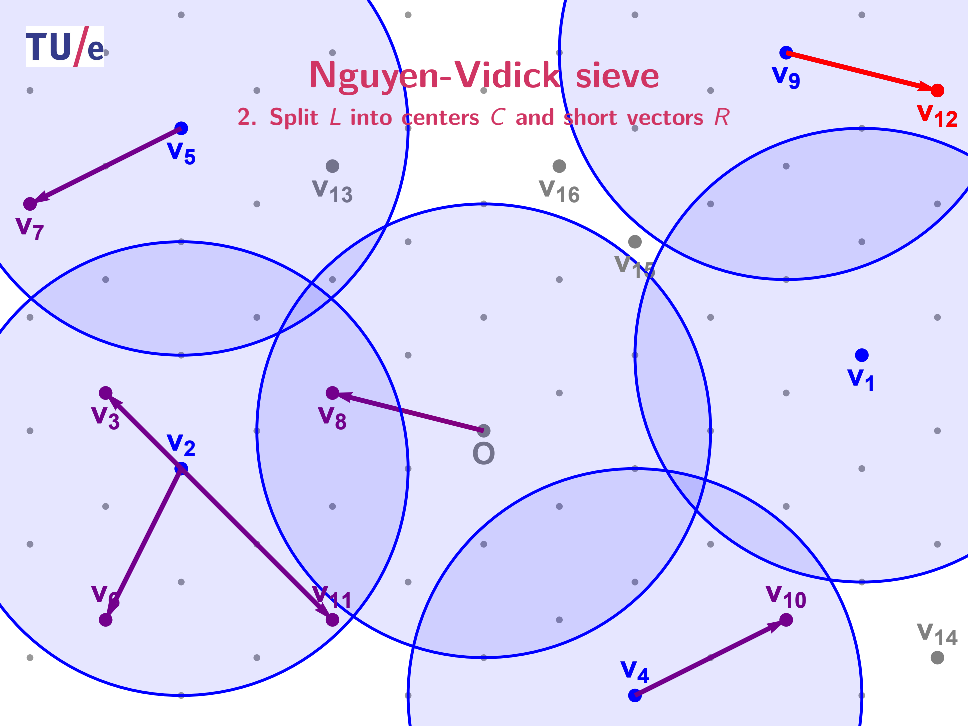

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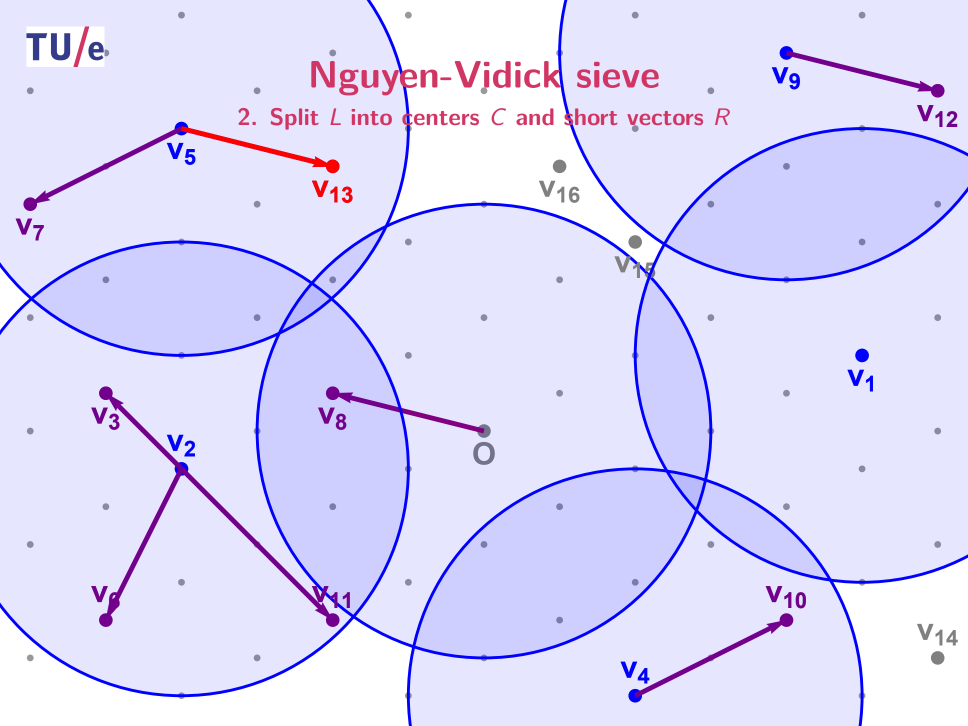

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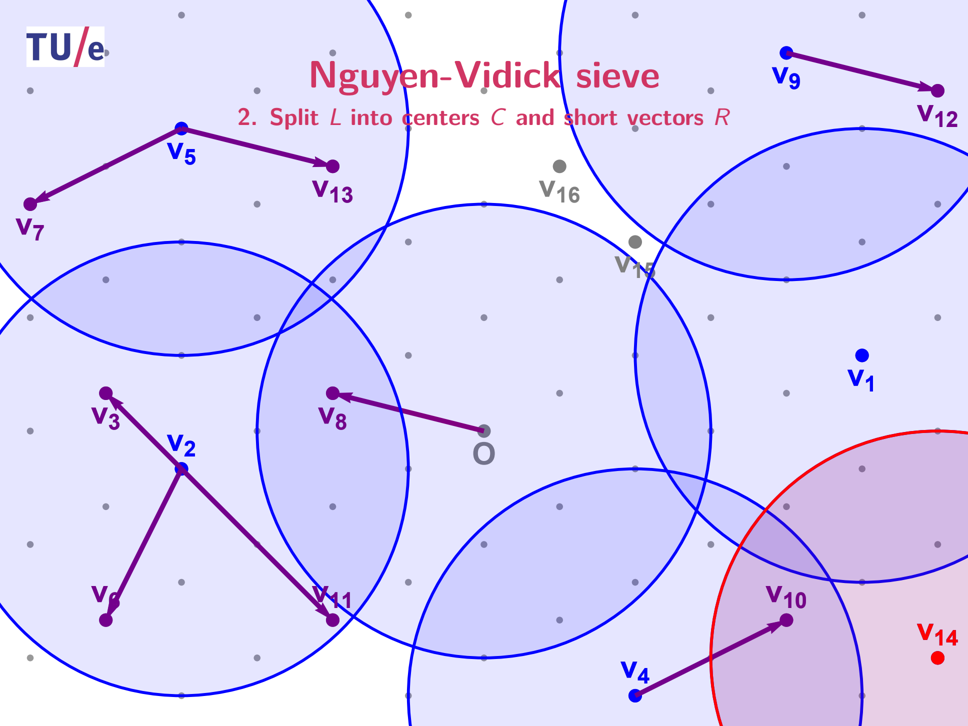

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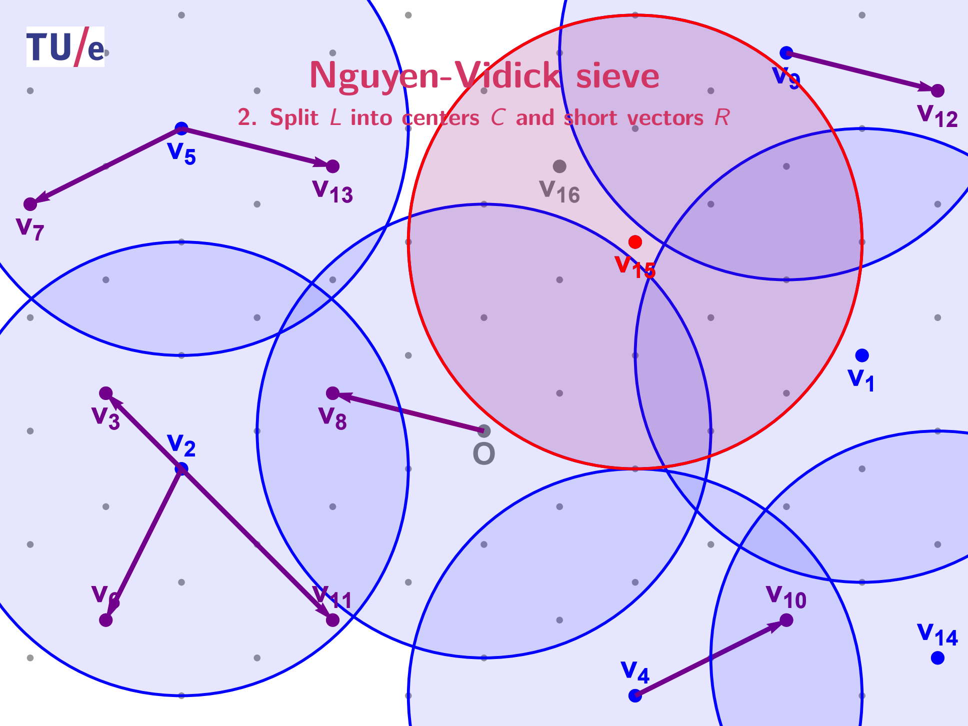

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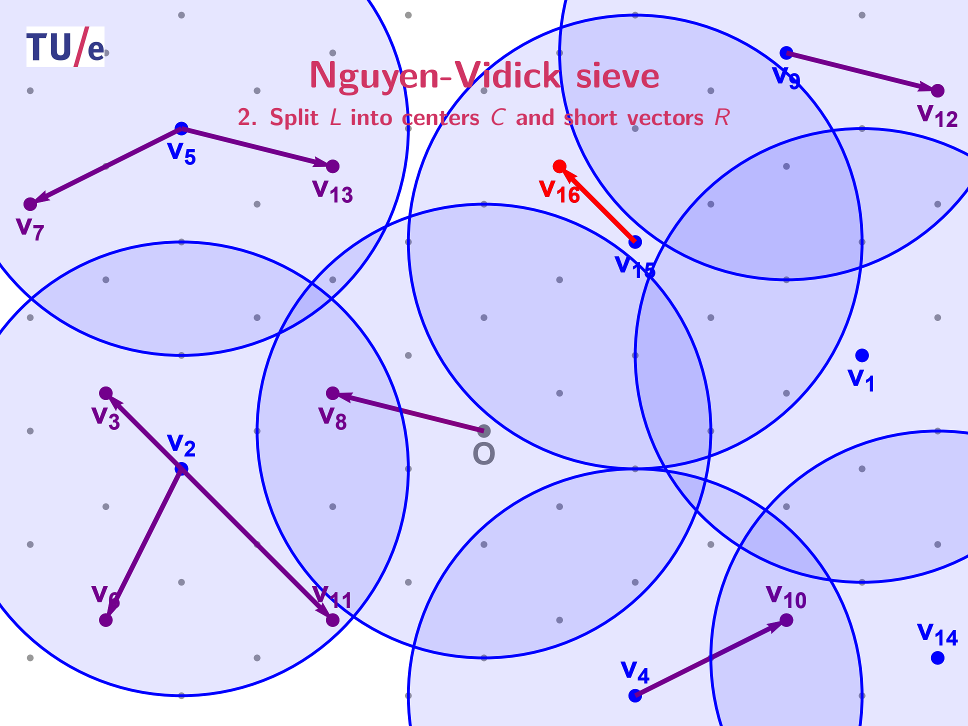

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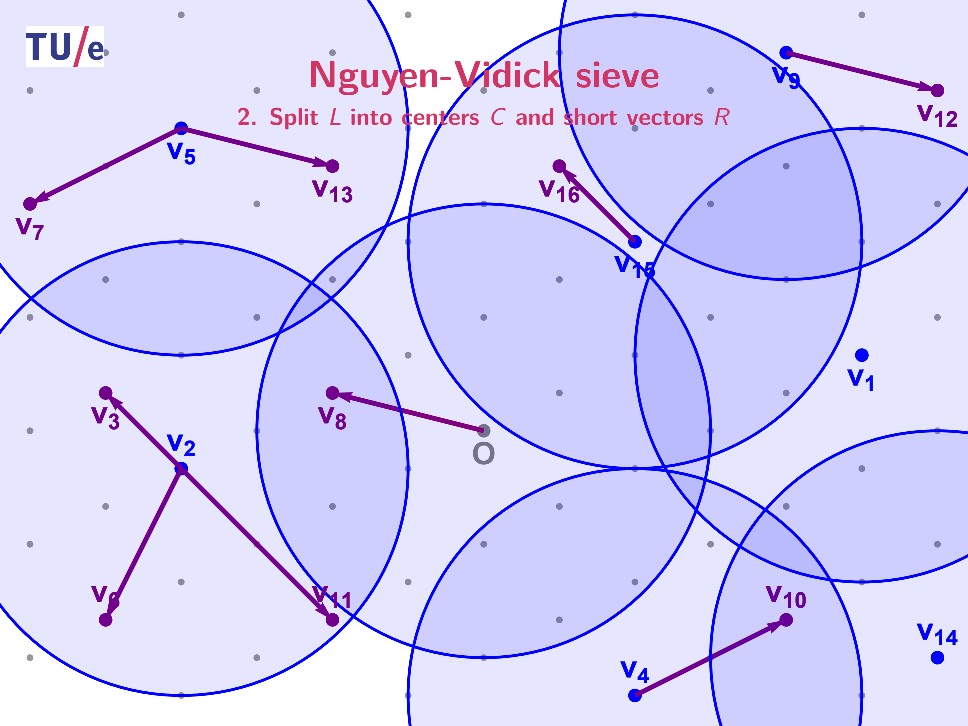

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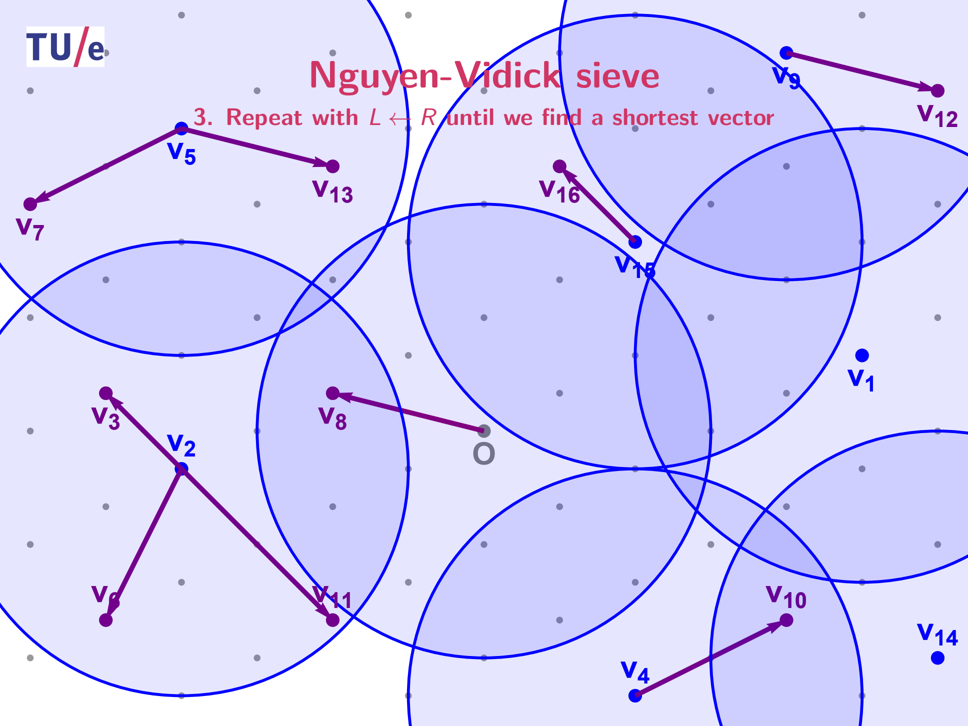

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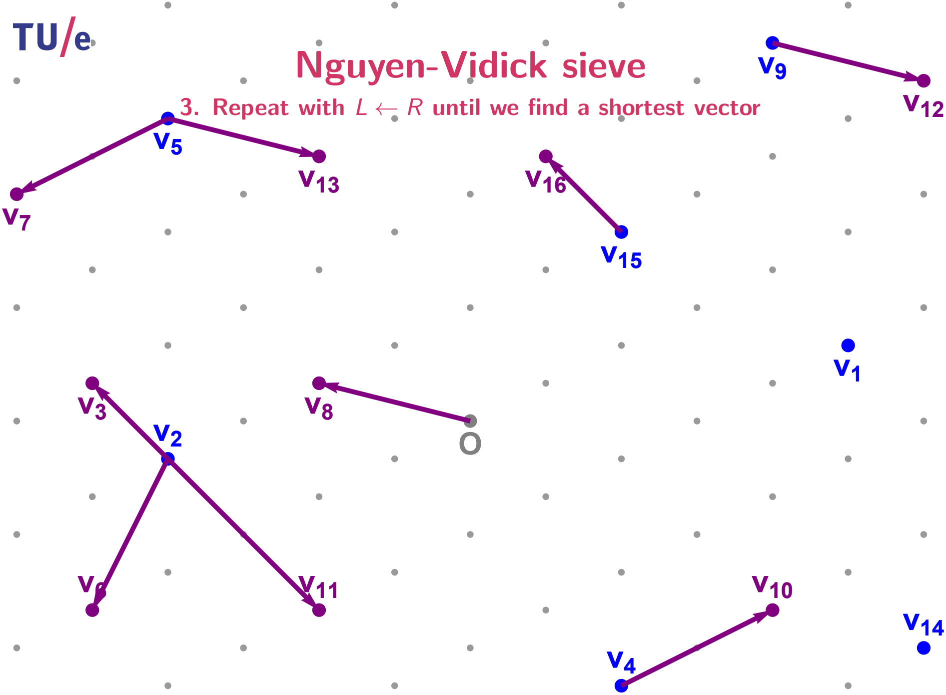

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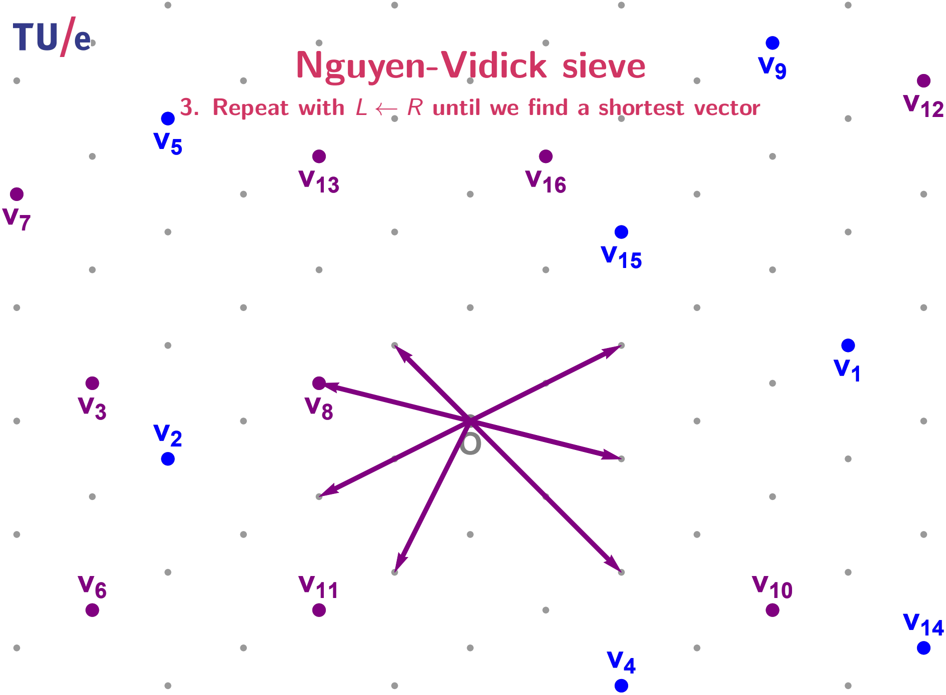

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Nguyen-Vidick sieve

Overview

Nguyen-Vidick sieve

Overview

Heuristic (Nguyen and Vidick, J. Math. Crypt. '08)

The Nguyen-Vidick sieve runs in time $(4/3)^n$ and space $\sqrt{4/3}^n$.

Two-level sieve

1. Sample a list L of random lattice vectors

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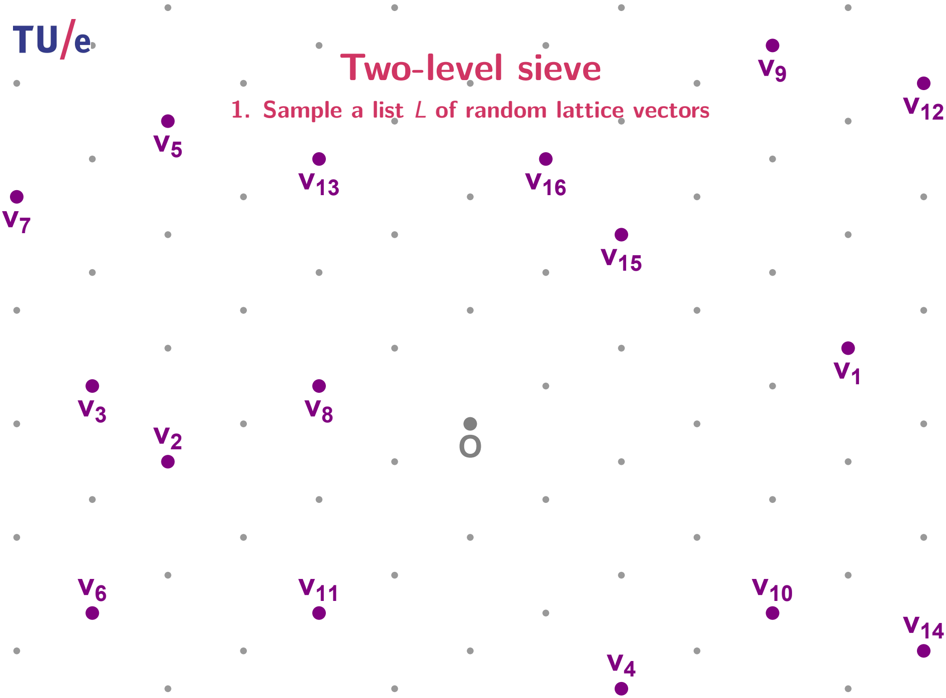
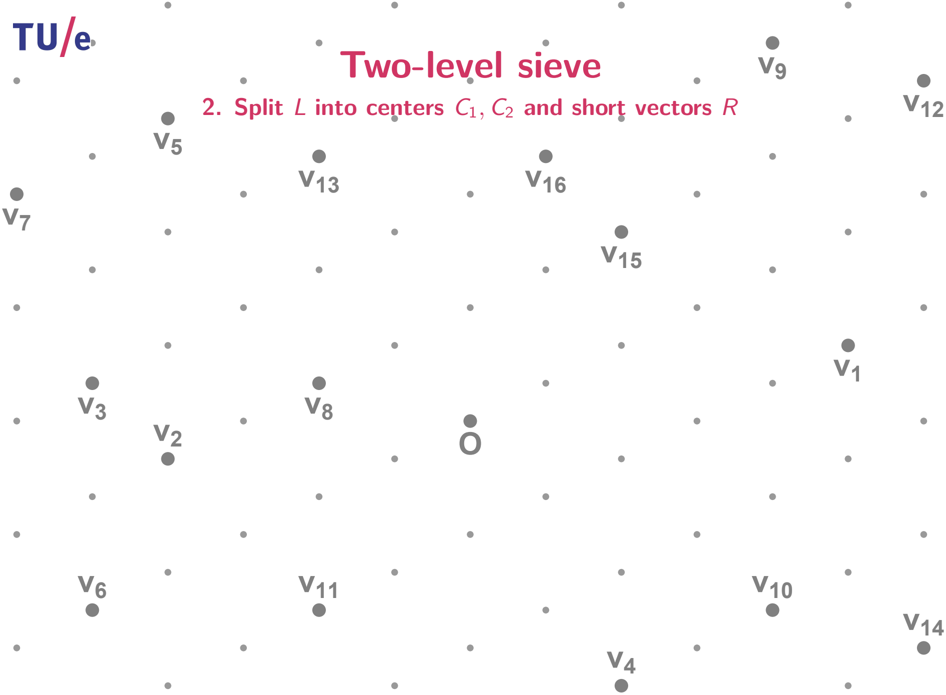


Two-level sieve

2. Split L into centers C_1, C_2 and short vectors R



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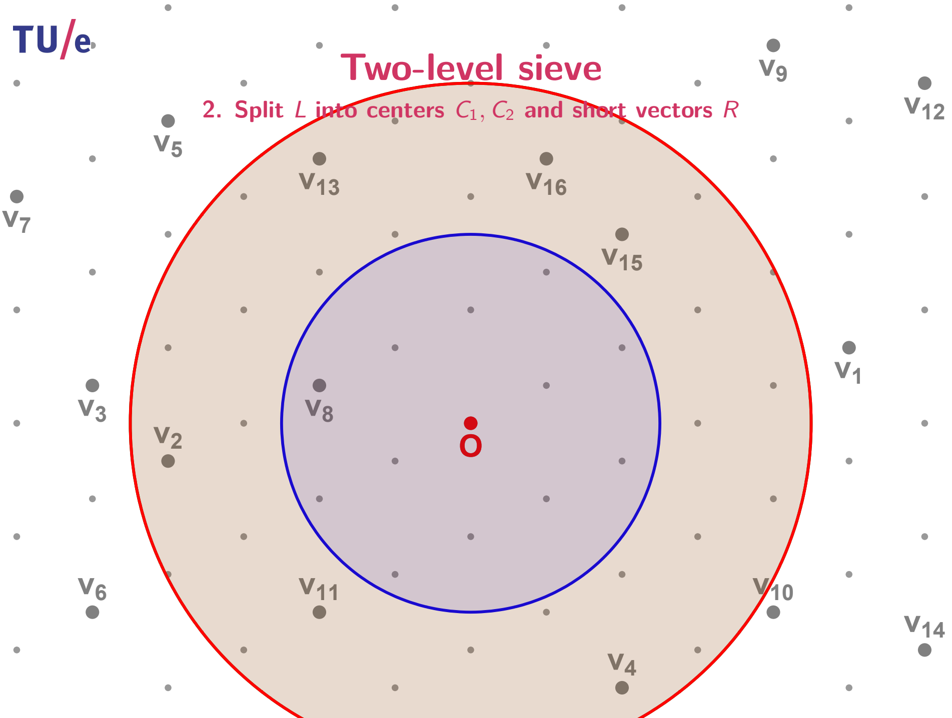

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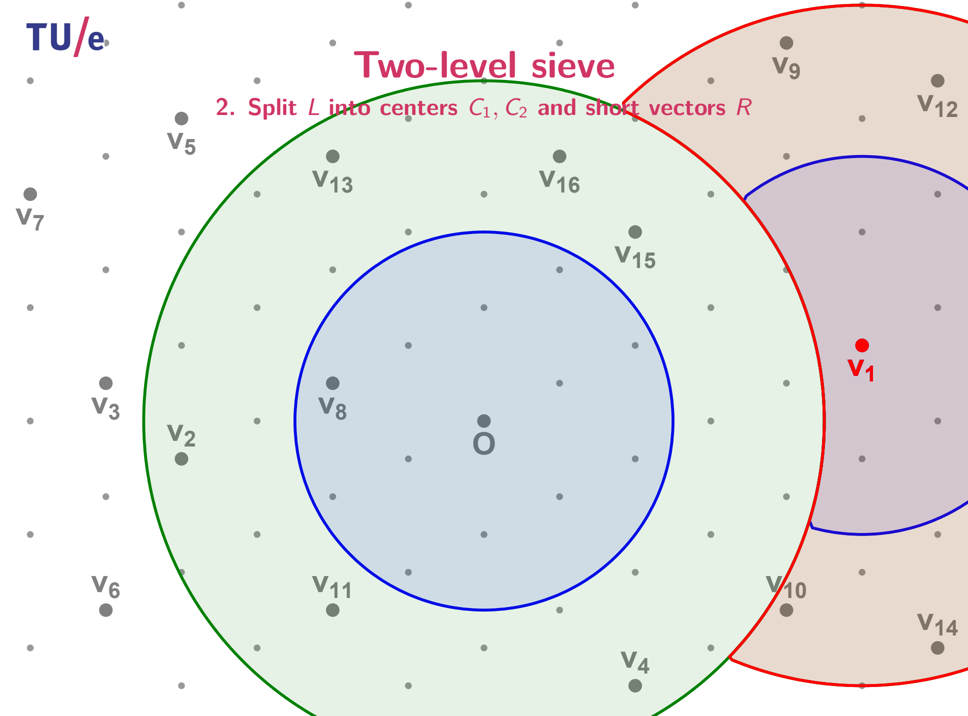

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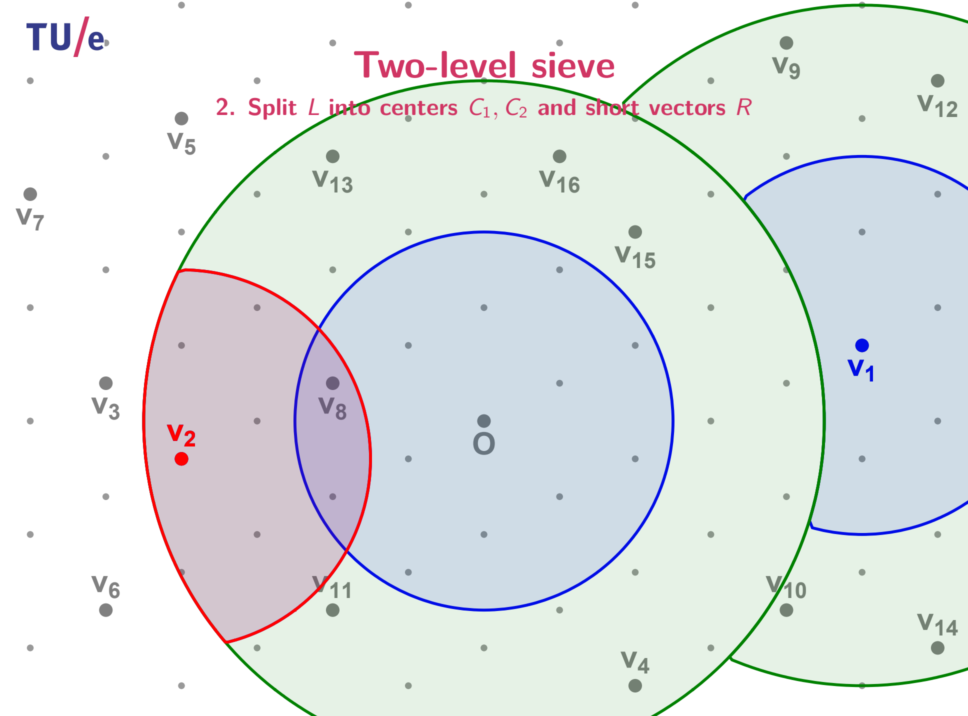

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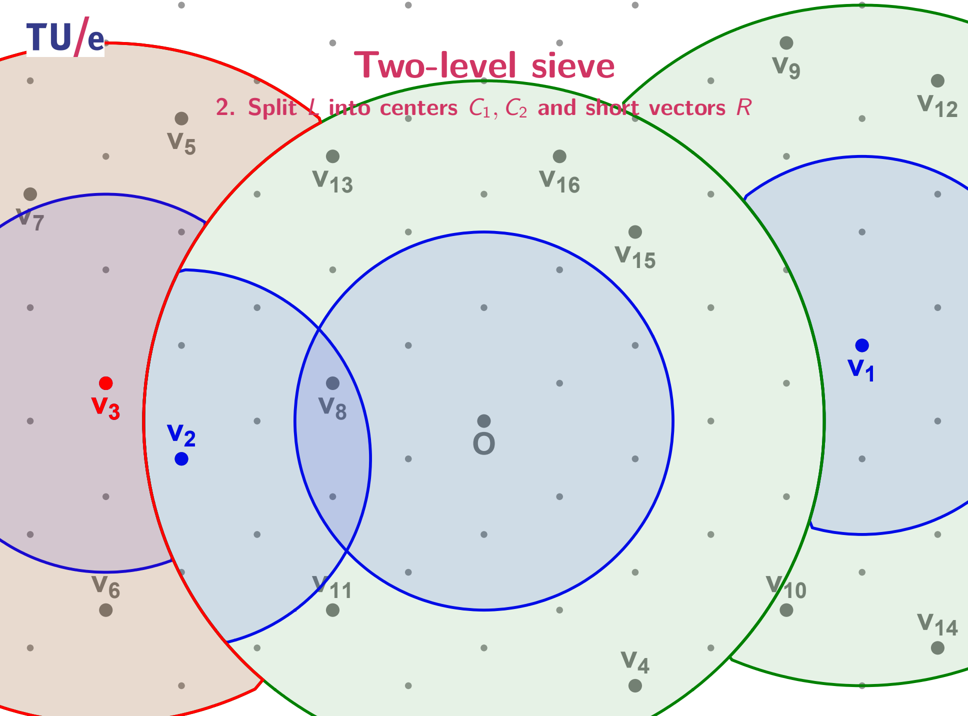

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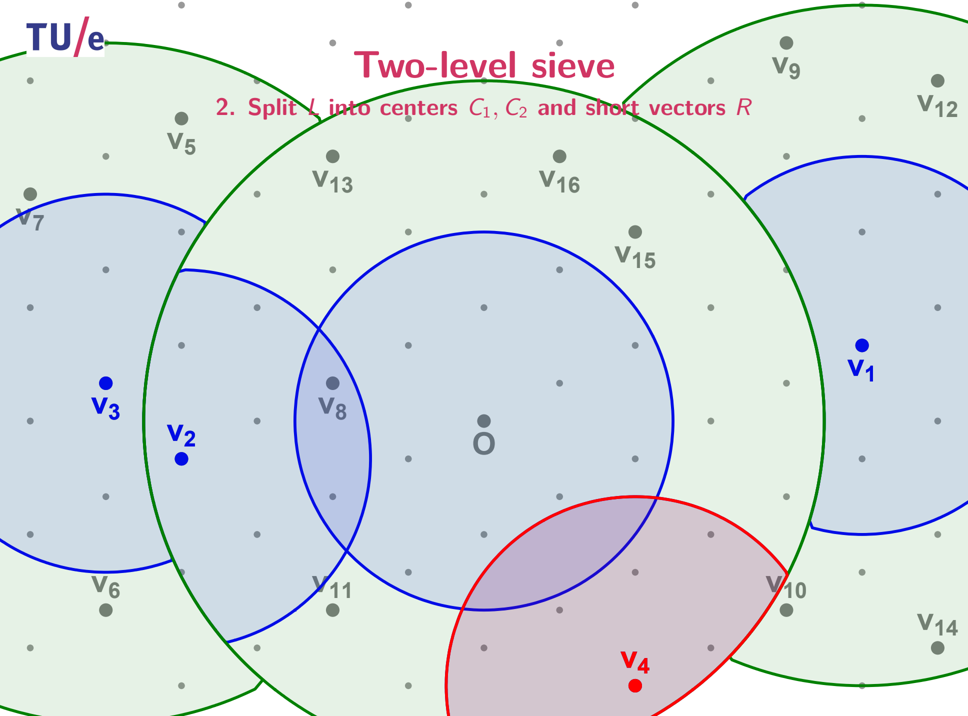

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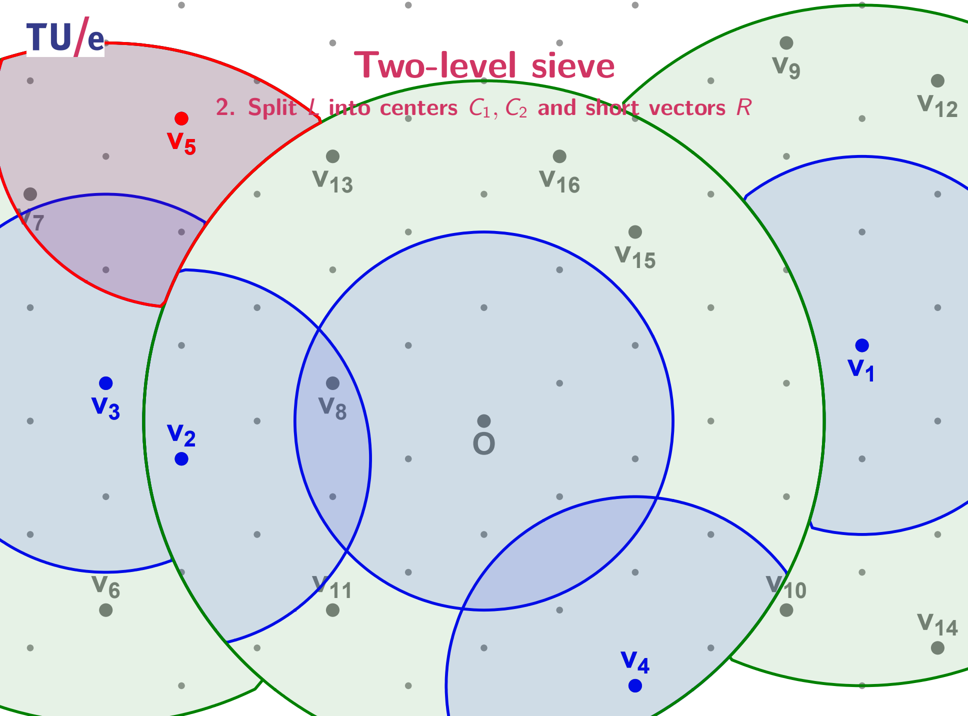

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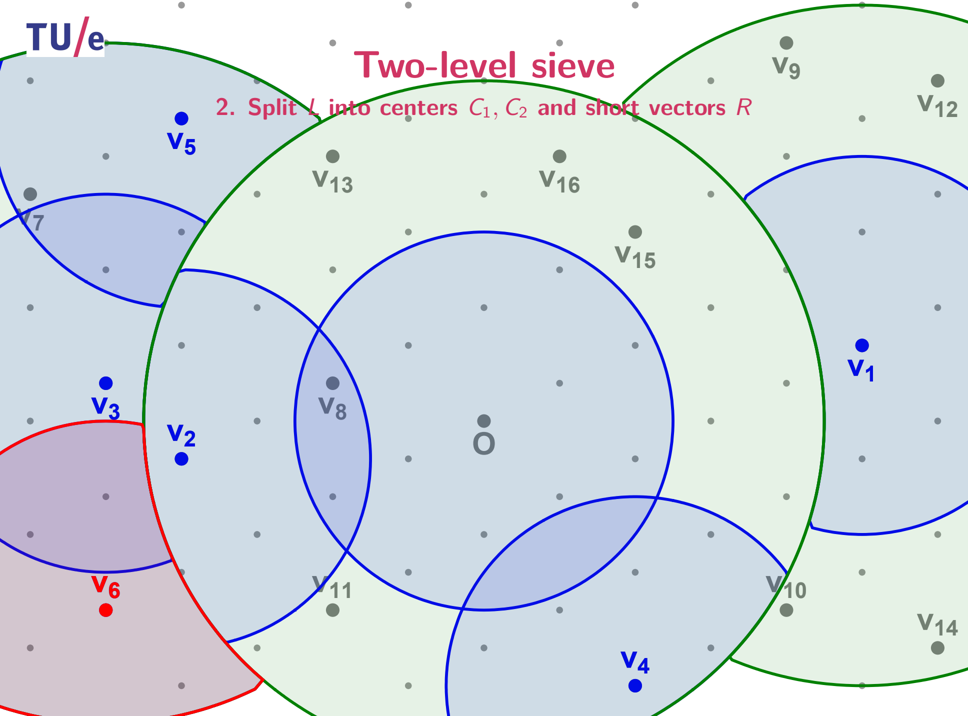

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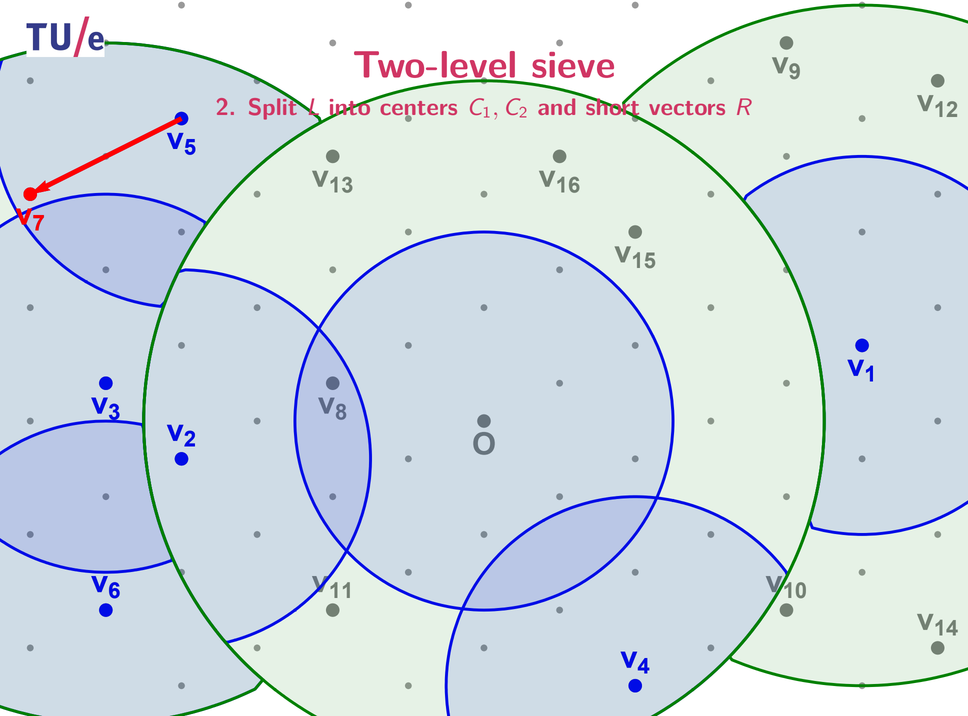

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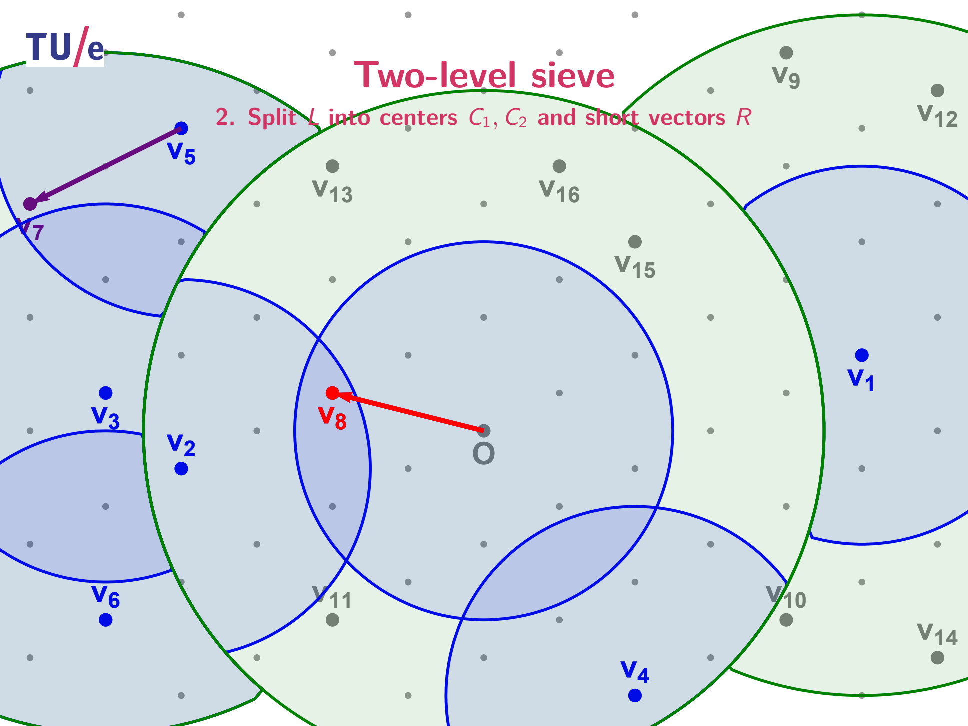

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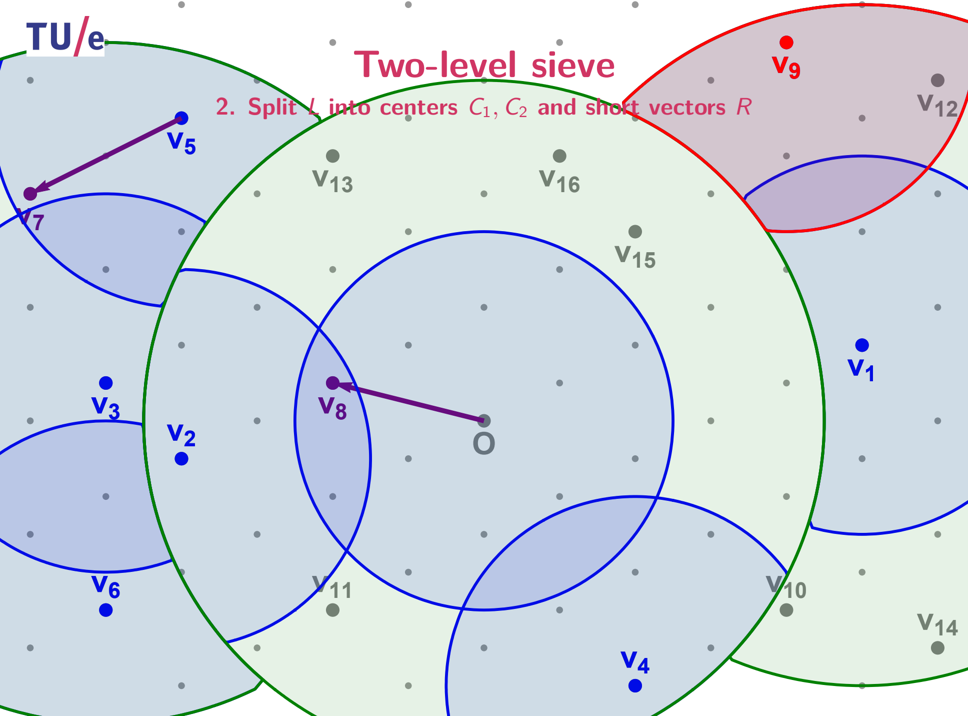

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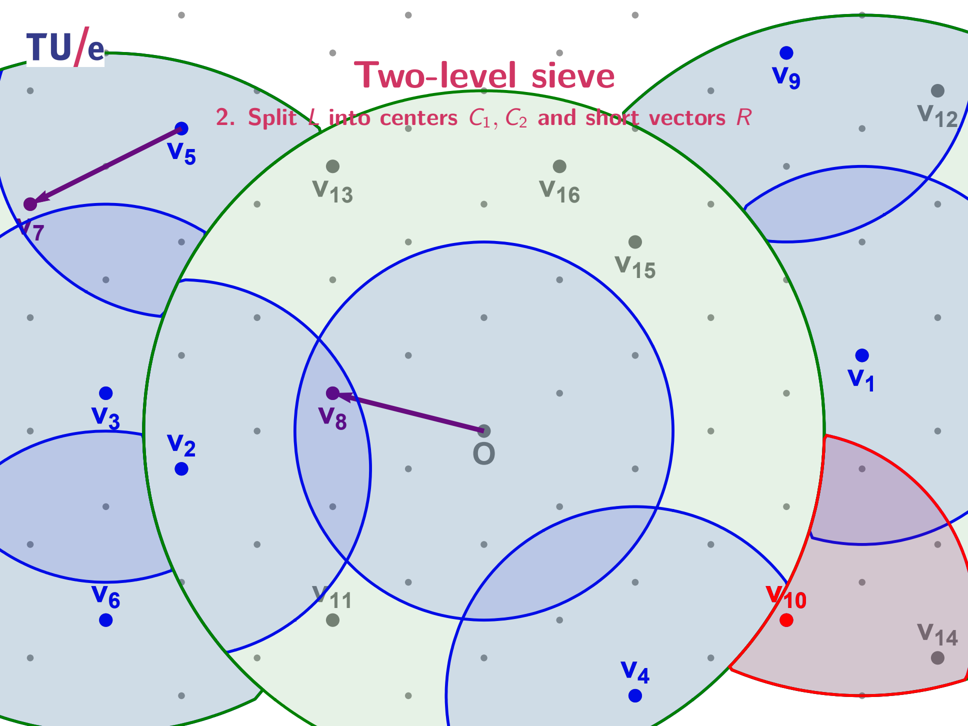

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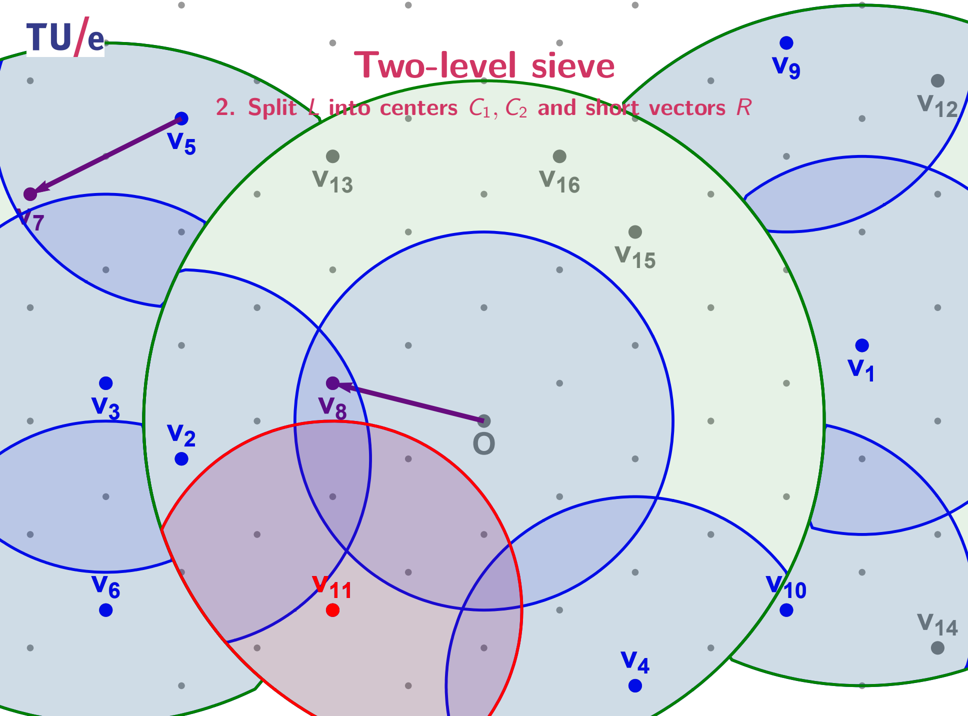

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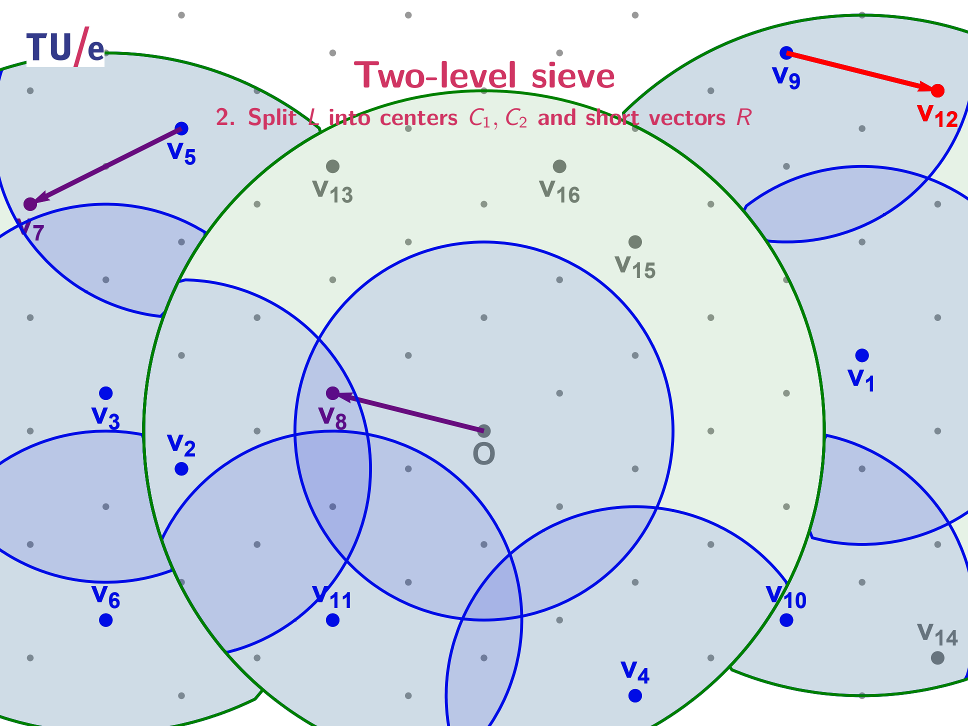

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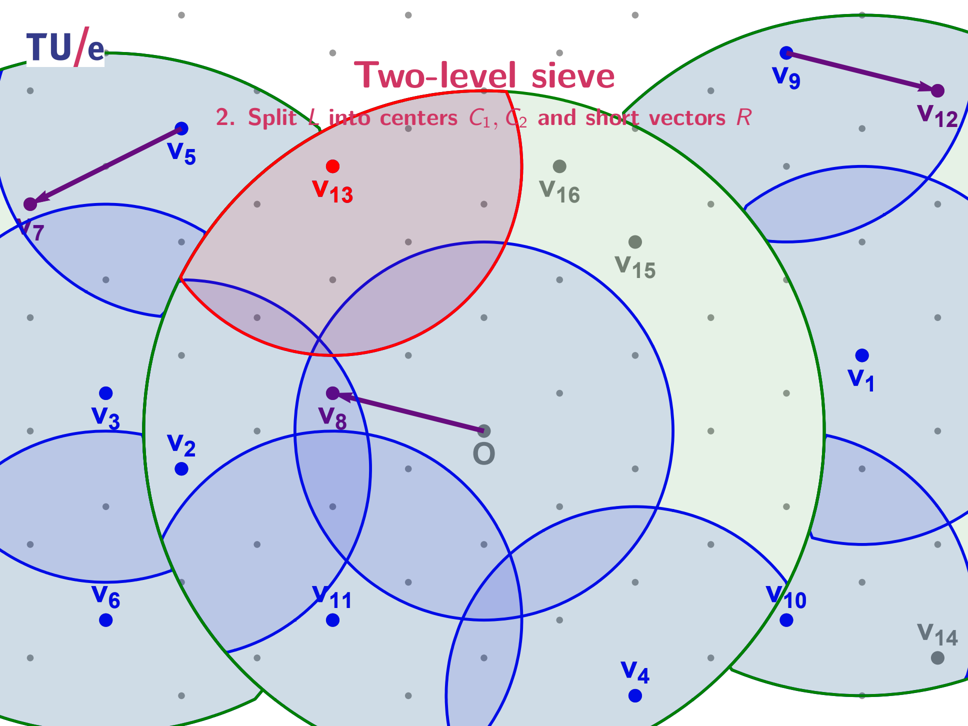

TU/e

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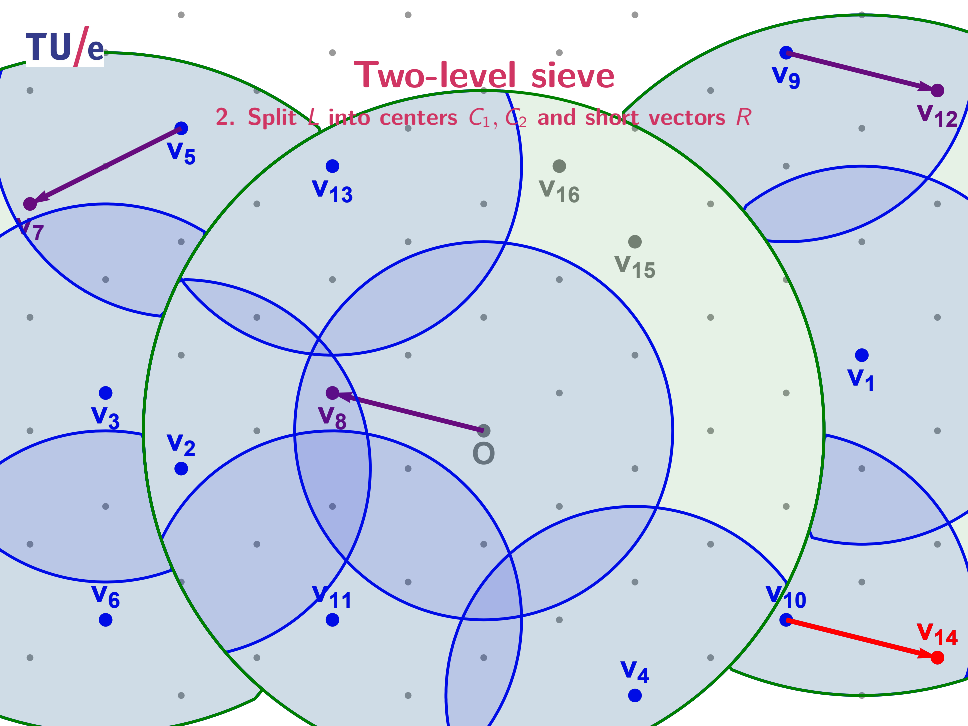
2. Split L into centers C_1, C_2 and short vectors R



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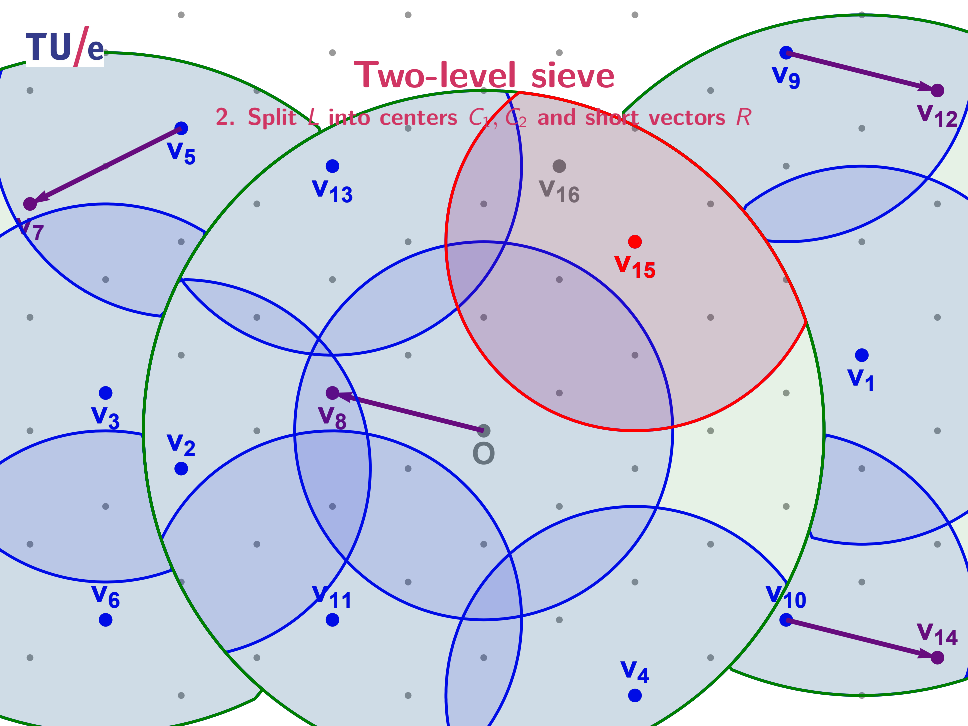

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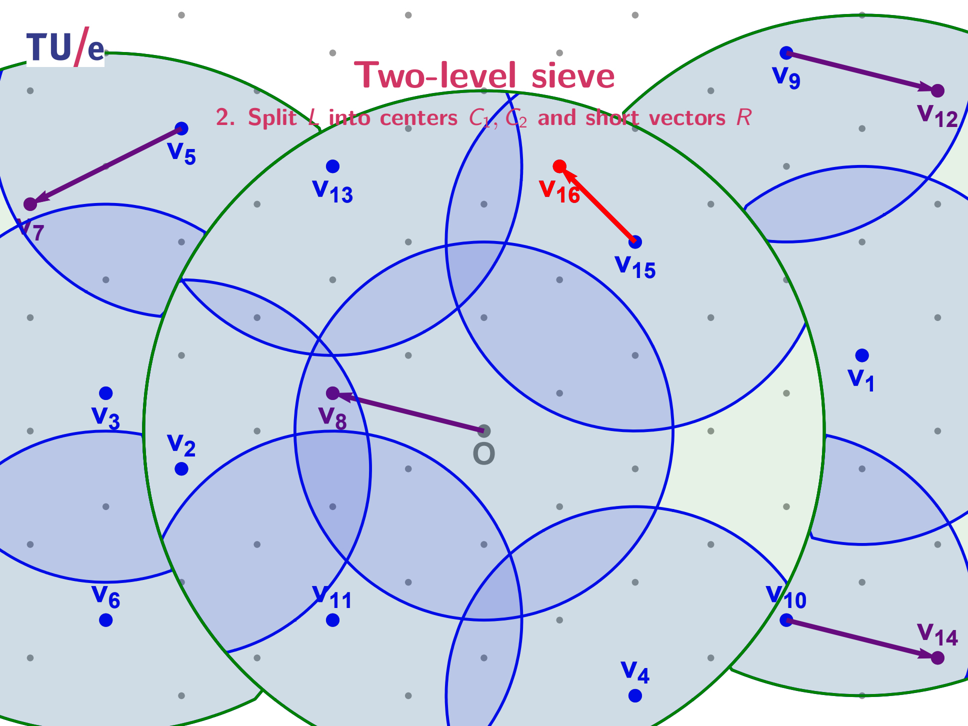

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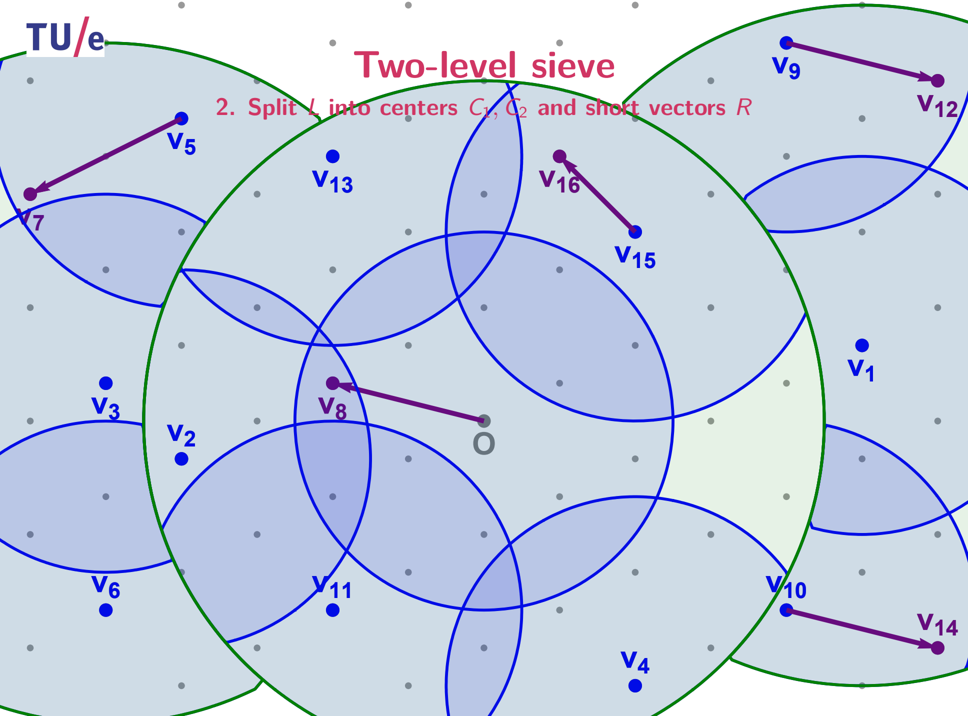

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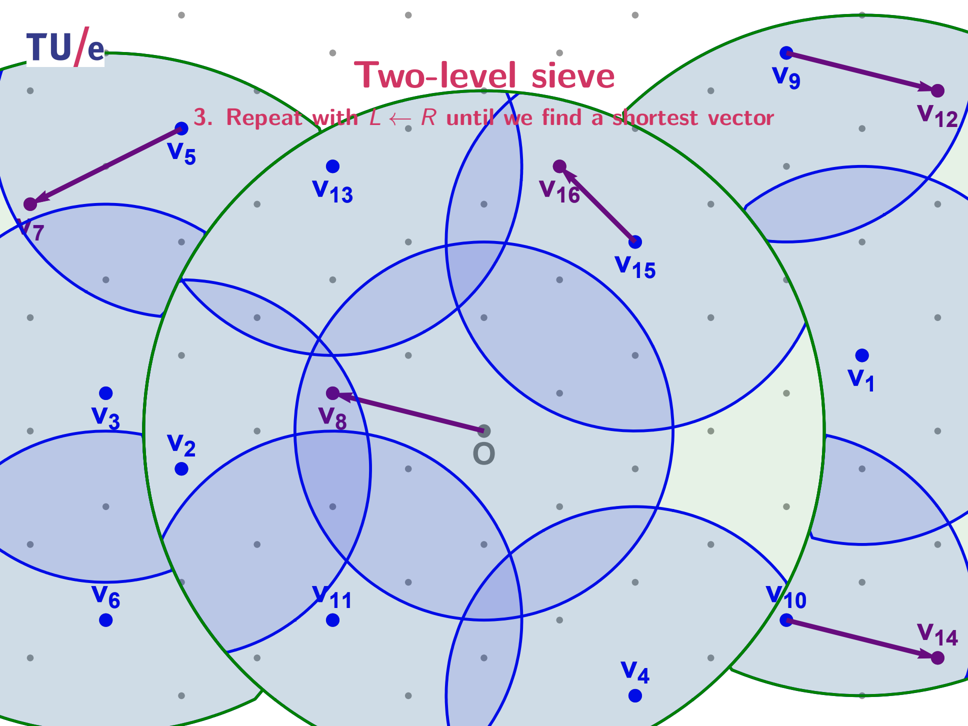

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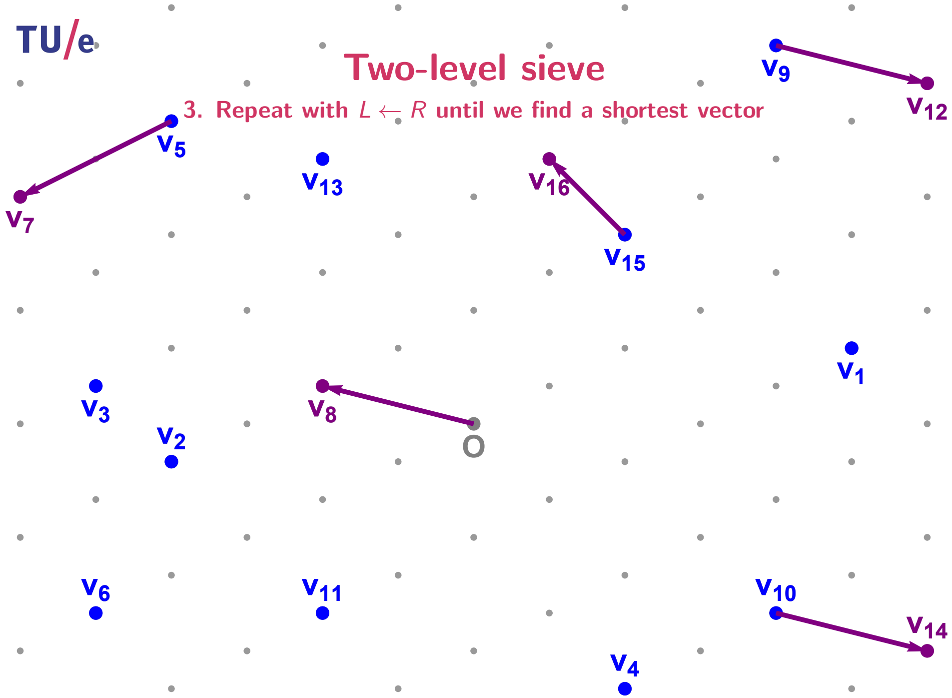

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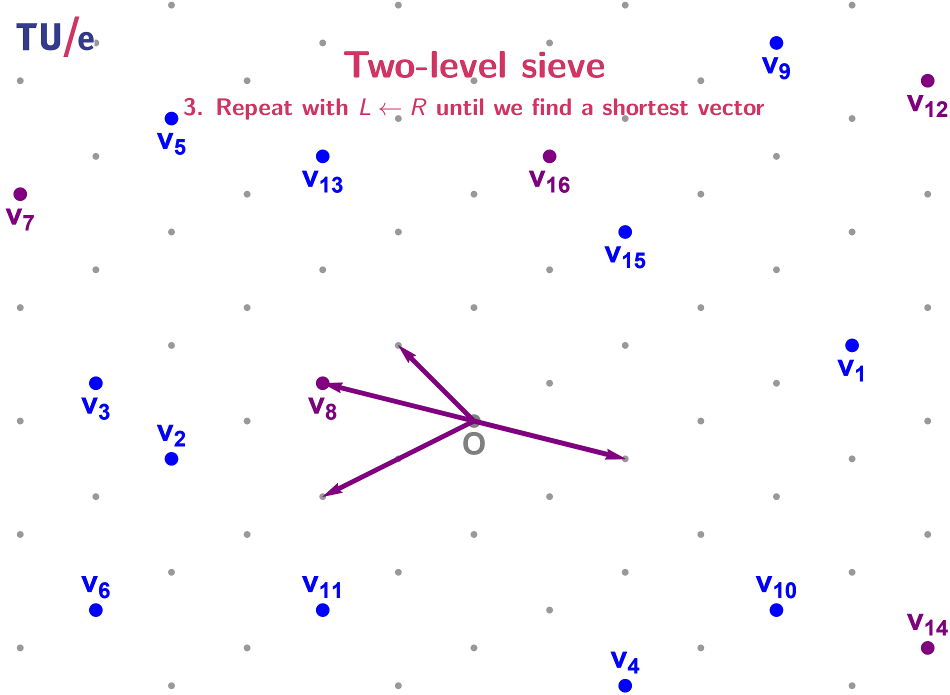

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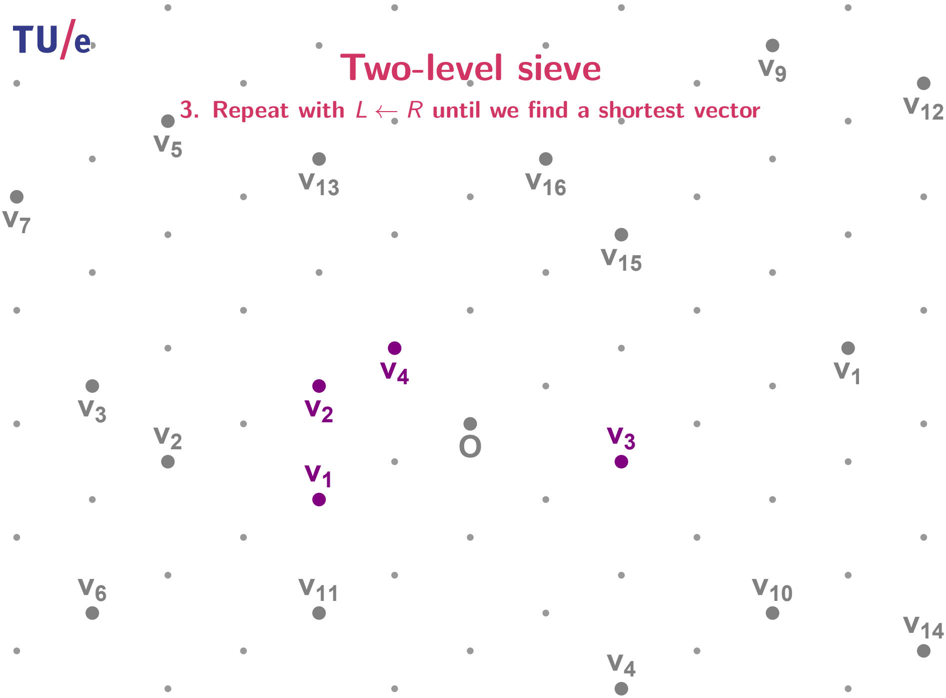
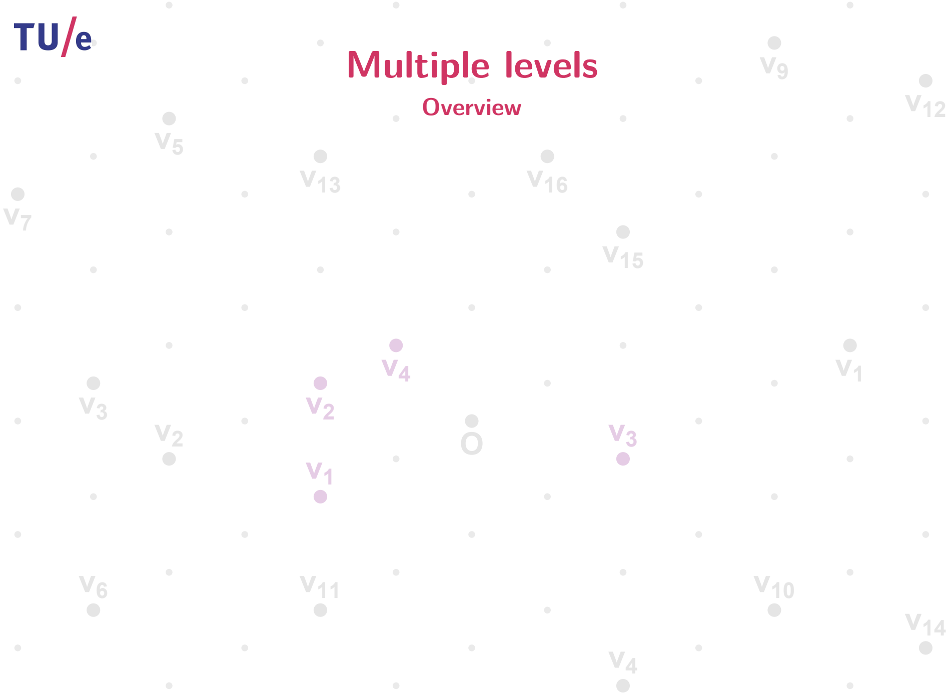


Multiple levels

Overview



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Heuristic (Nguyen and Vidick, J. Math. Crypt. '08)

The one-level sieve runs in time $2^{0.4150n}$ and space $2^{0.2075n}$.

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Heuristic (Zhang et al., SAC'13)

The three-level sieve runs in time $2^{0.3778n}$ and space $2^{0.2833n}$.

GaussSieve

1. Generate random lattice vectors

GaussSieve

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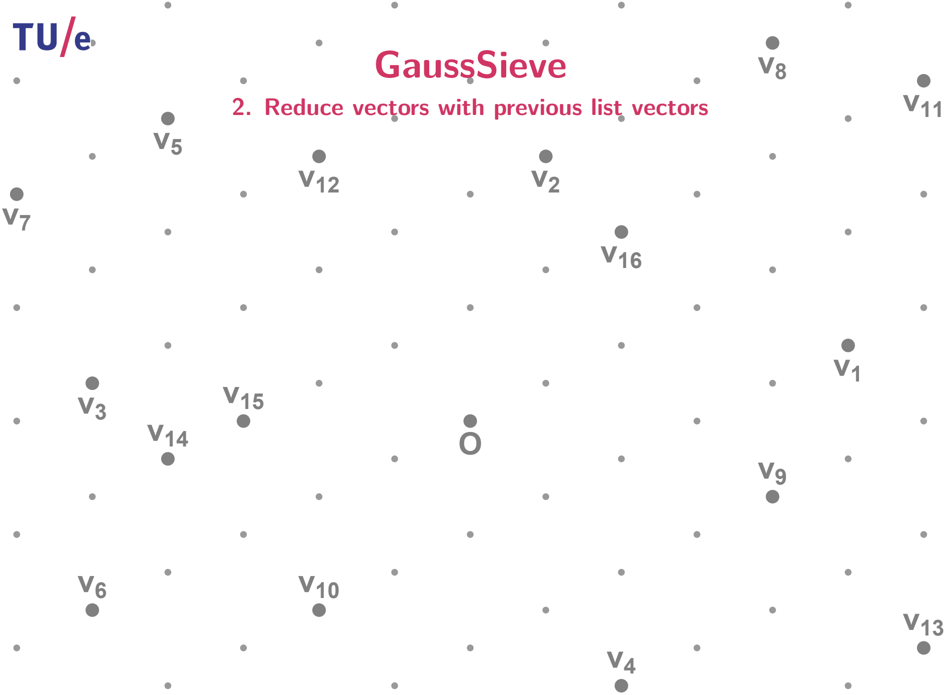
GaussSieve

2. Reduce vectors with previous list vectors



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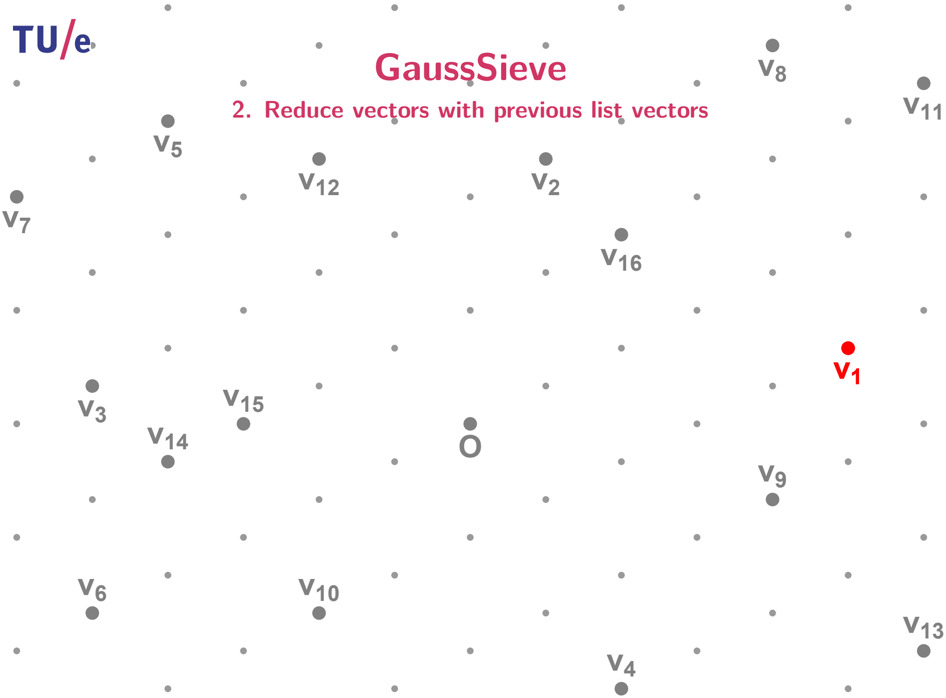

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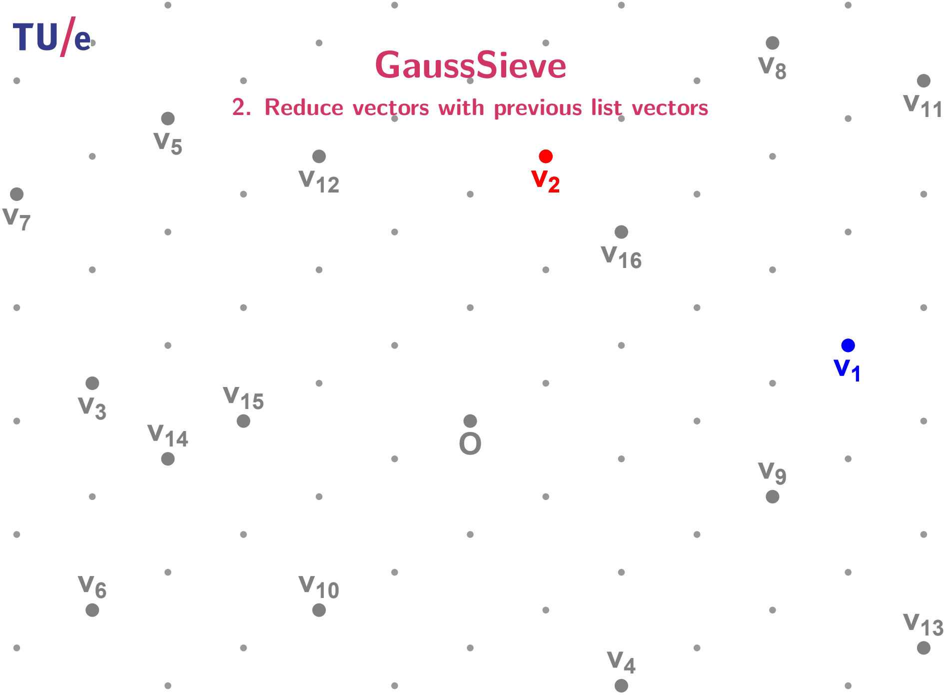

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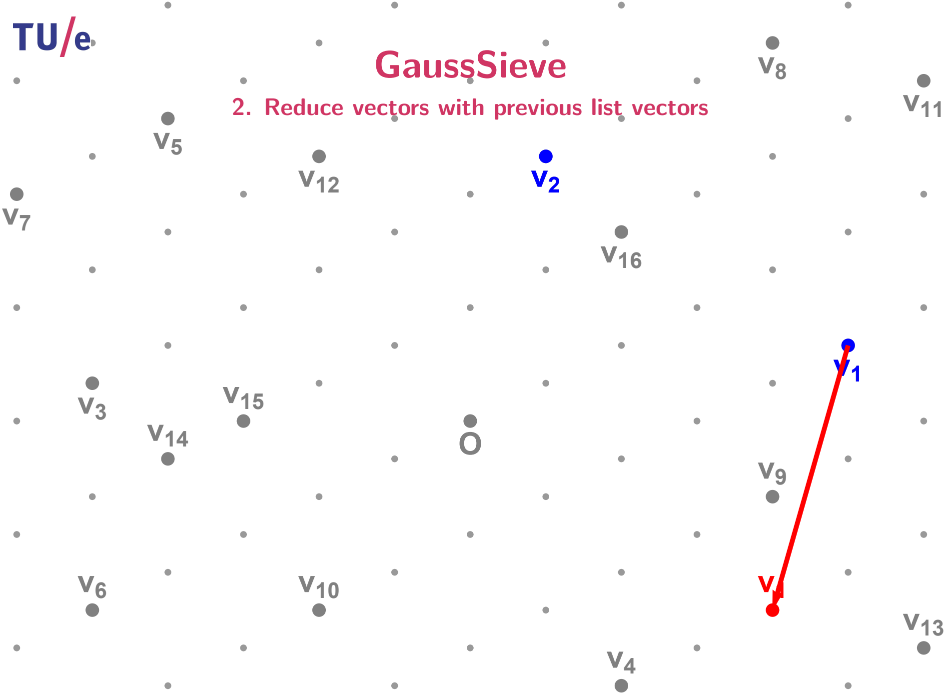

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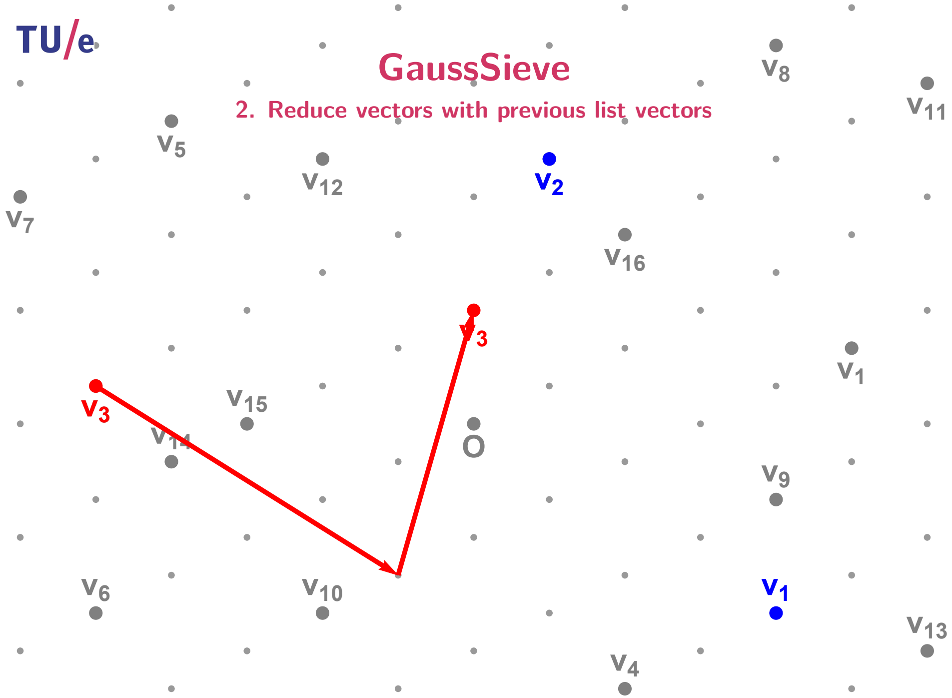

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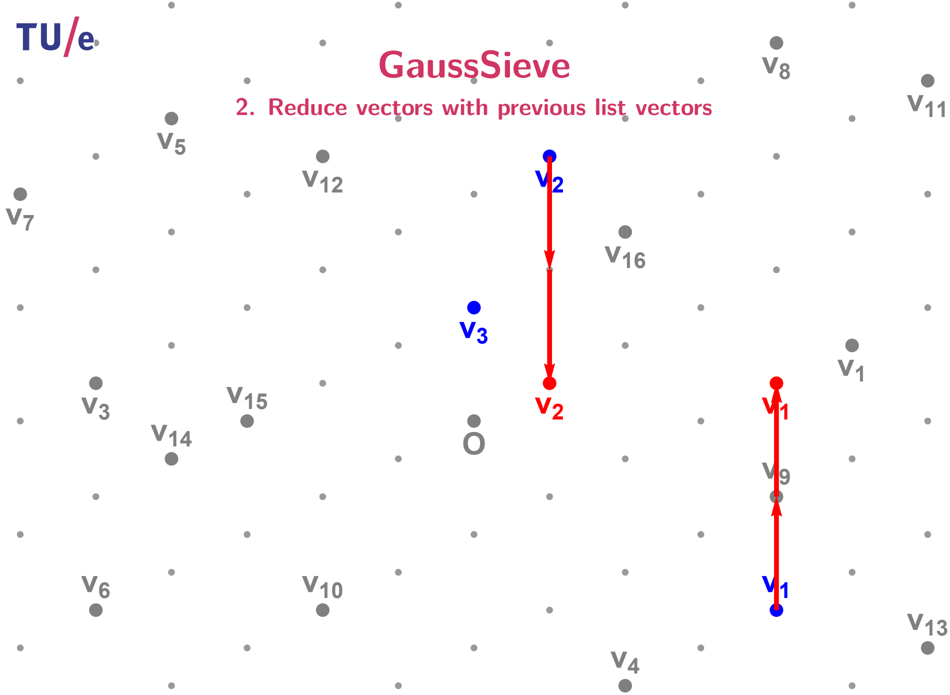

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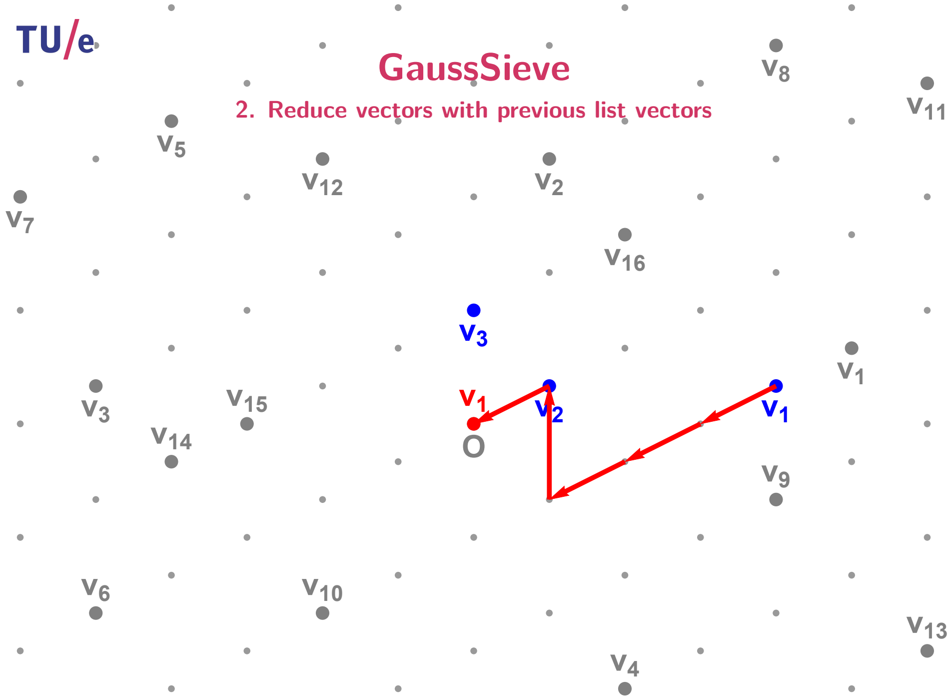

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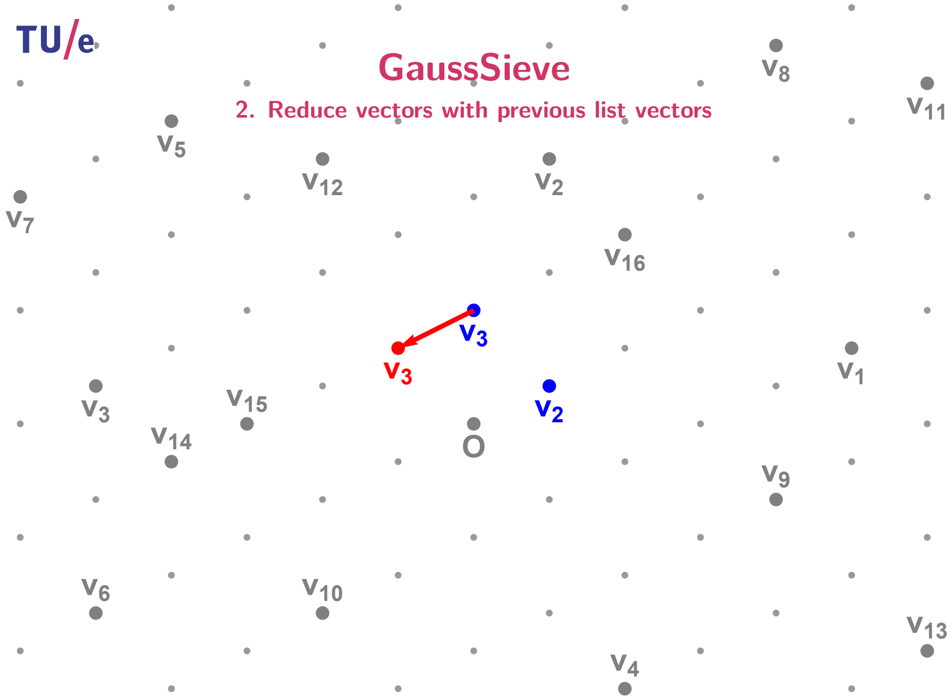

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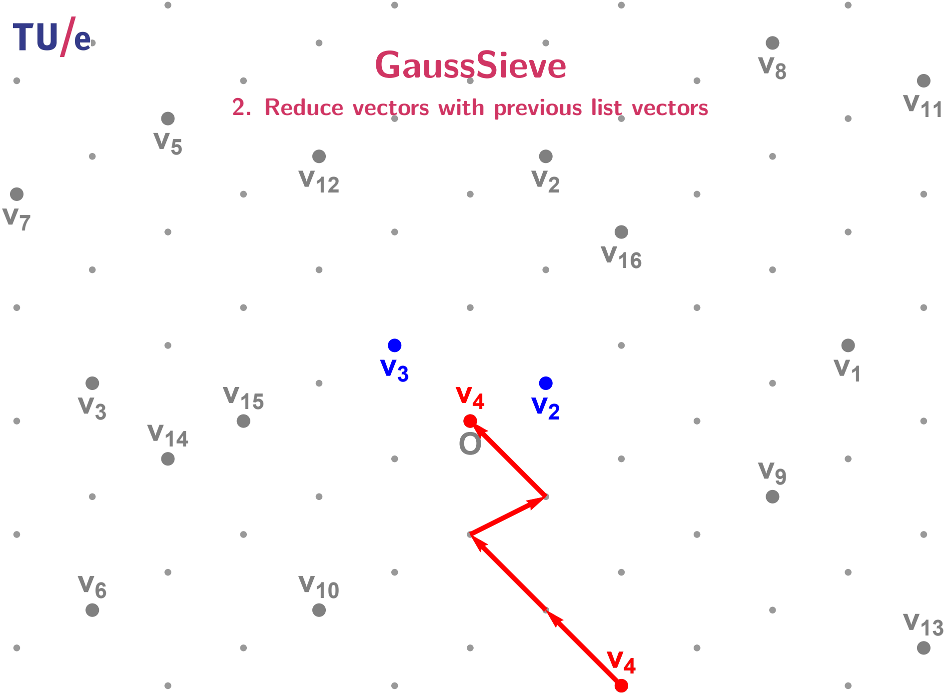

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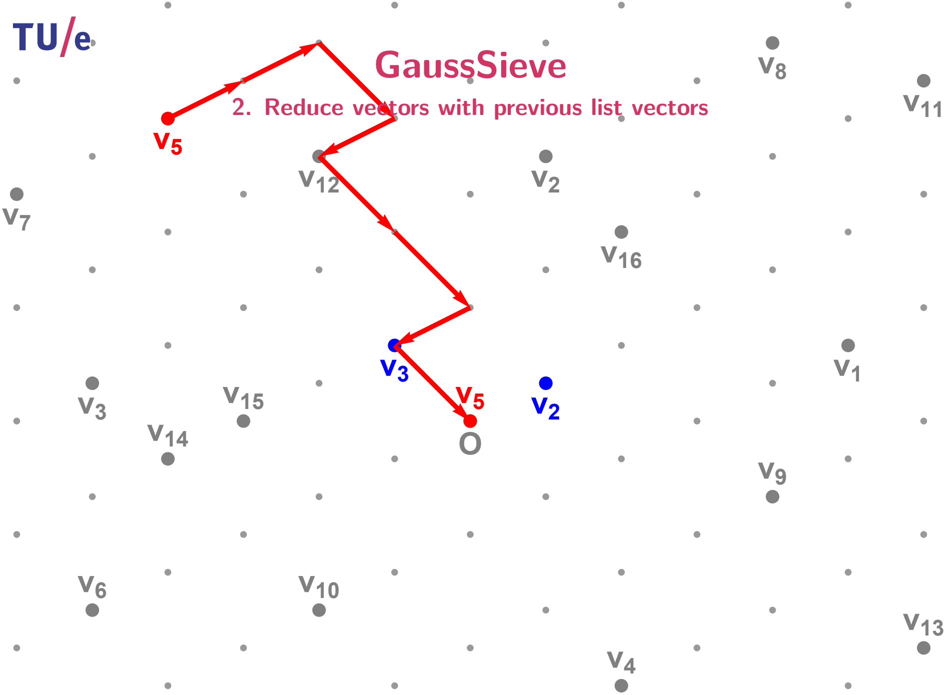

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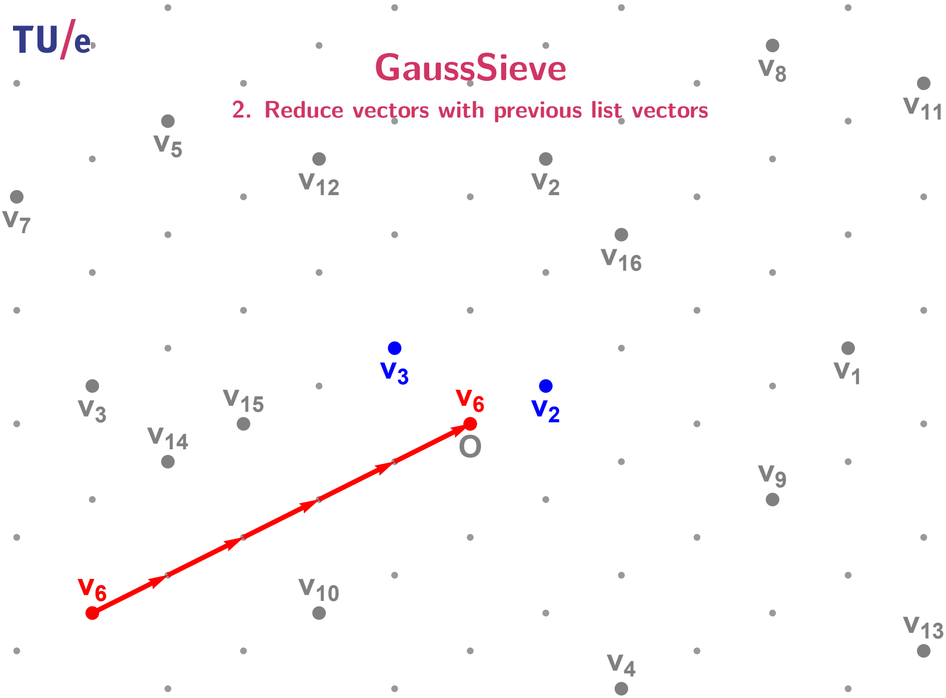

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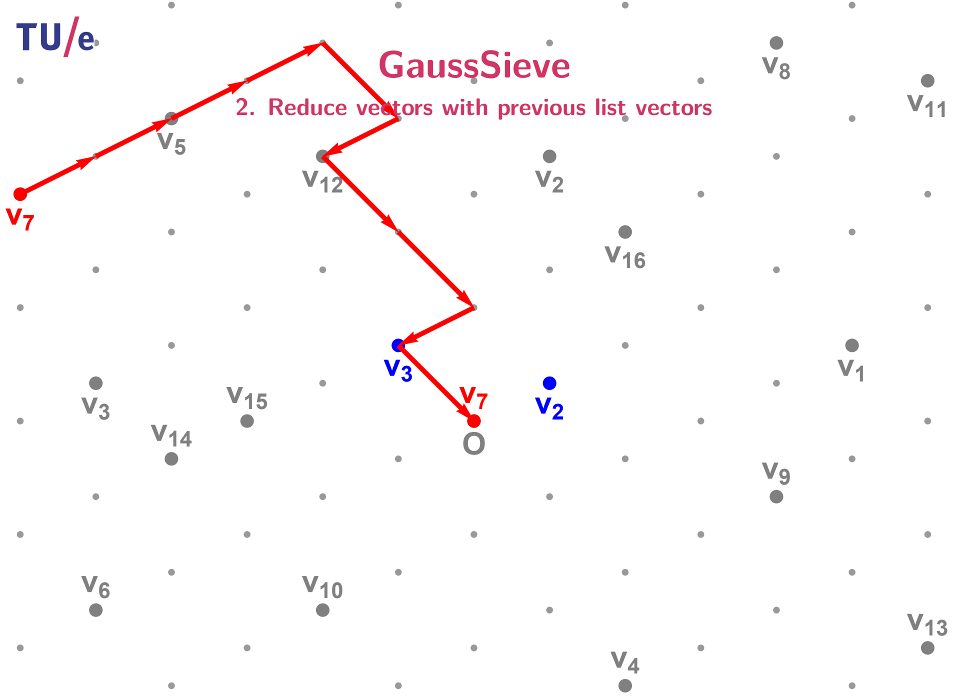

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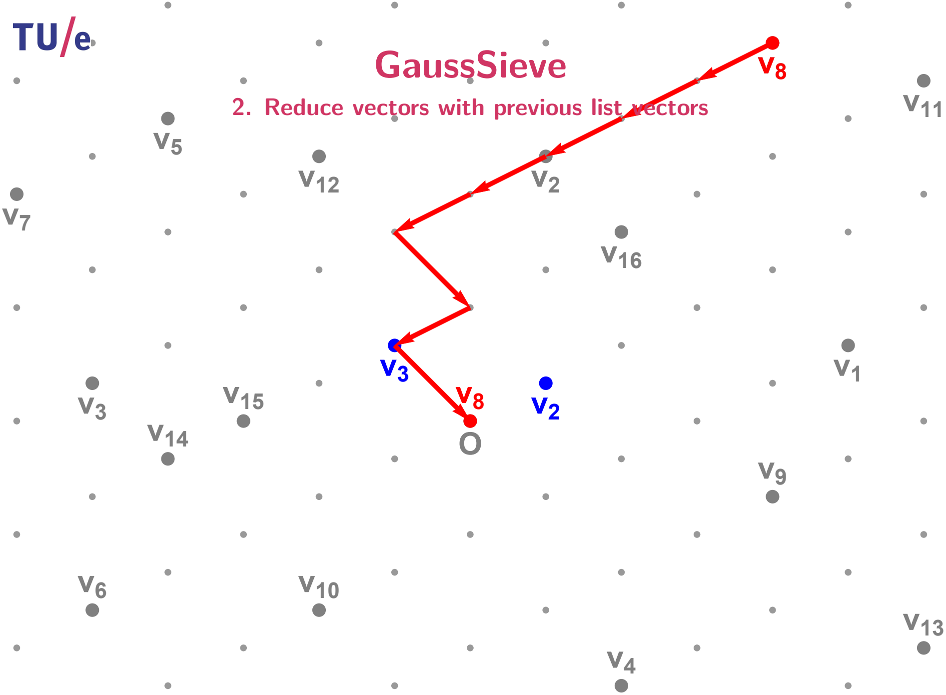

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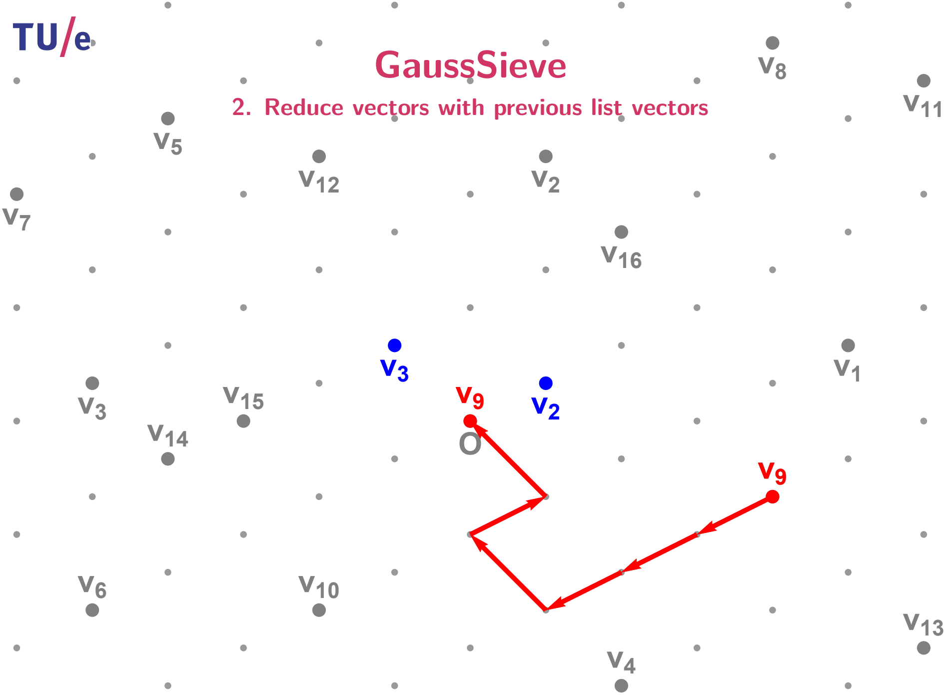

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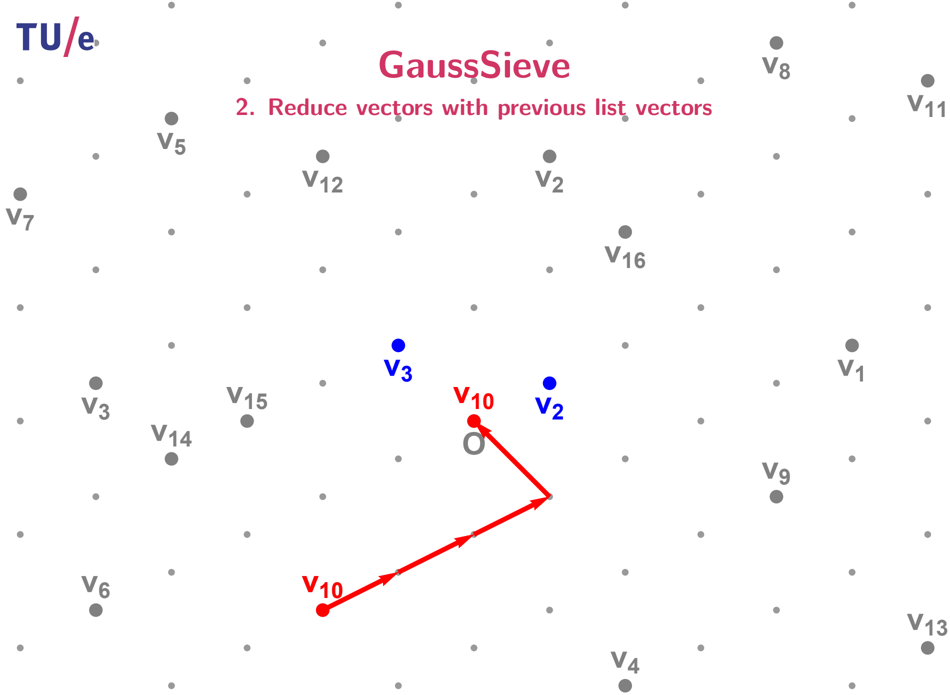

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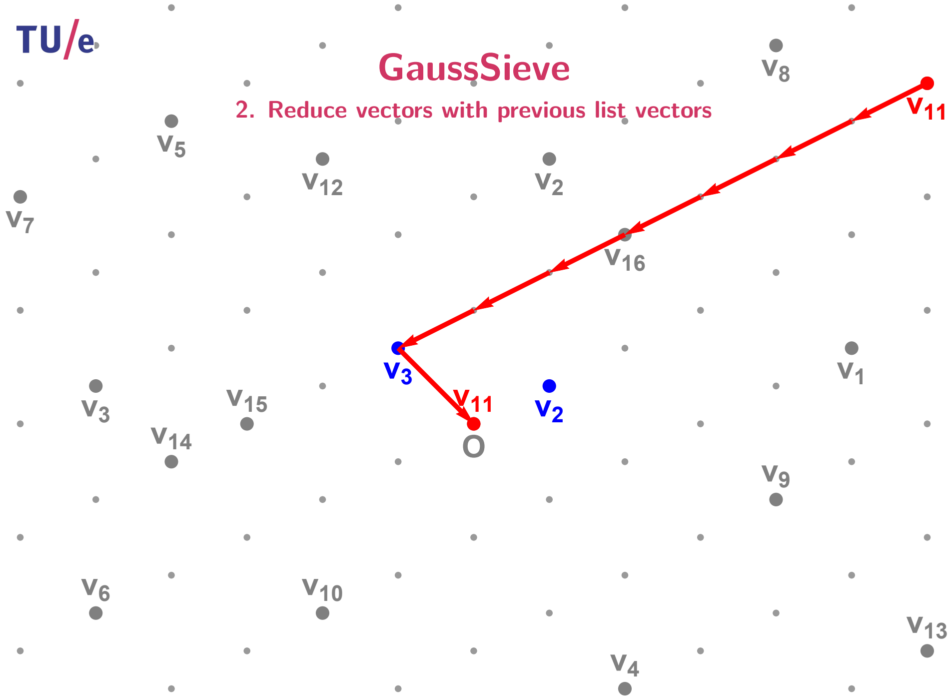

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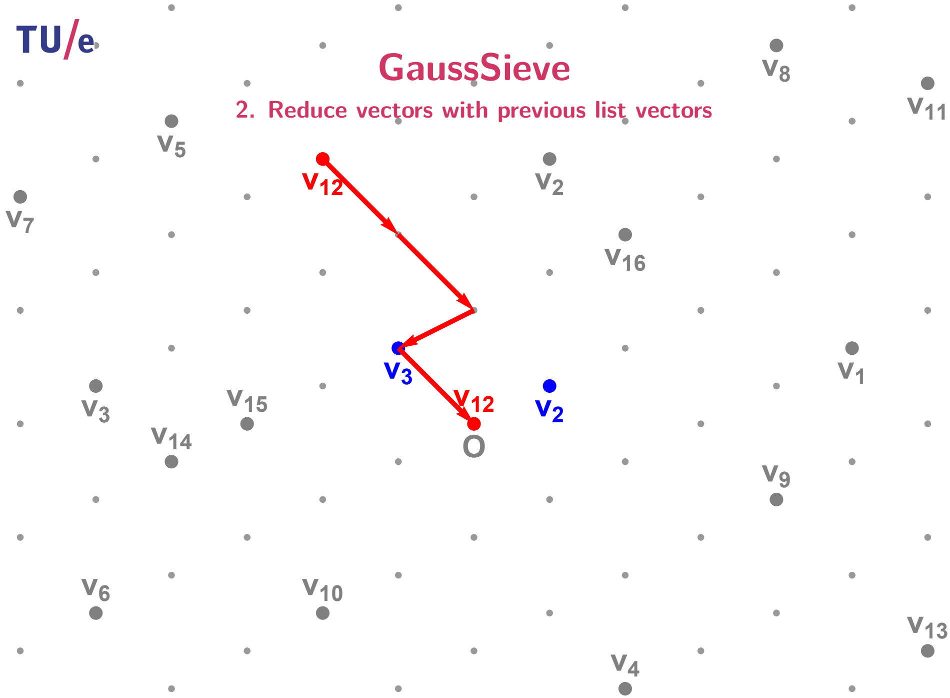

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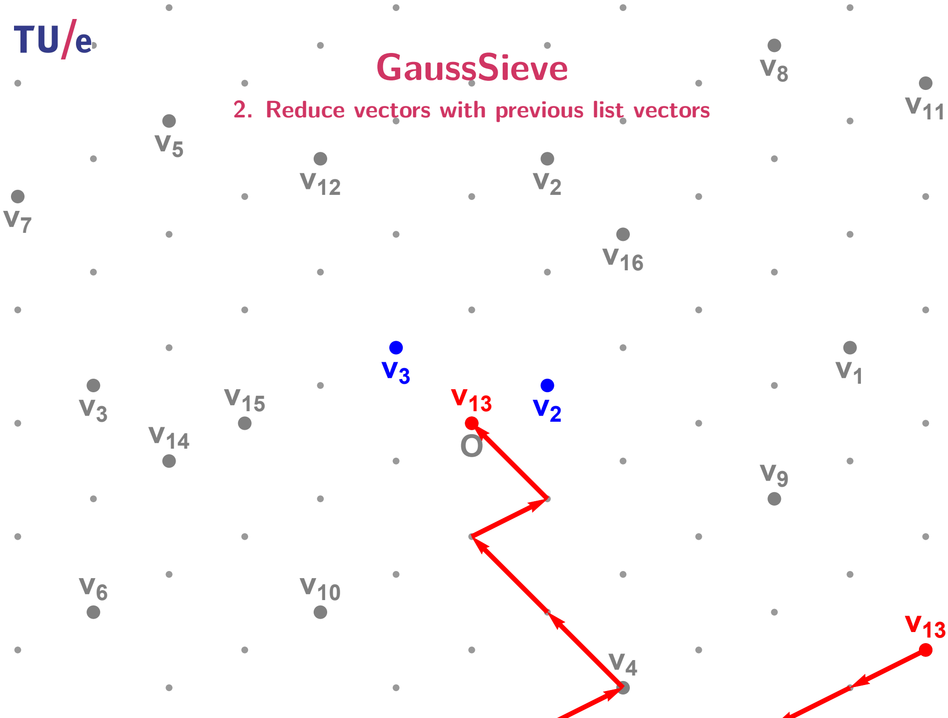

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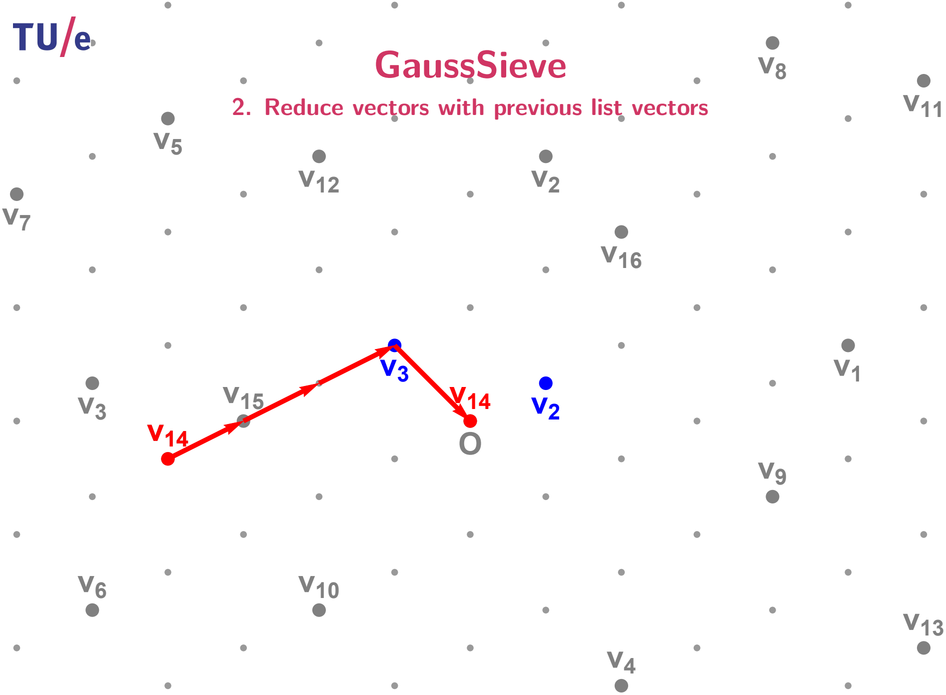

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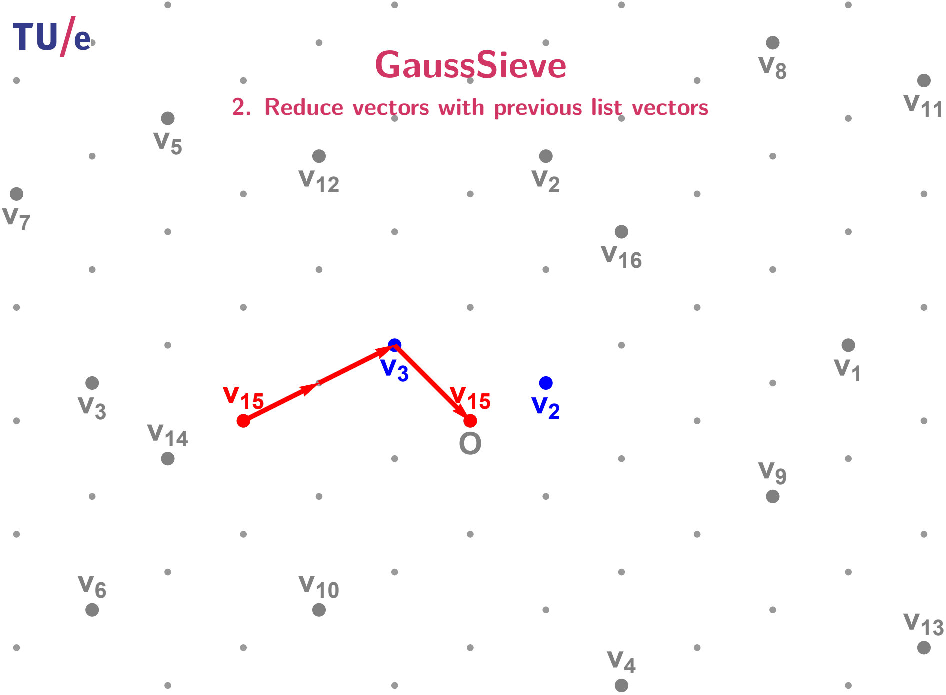

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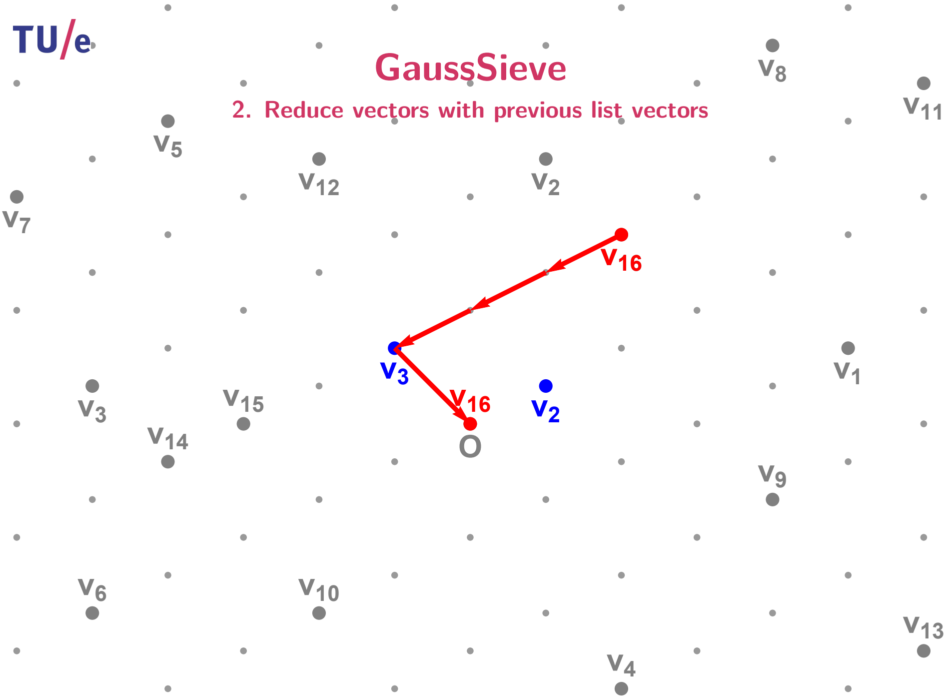

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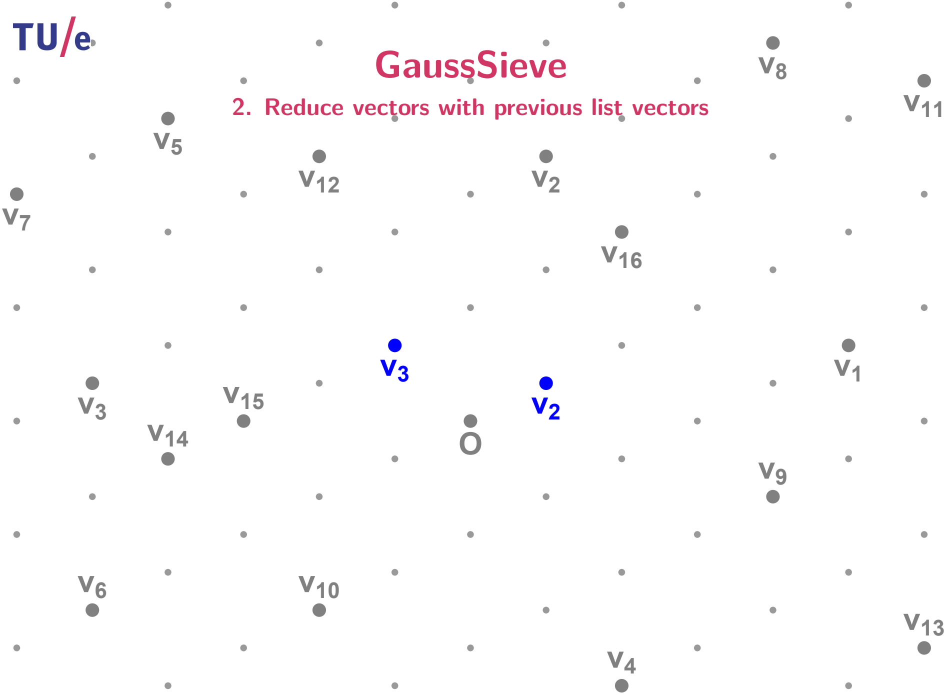
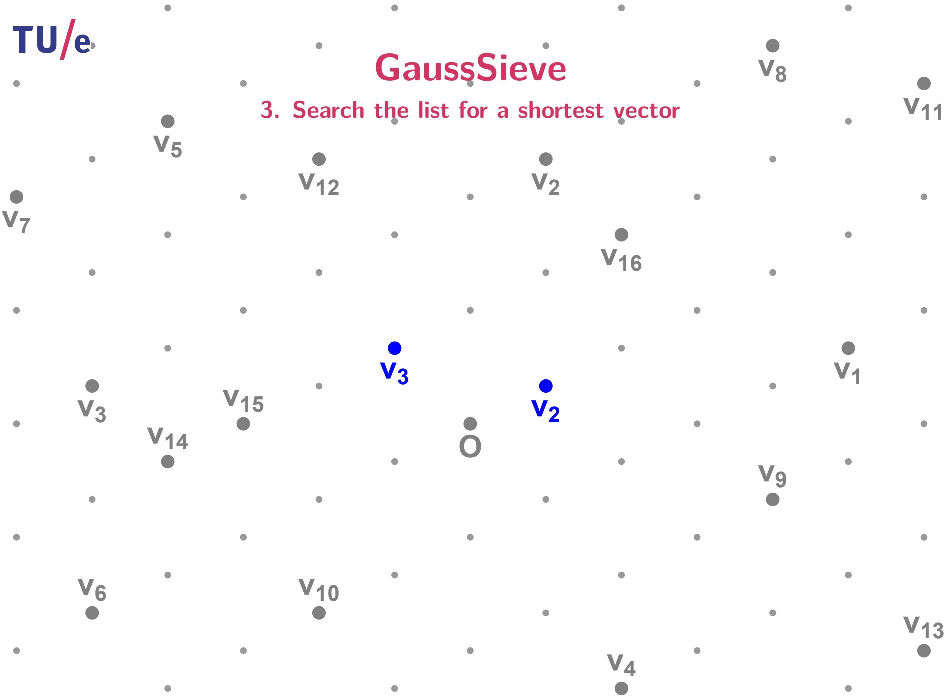


GaussSieve

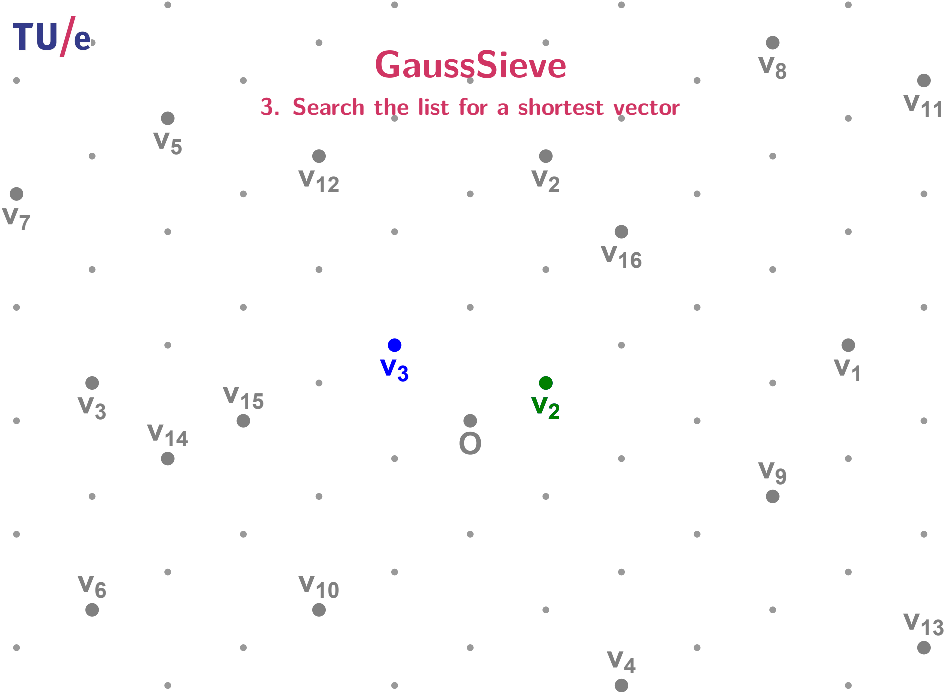
3. Search the list for a shortest vector



TU/e

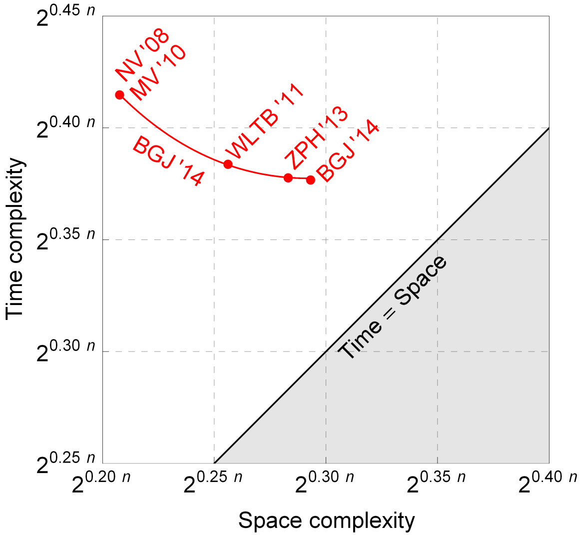
GaussSieve

3. Search the list for a shortest vector



Sieving

Space/time trade-off



Outline

Lattices

Enumeration algorithms
- Fincke-Pohst enumeration
- Kannan enumeration
- Pruning the enumeration tree

The Voronoi cell algorithm

Sieving algorithms
- Nguyen-Vidick sieve
- Multiple levels
- GaussSieve

Sieving using angular locality-sensitive hashing
- Nguyen-Vidick sieve
- GaussSieve

Locality-sensitive hashing

Introduction

"The key idea is to use hash functions such that the probability of collision is much higher for objects that are close to each other than for those that are far apart."

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"Unfortunately, the estimated improvement is too small to outweigh the increase of the constants for dimensions of practical interest. [...] We implemented the LSH algorithm of Datar et al. and found that it performed only marginally better than the naive algorithm for dimensions higher than 50*."*

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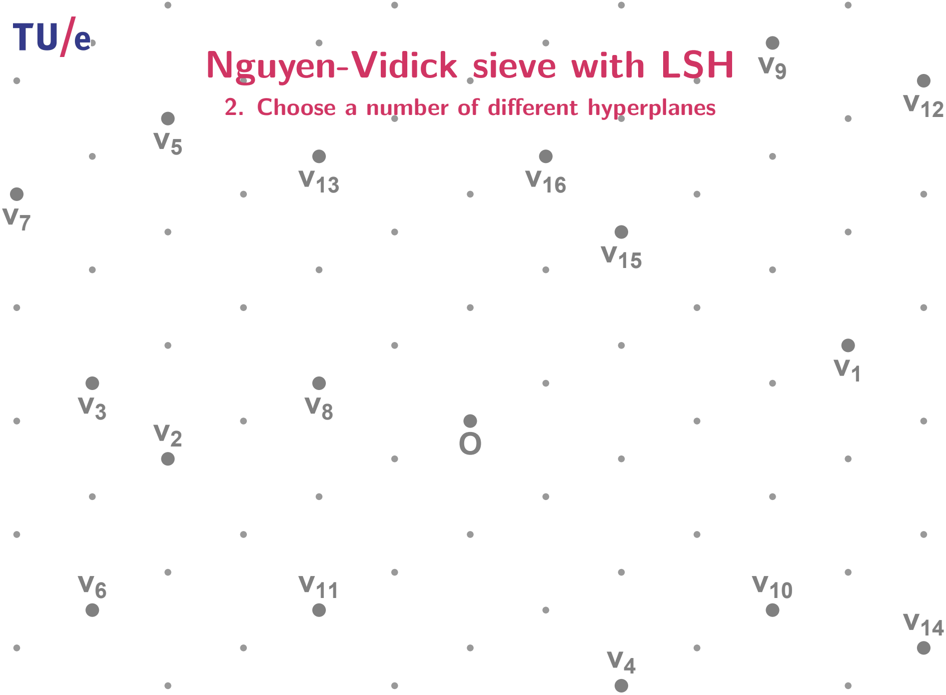

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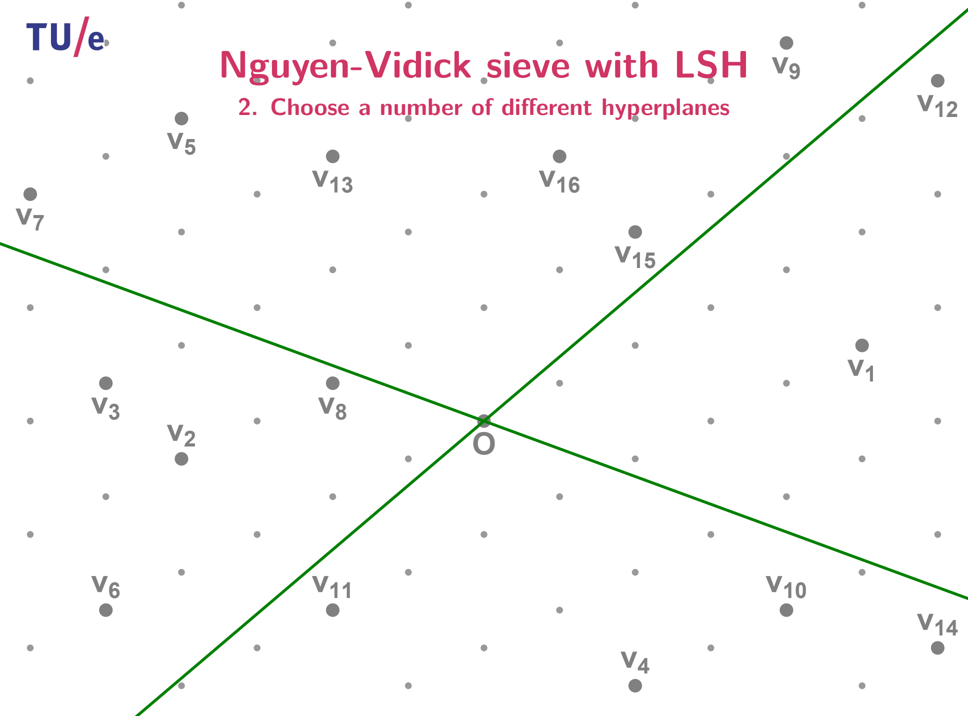

Nguyen-Vidick sieve with LSH

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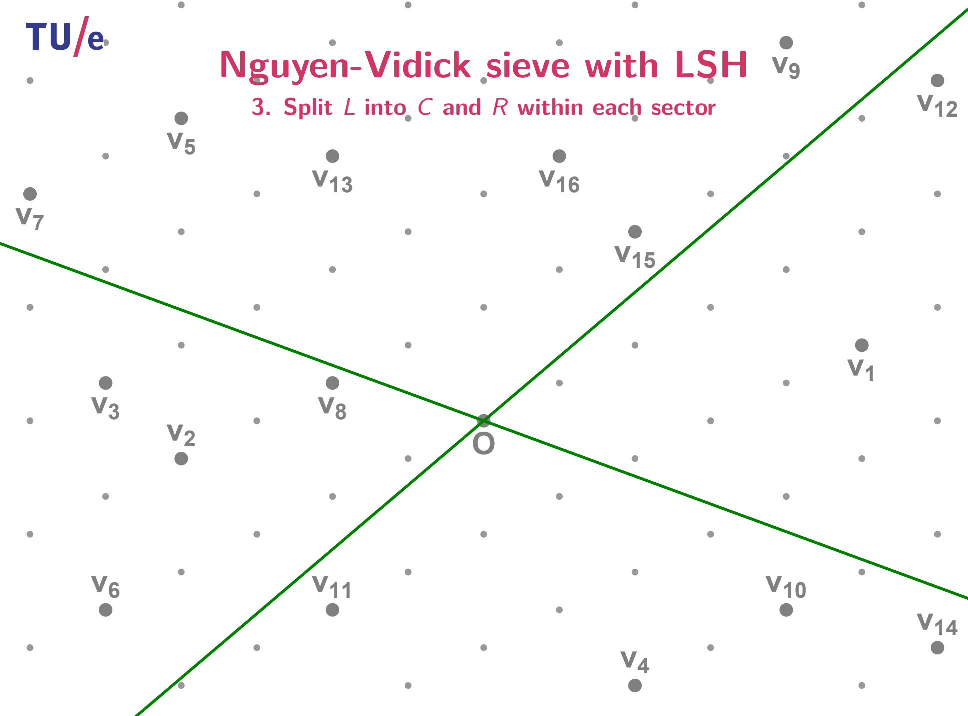

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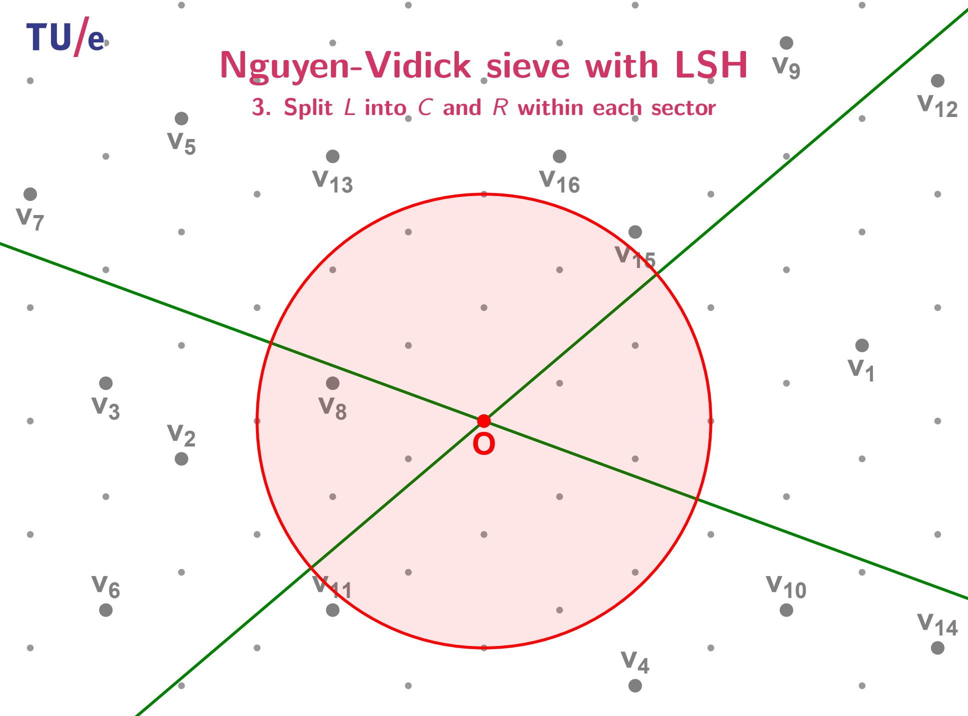

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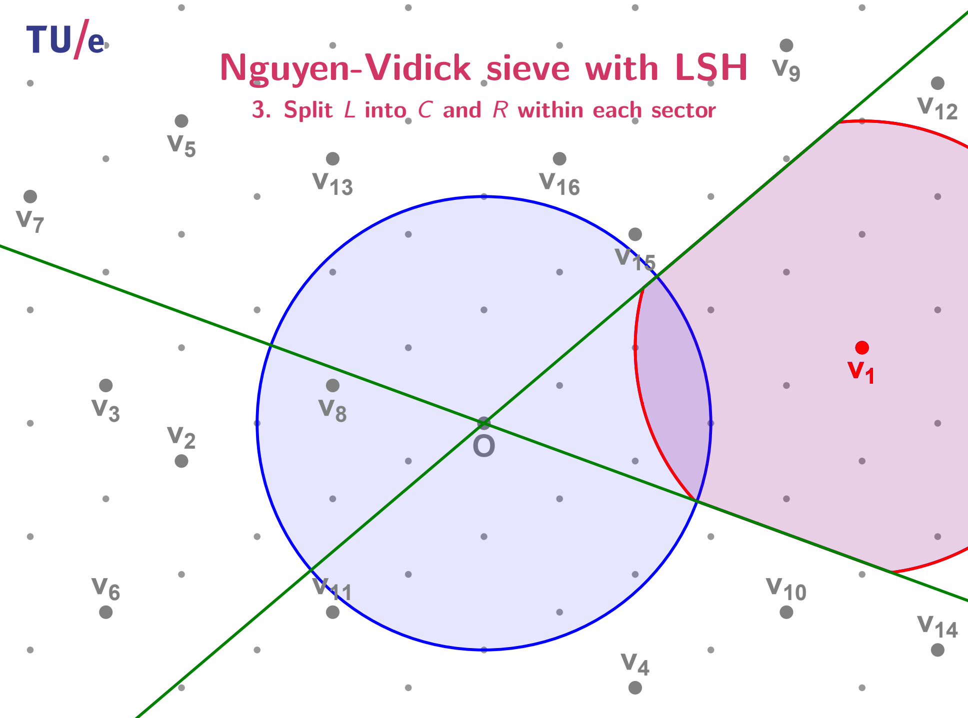

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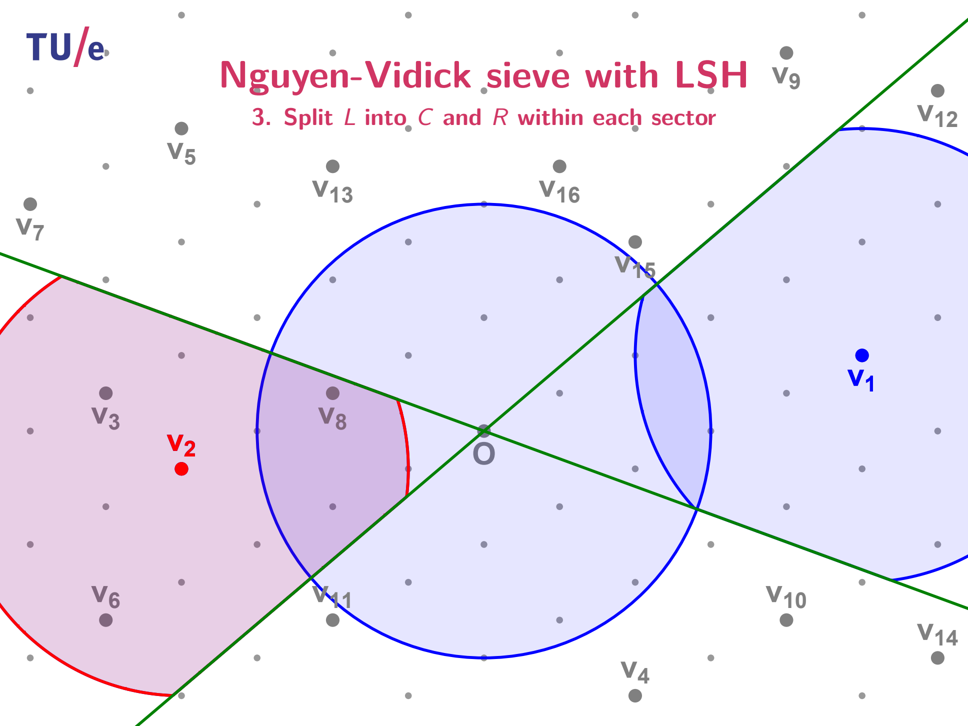

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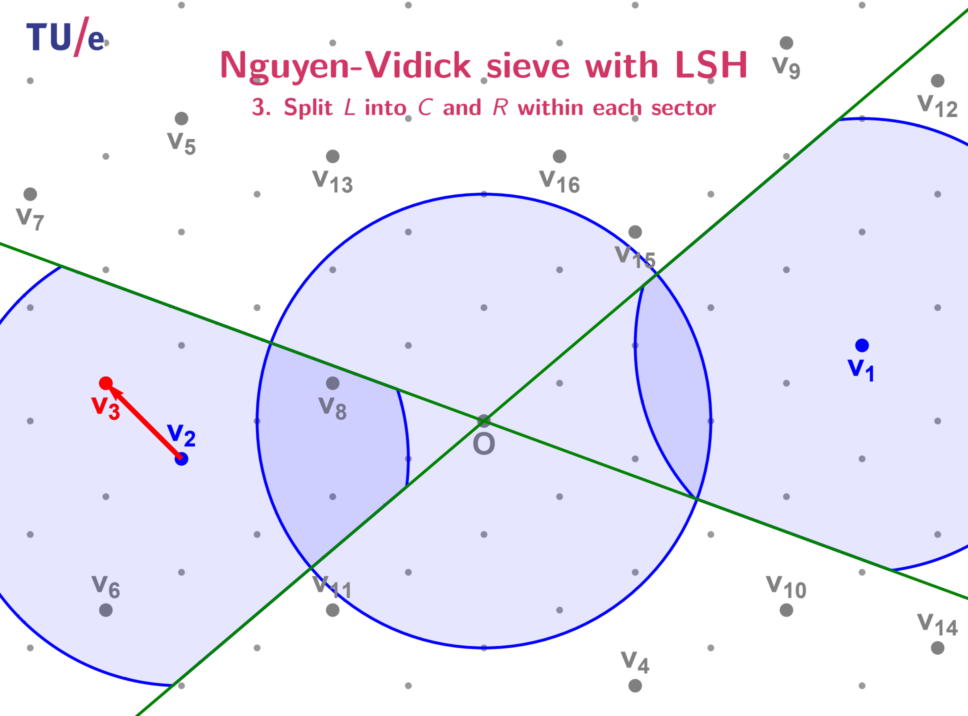

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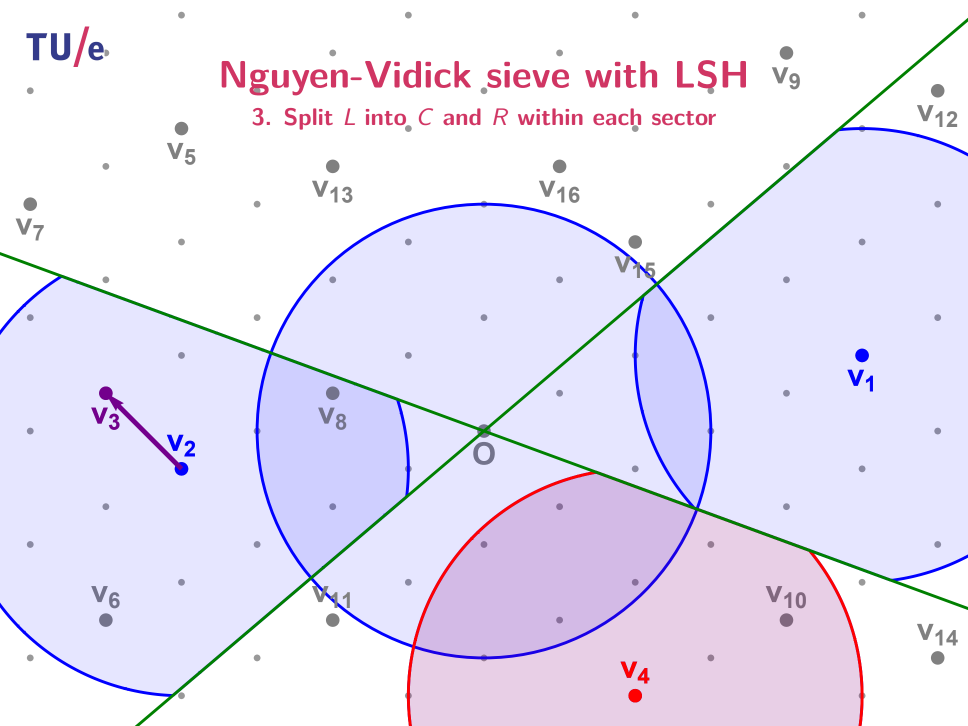

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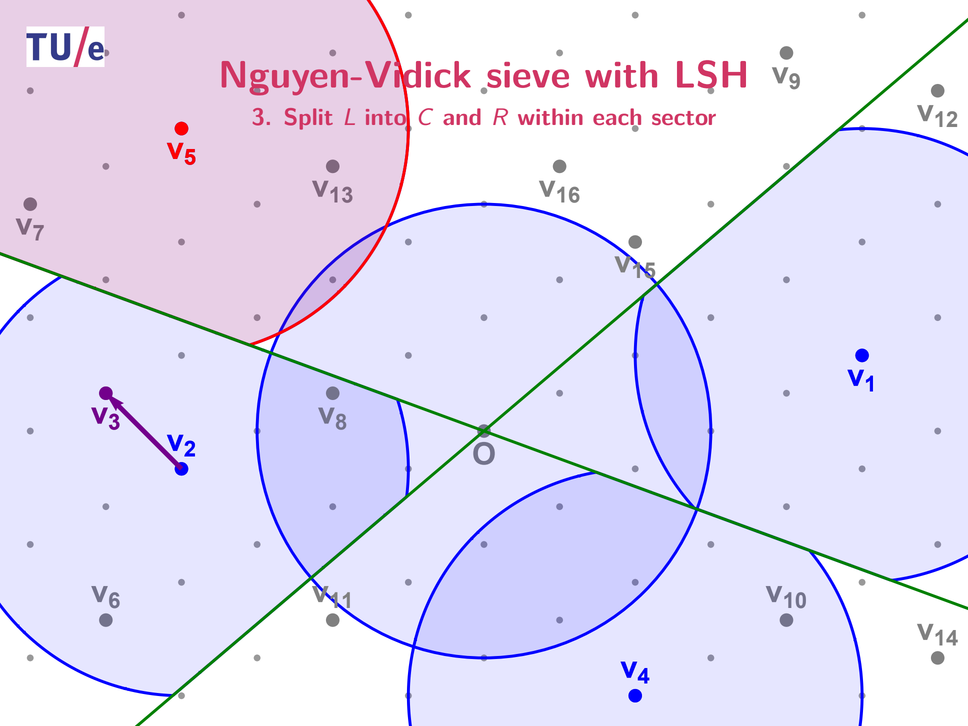

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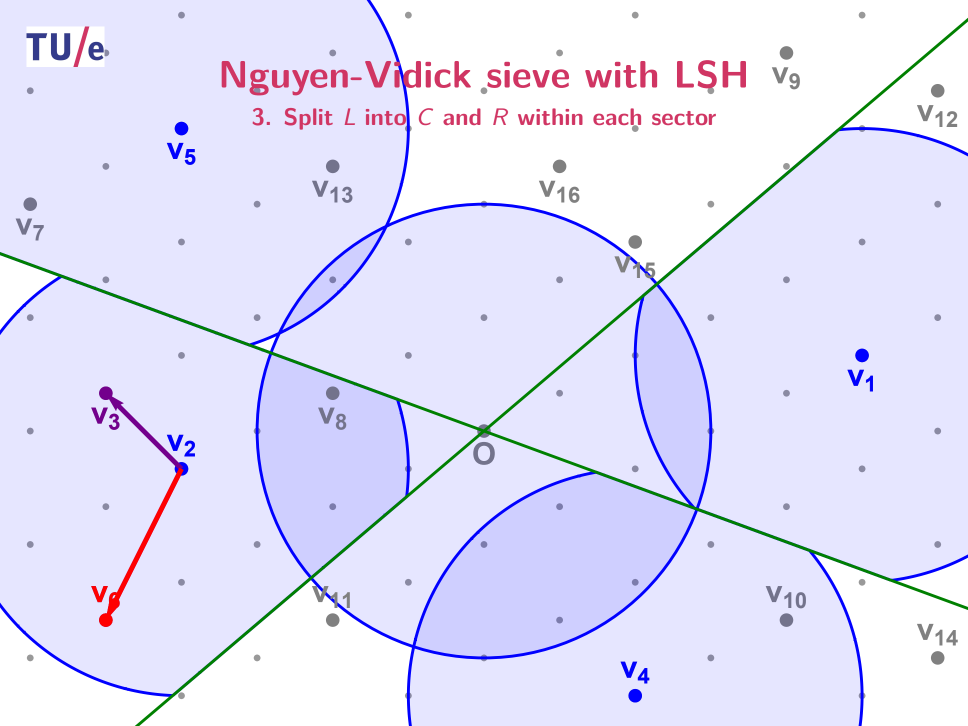

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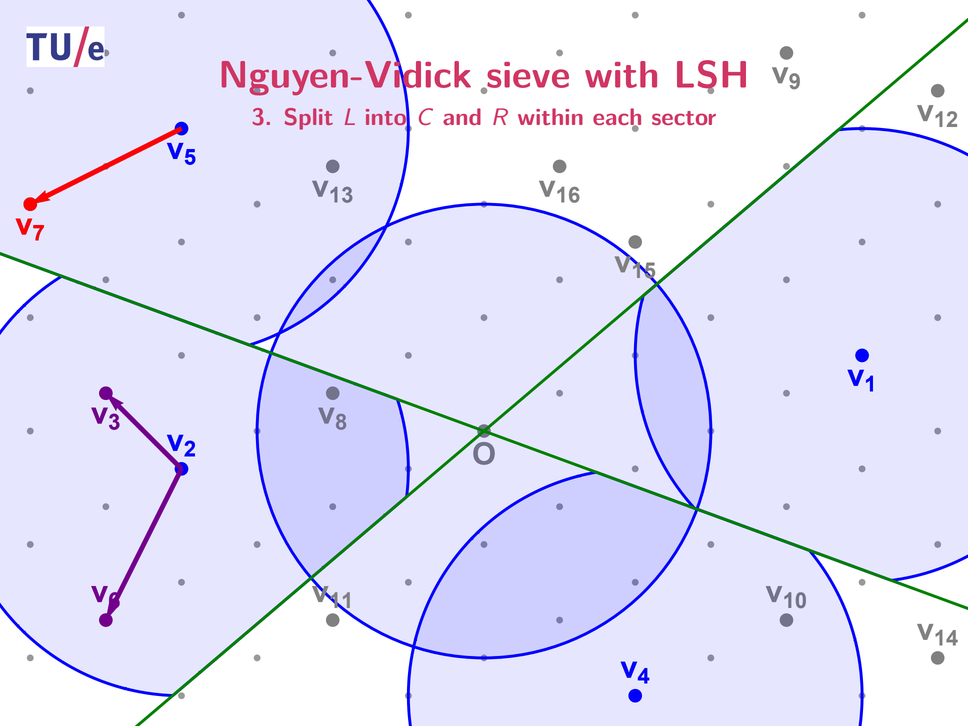

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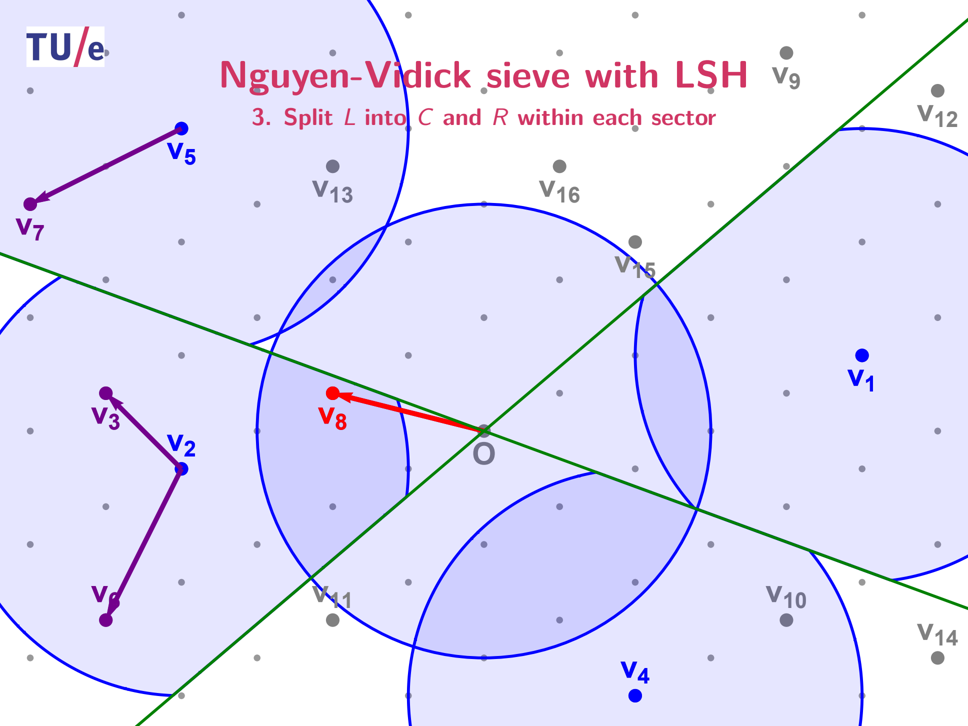

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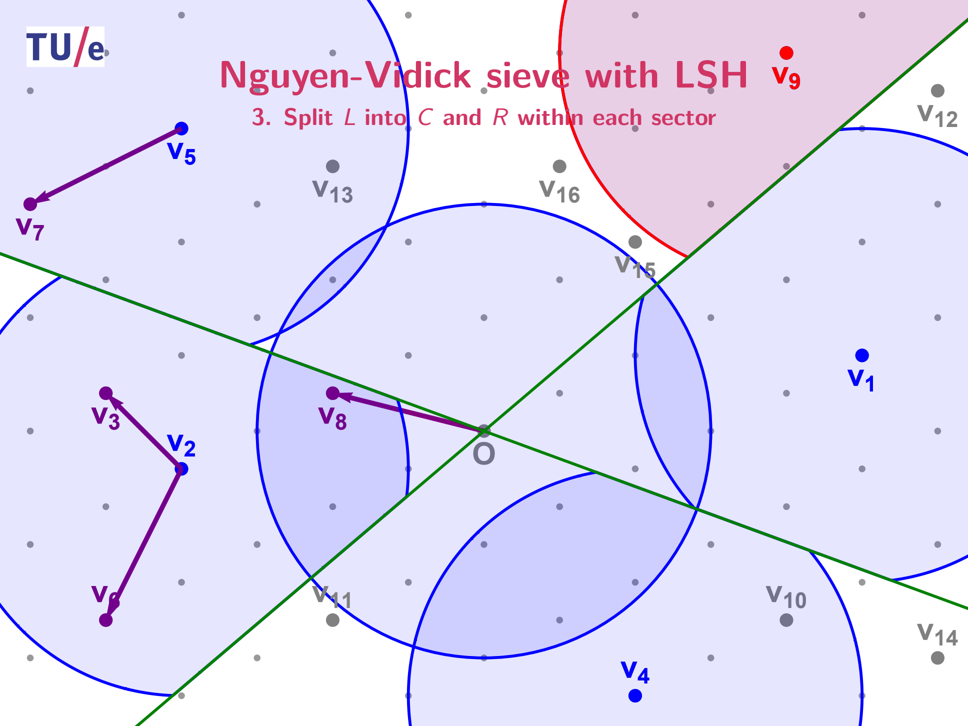

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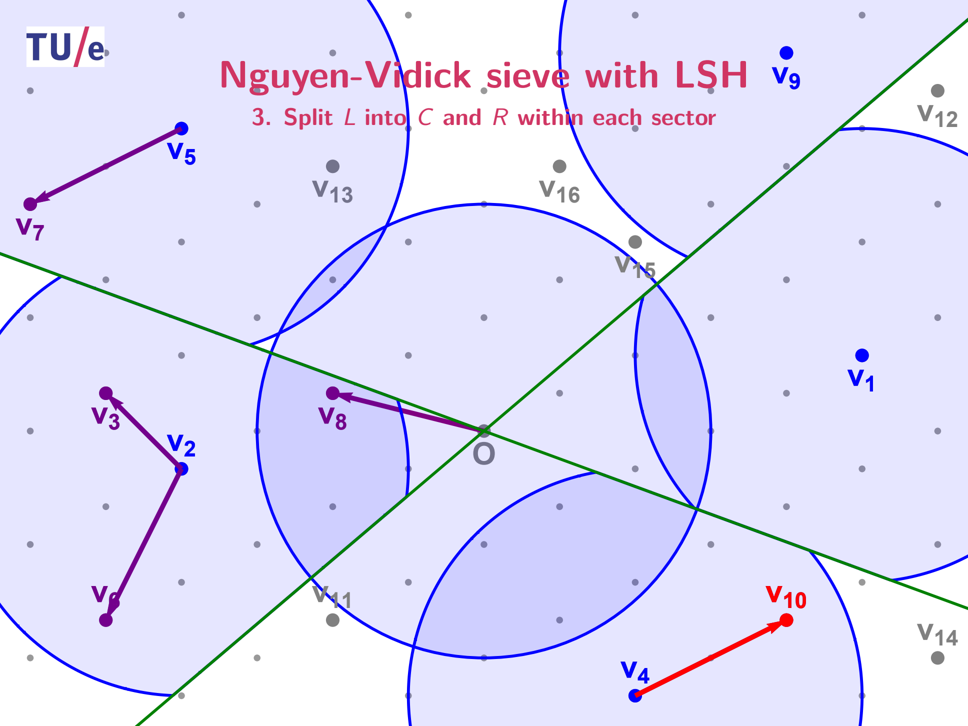

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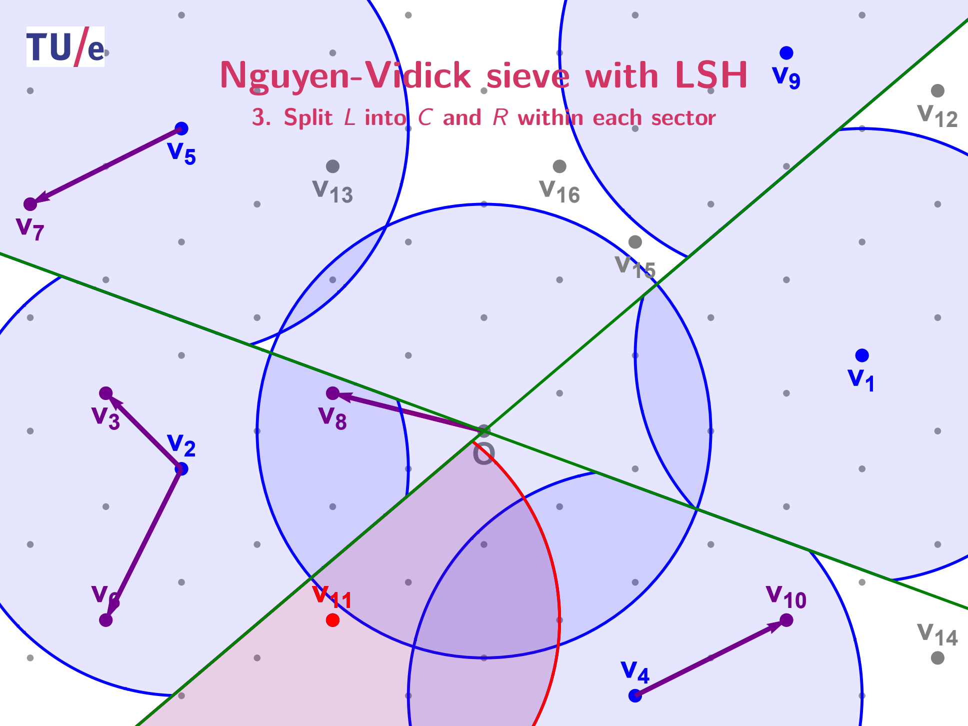

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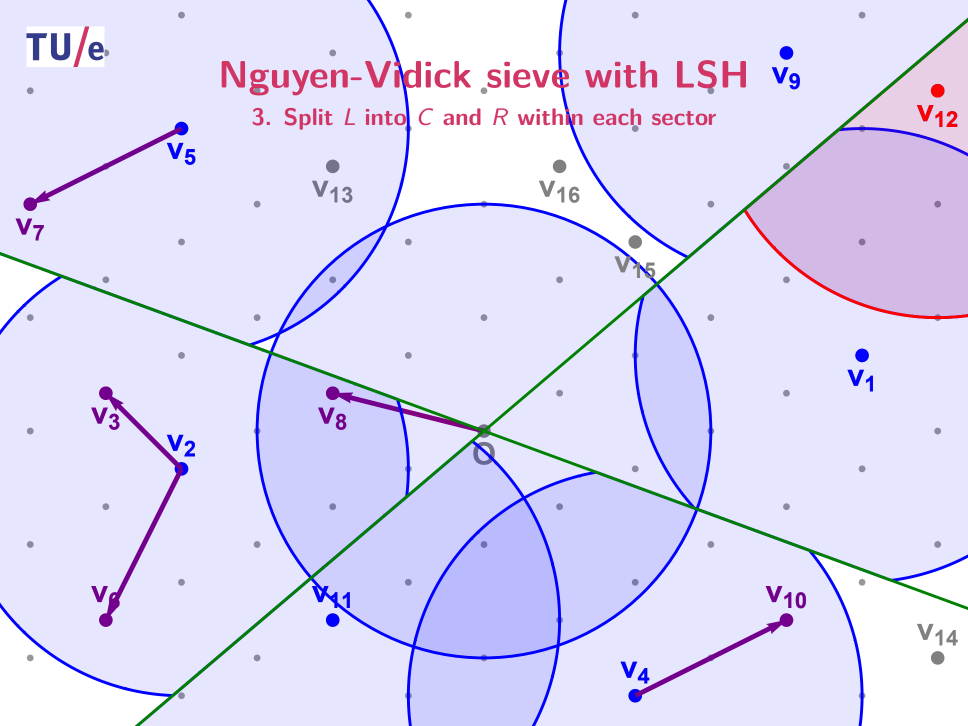

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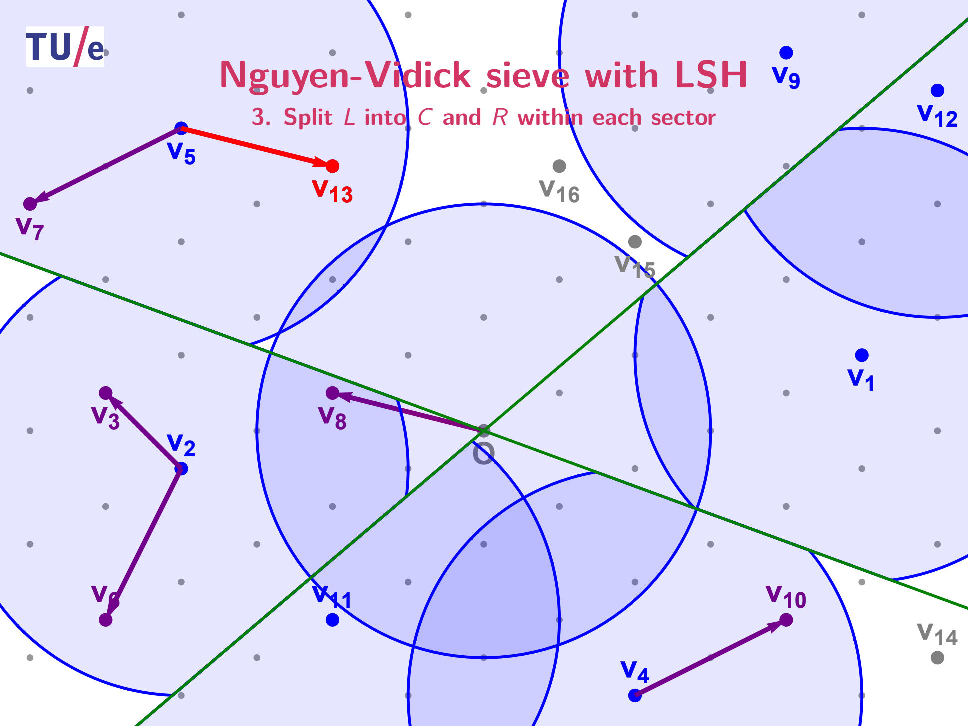

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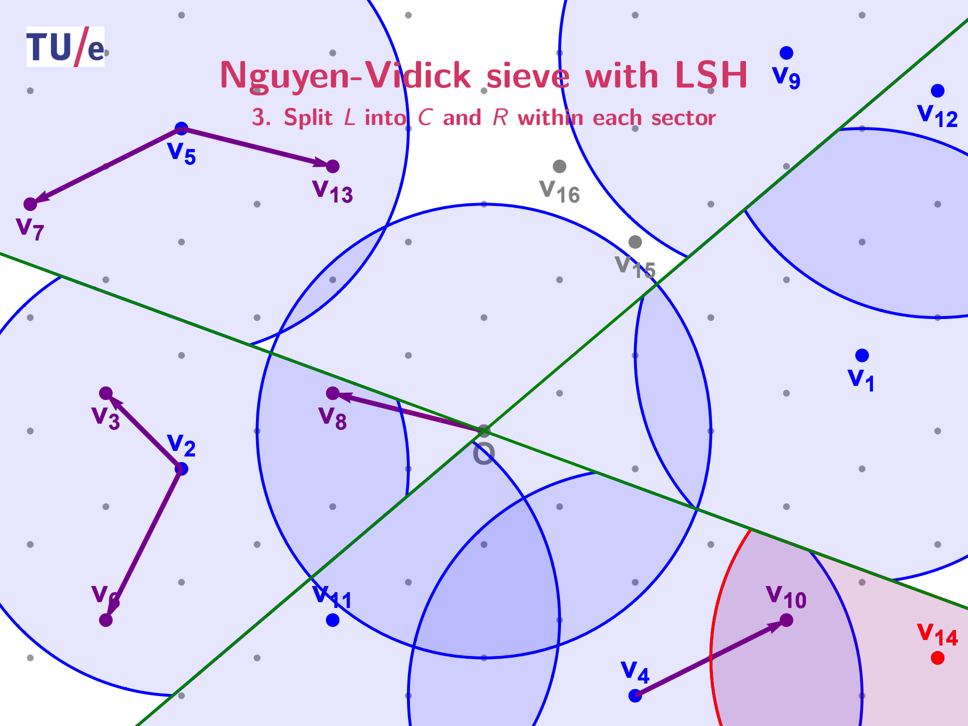

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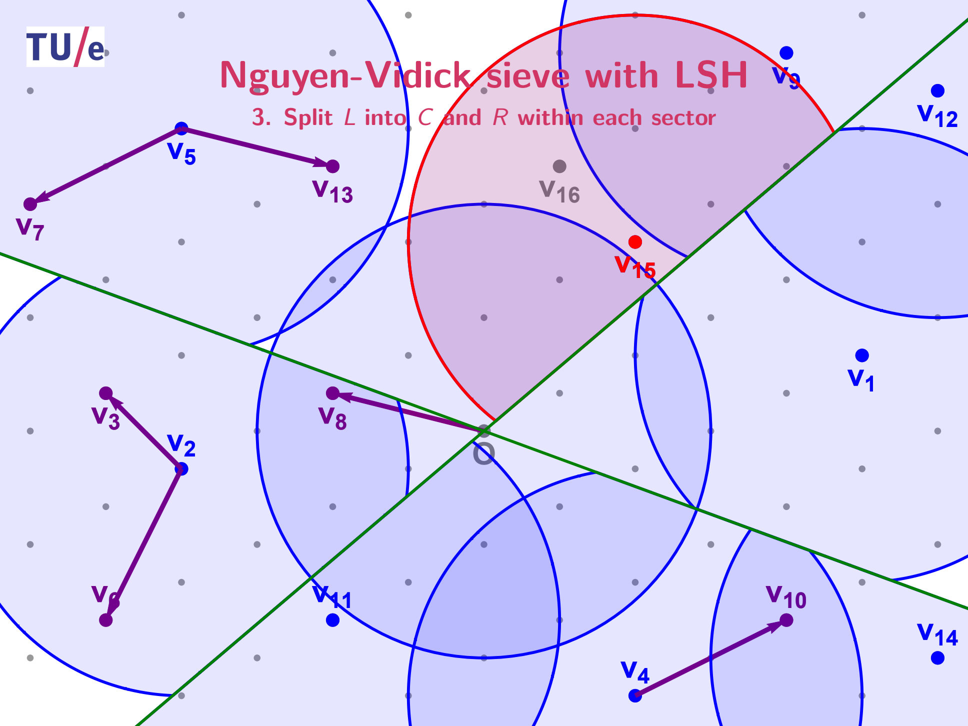

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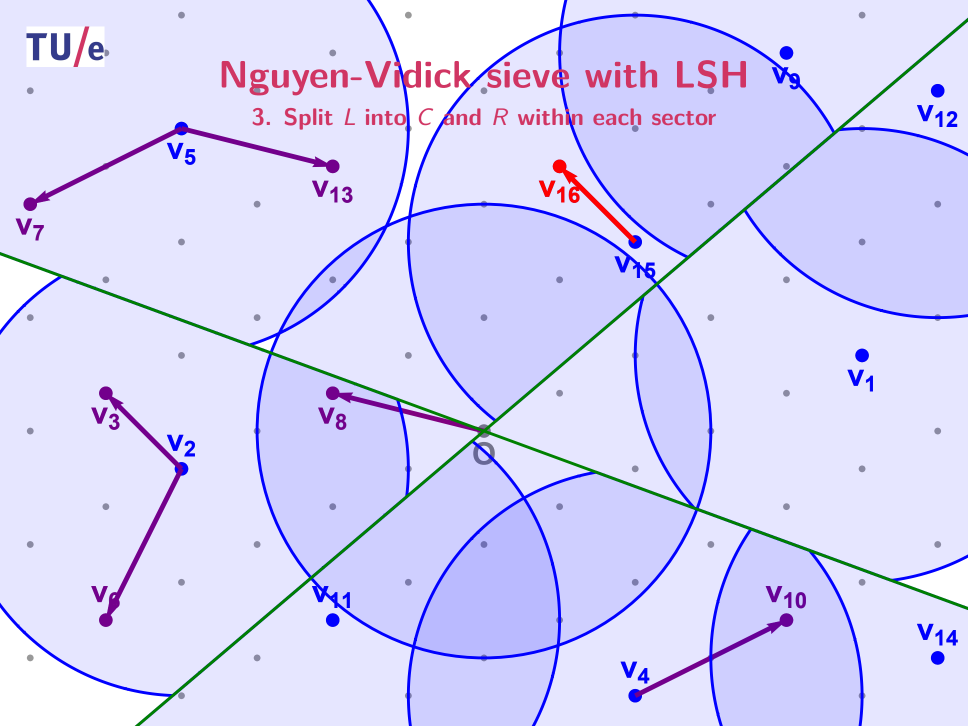

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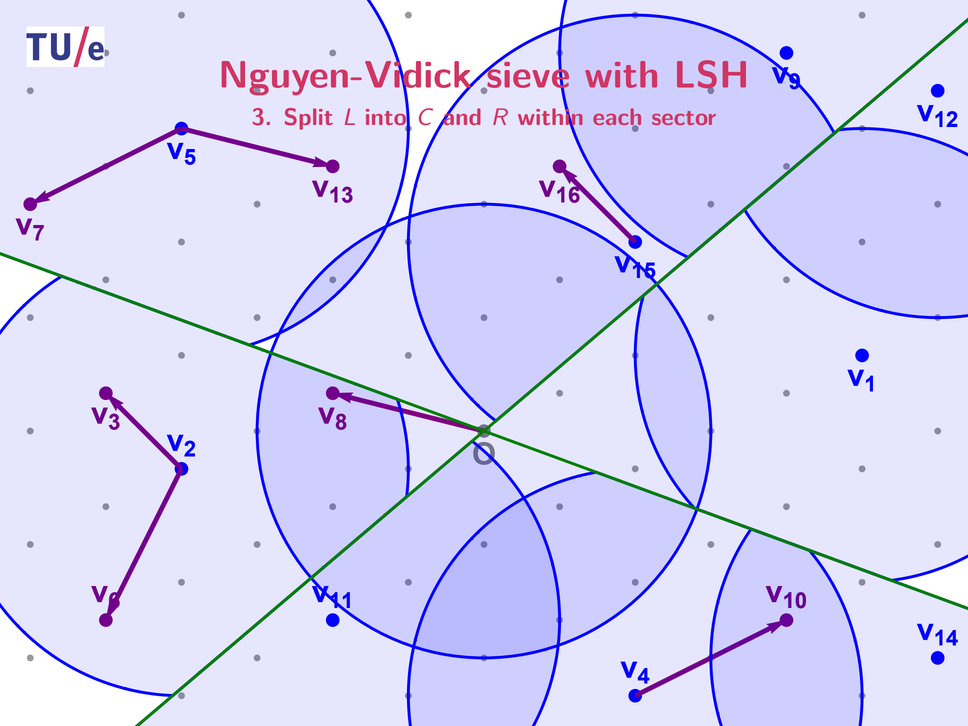

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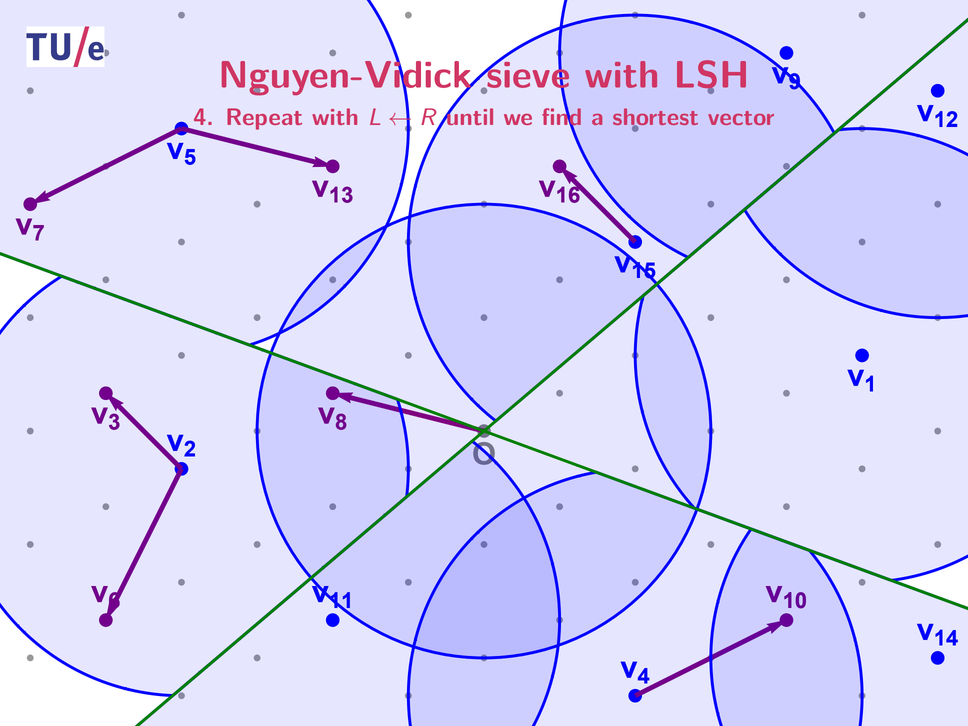

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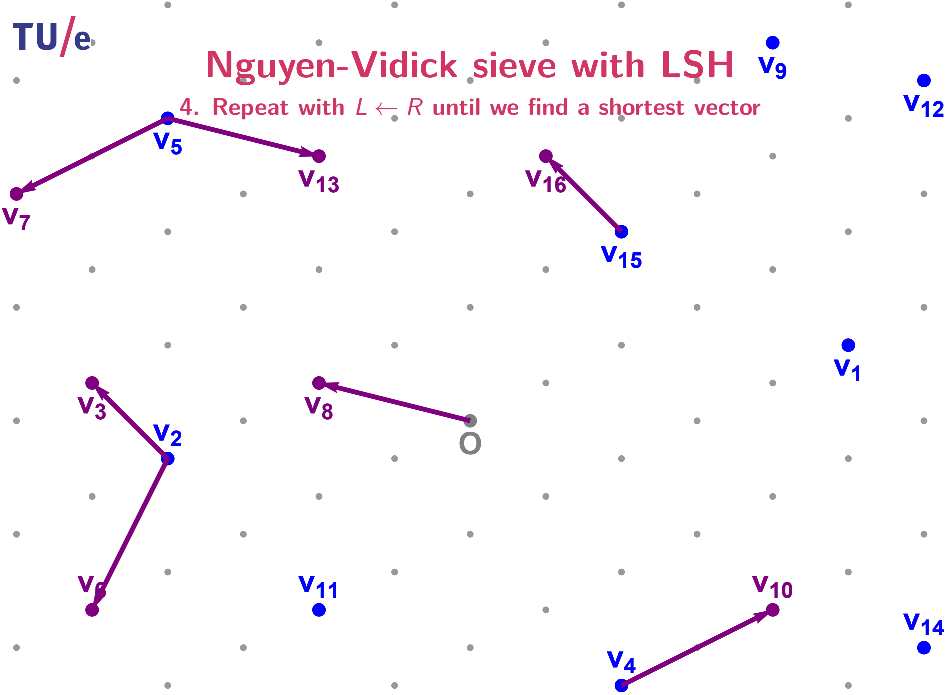

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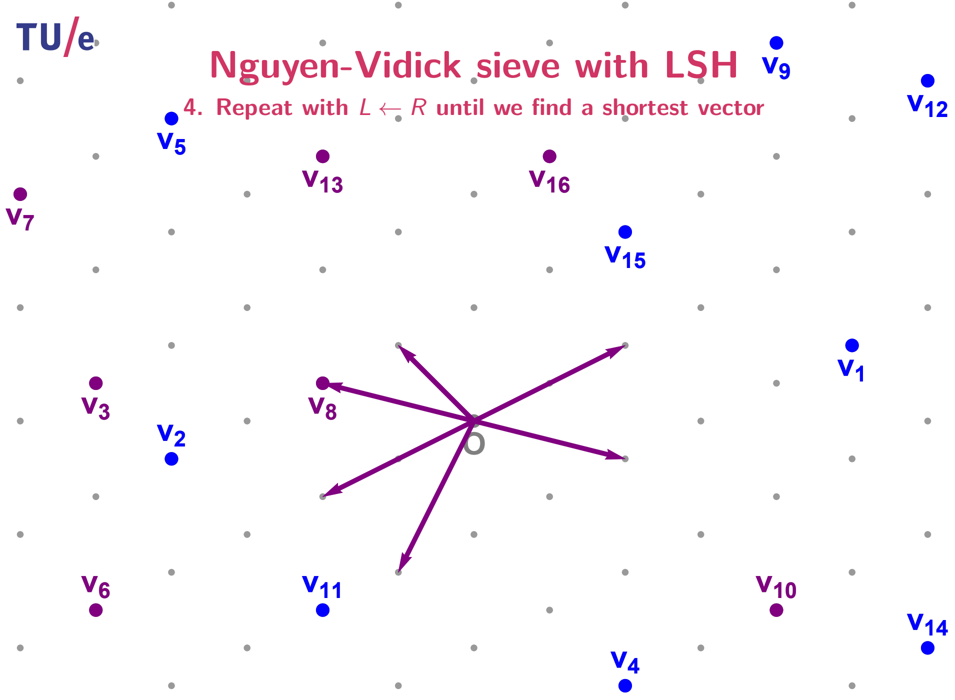

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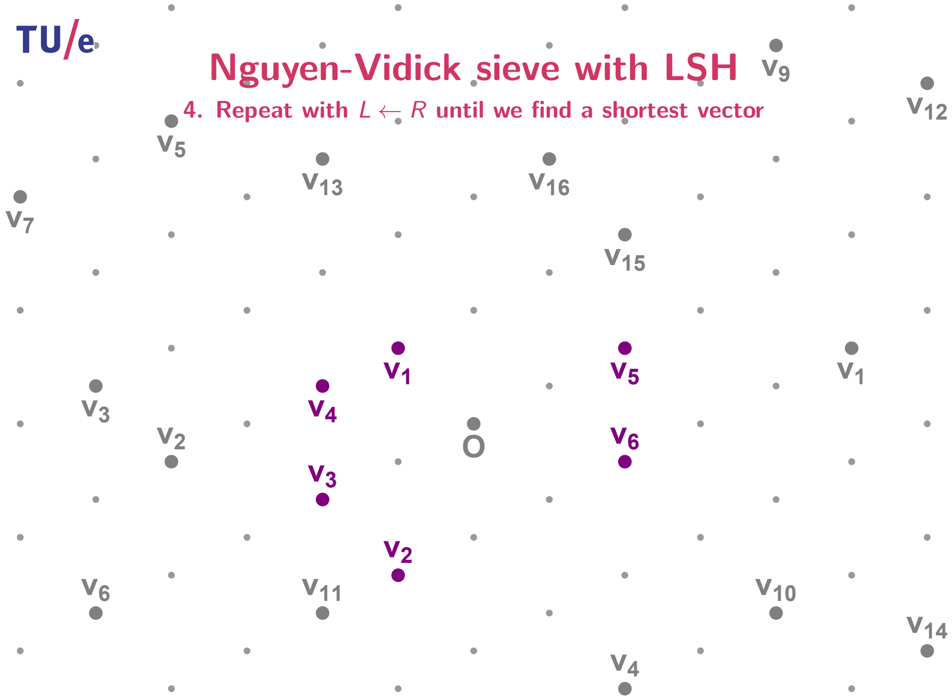
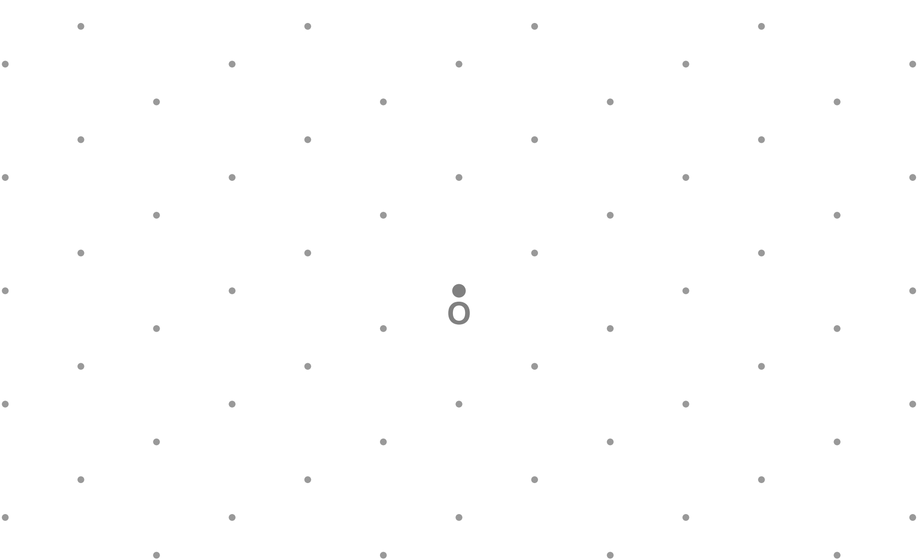


GaussSieve

1. Generate random lattice vectors

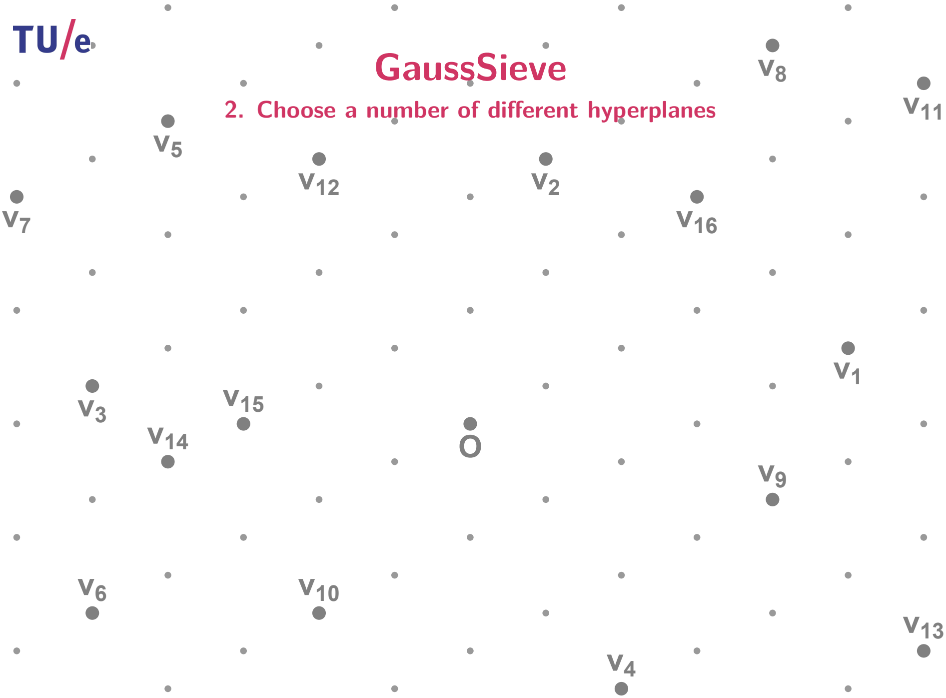


GaussSieve

1. Generate random lattice vectors

GaussSieve

2. Choose a number of different hyperplanes



GaussSieve

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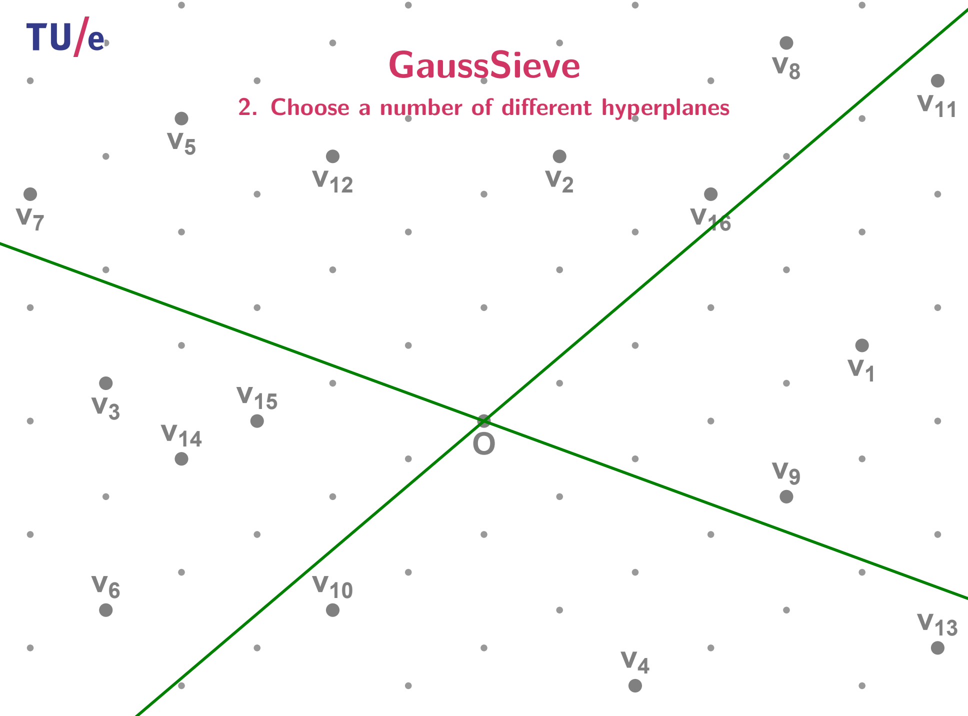

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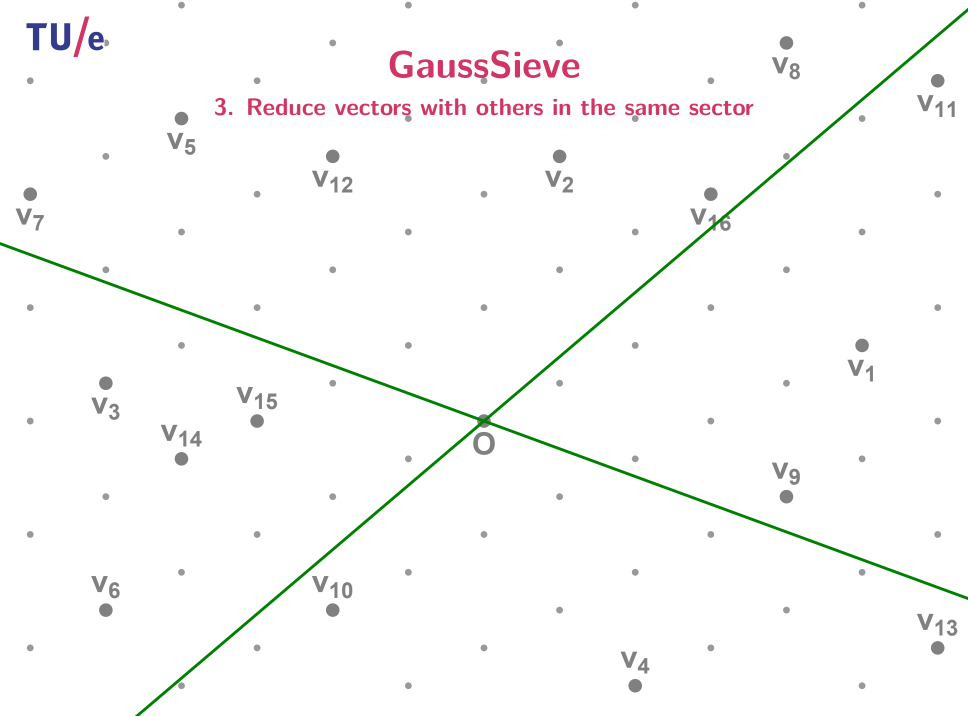
GaussSieve

3. Reduce vectors with others in the same sector



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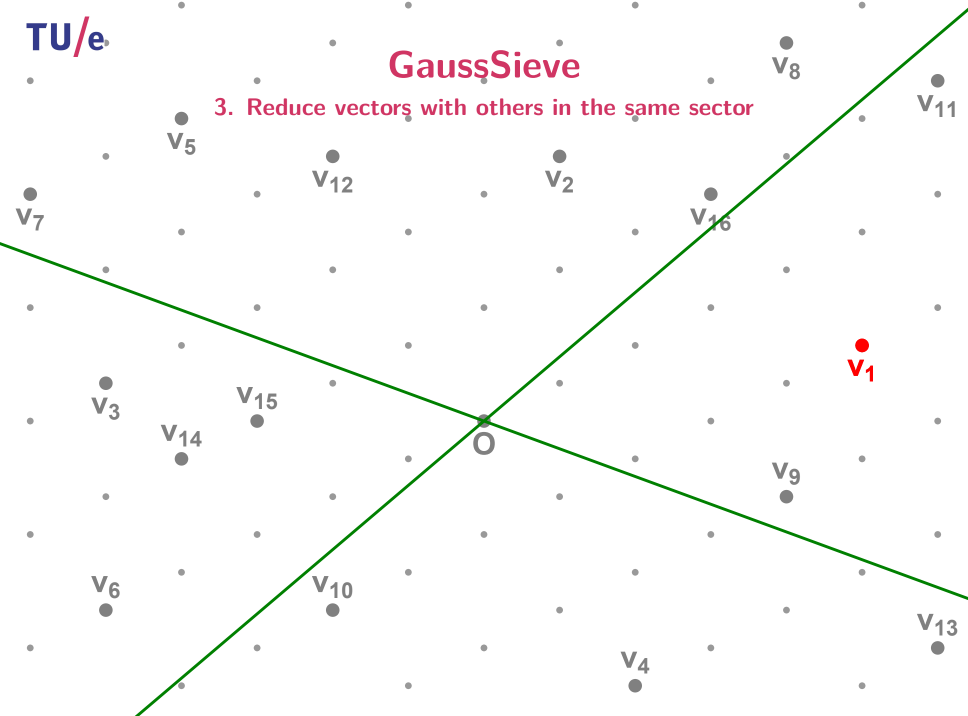

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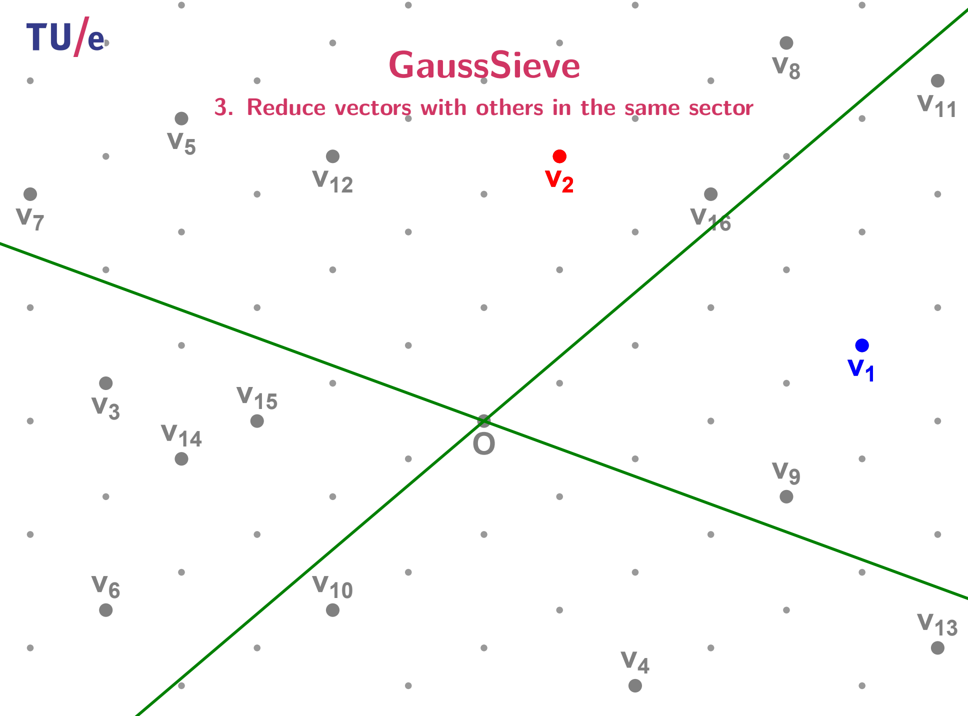

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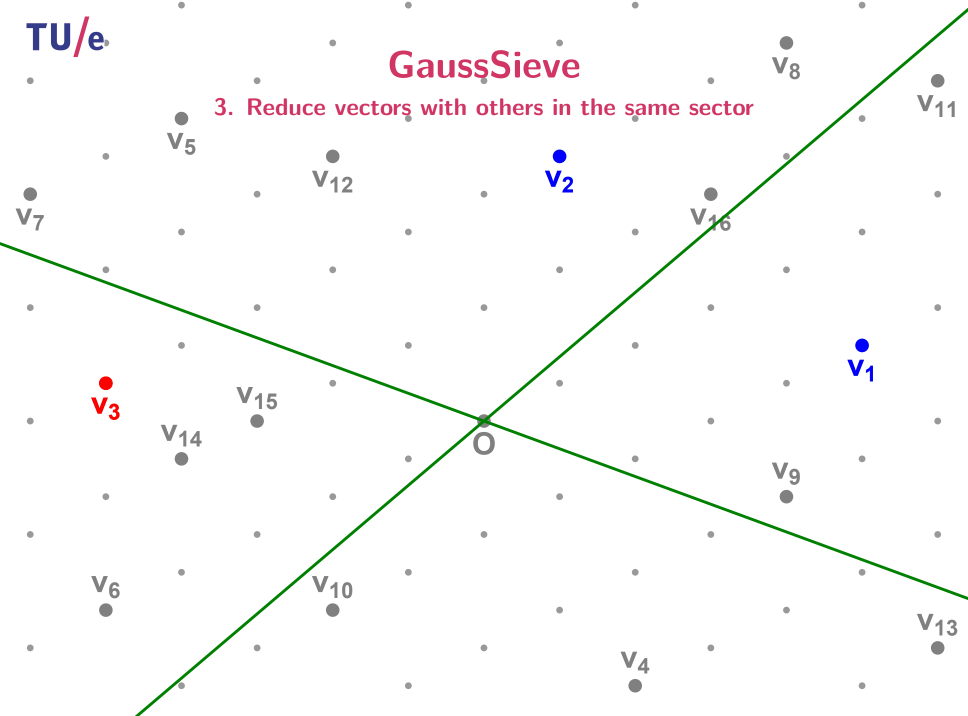

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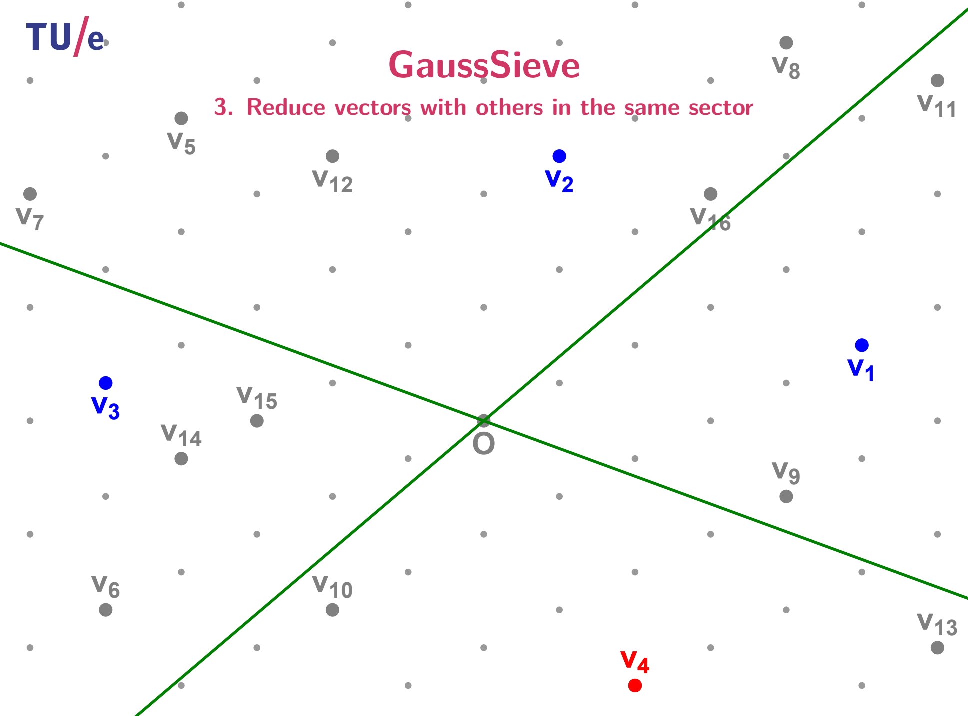

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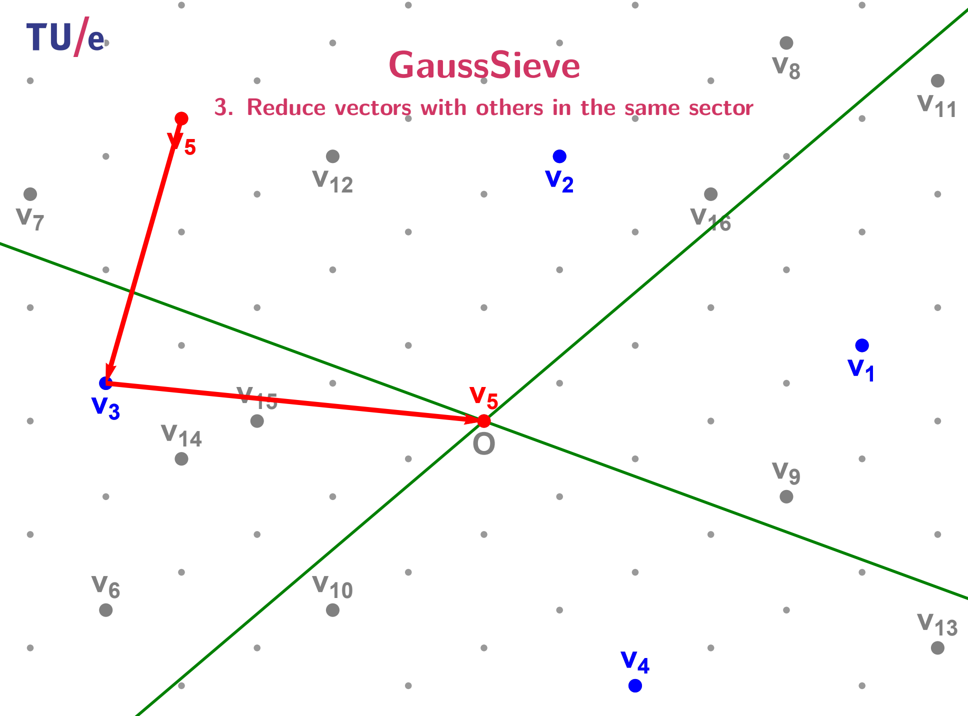

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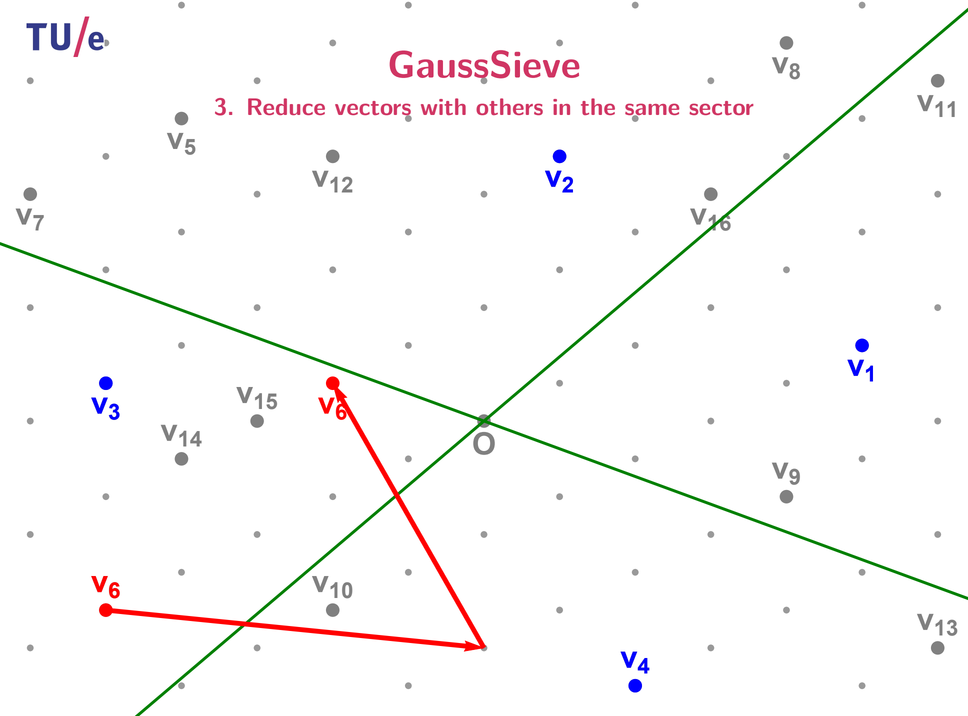

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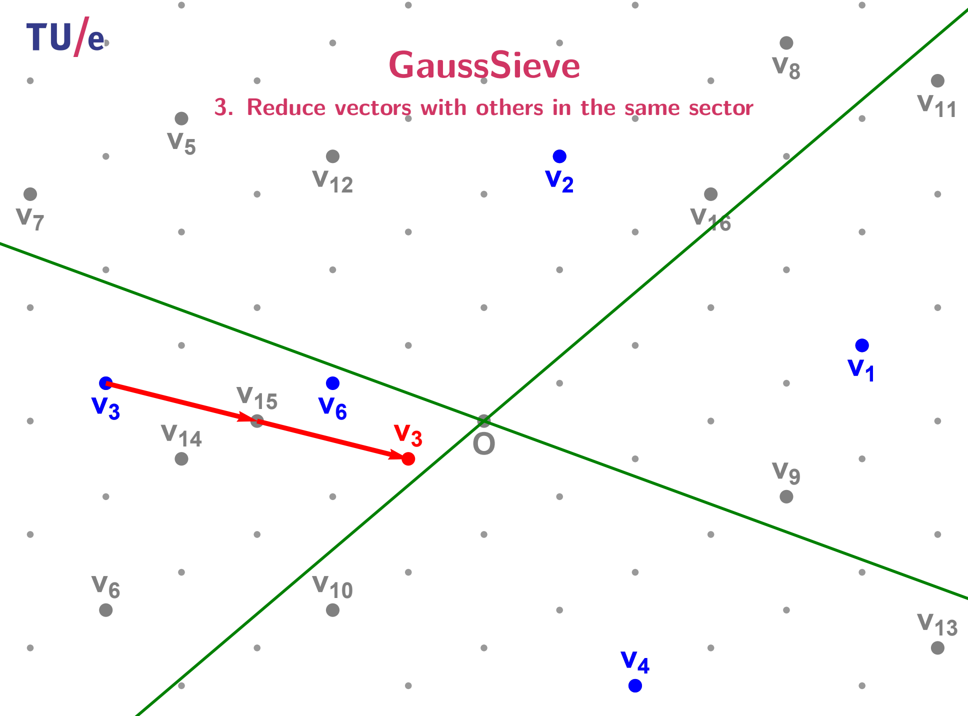

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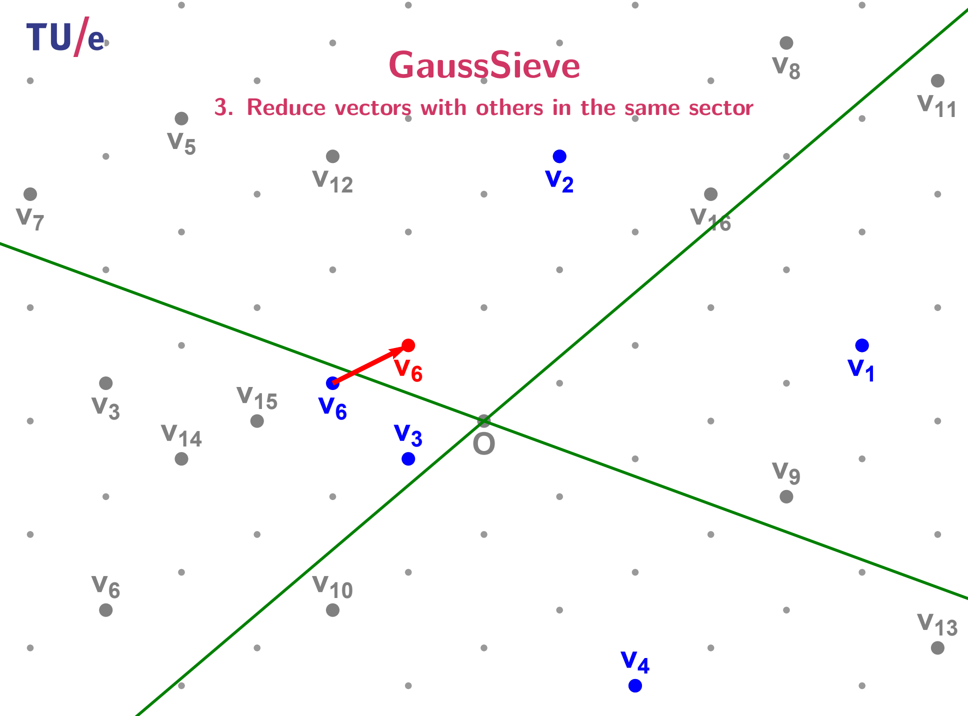

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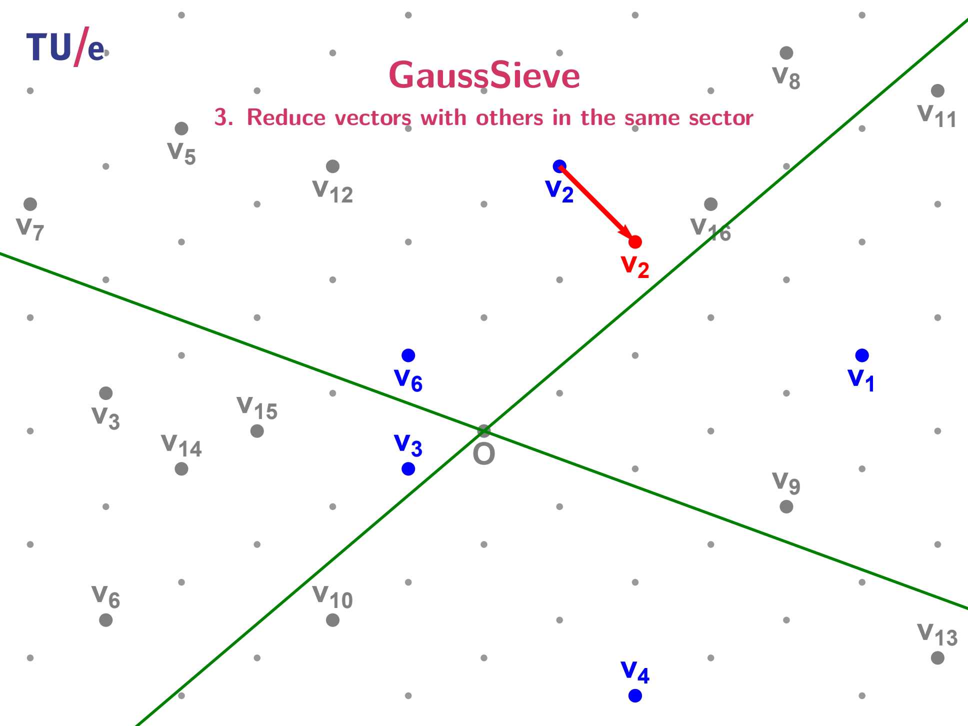

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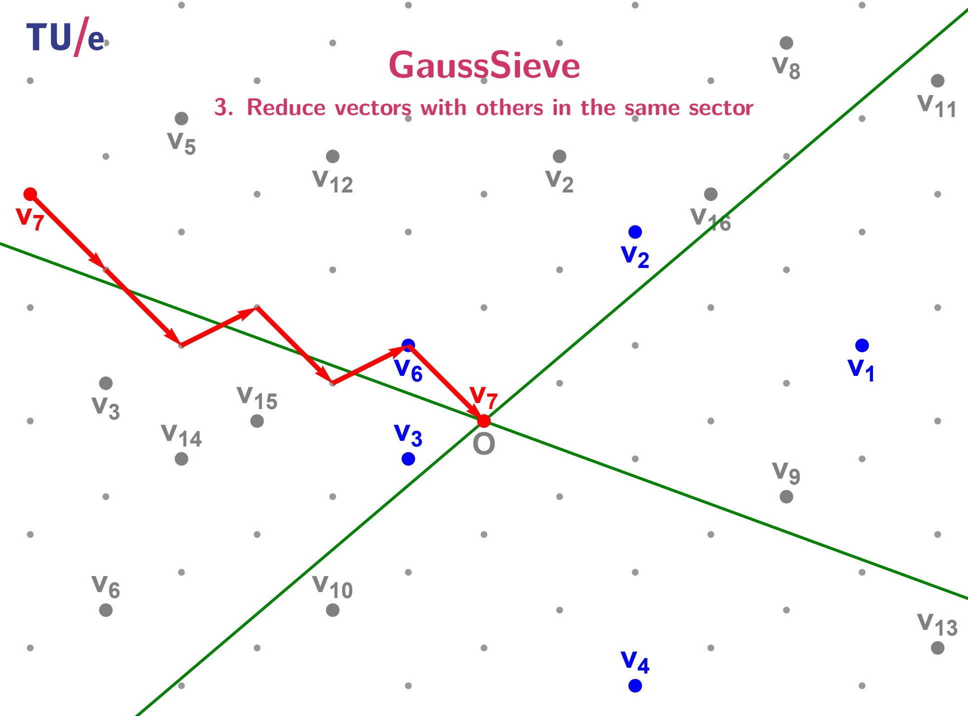

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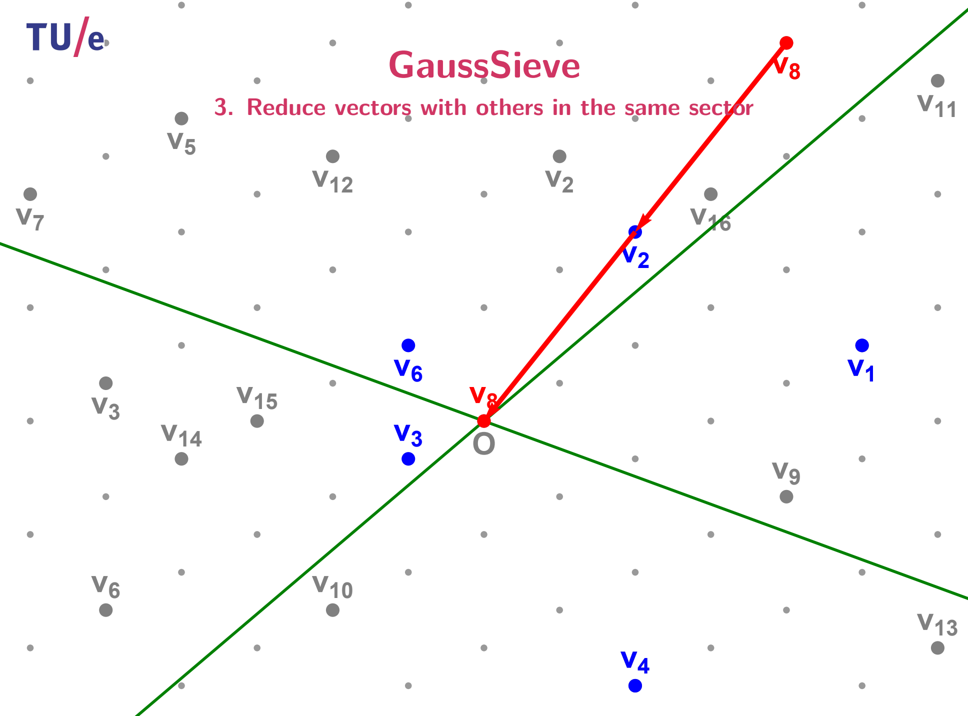

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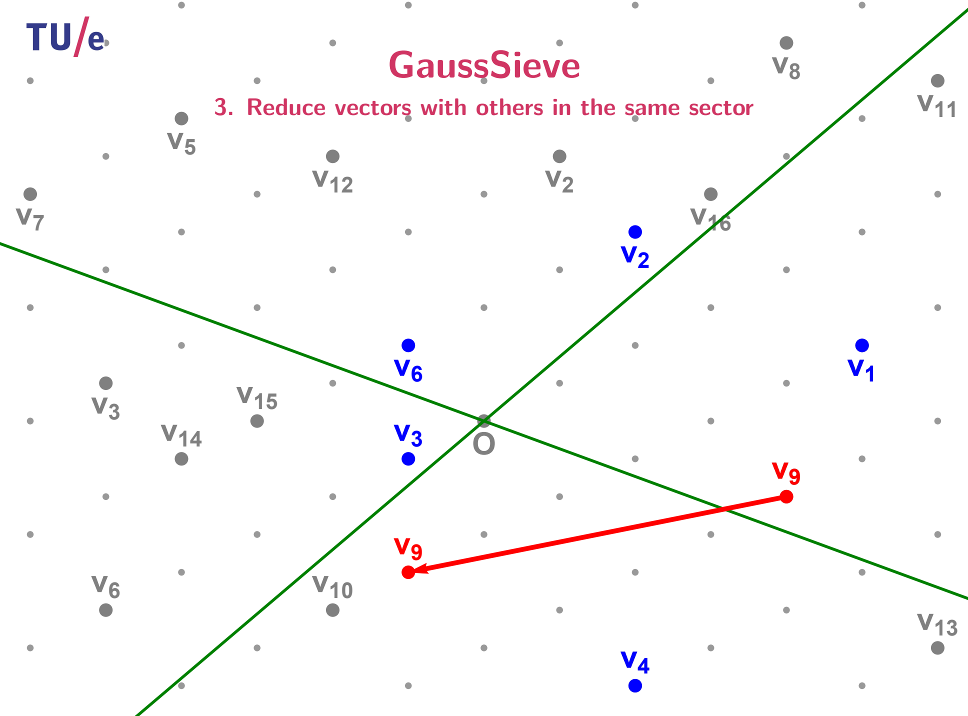

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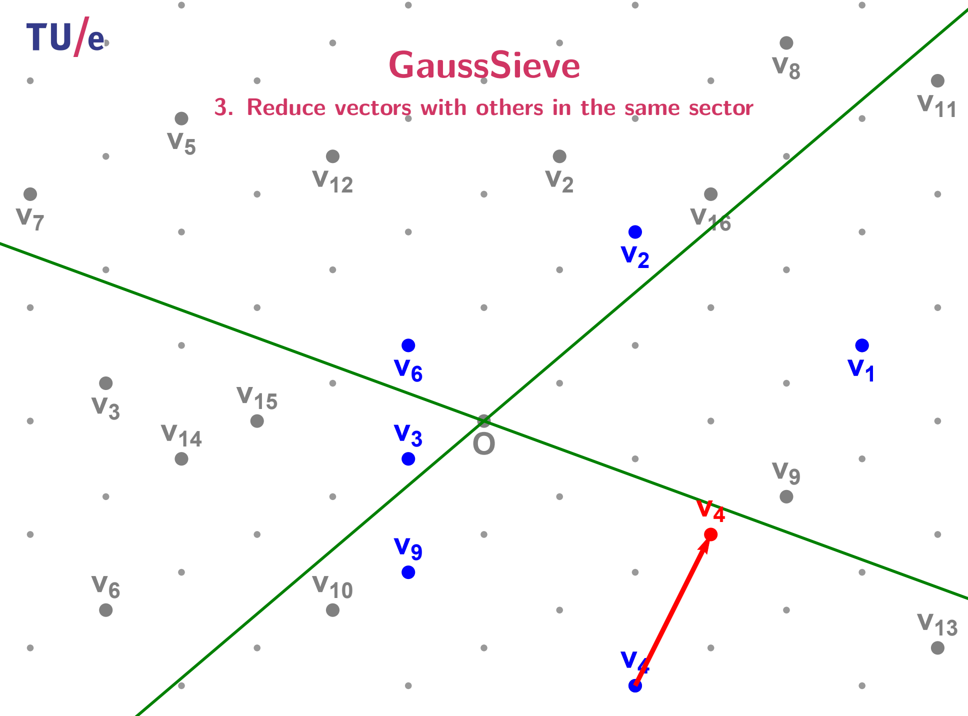

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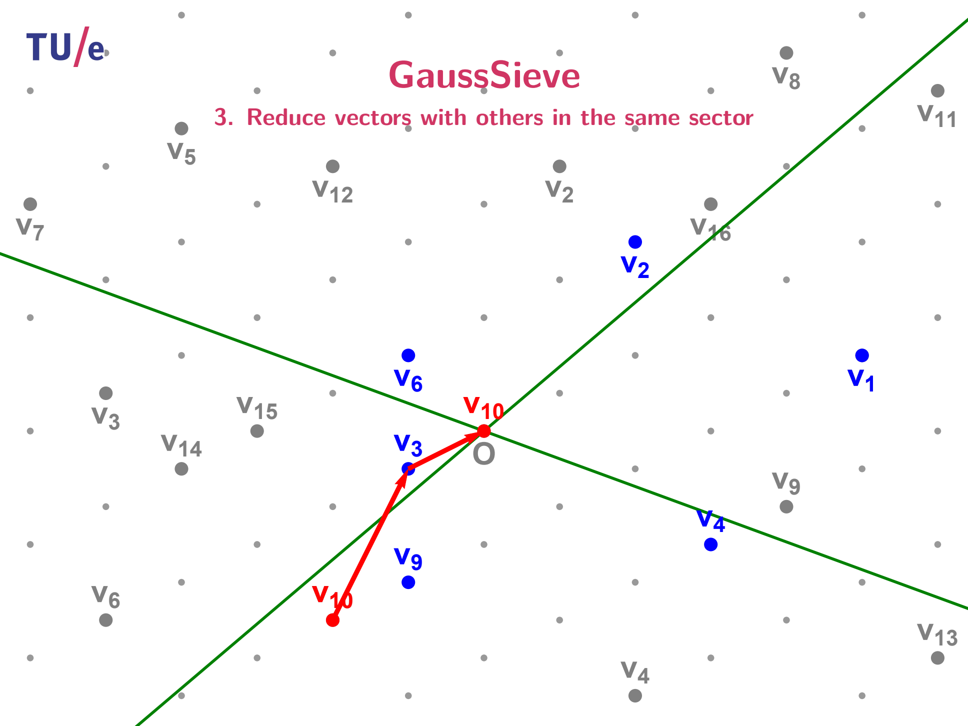

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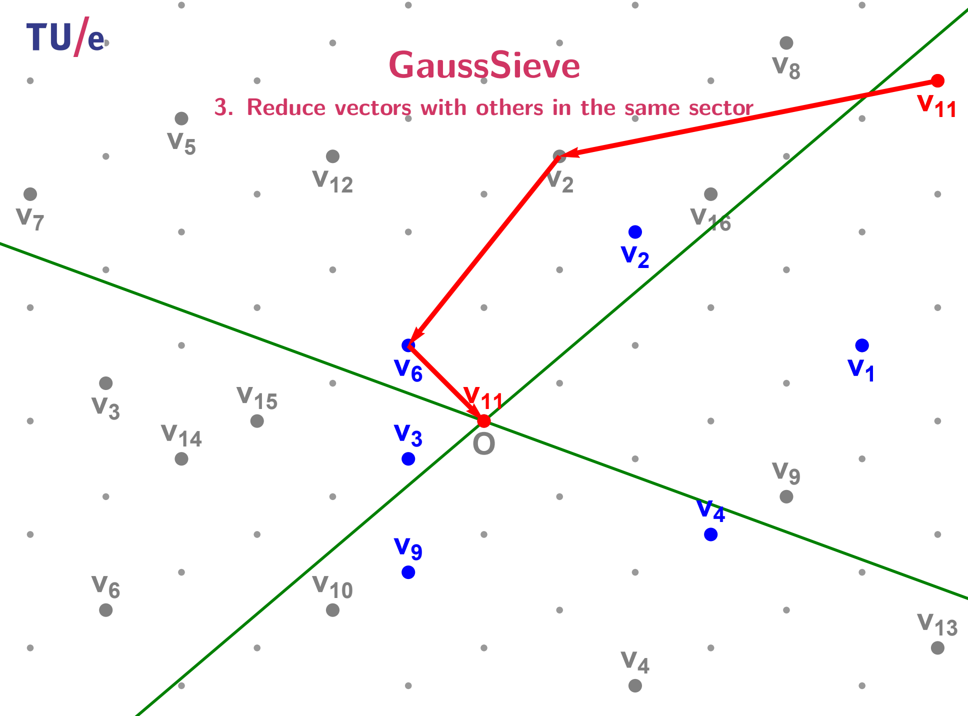

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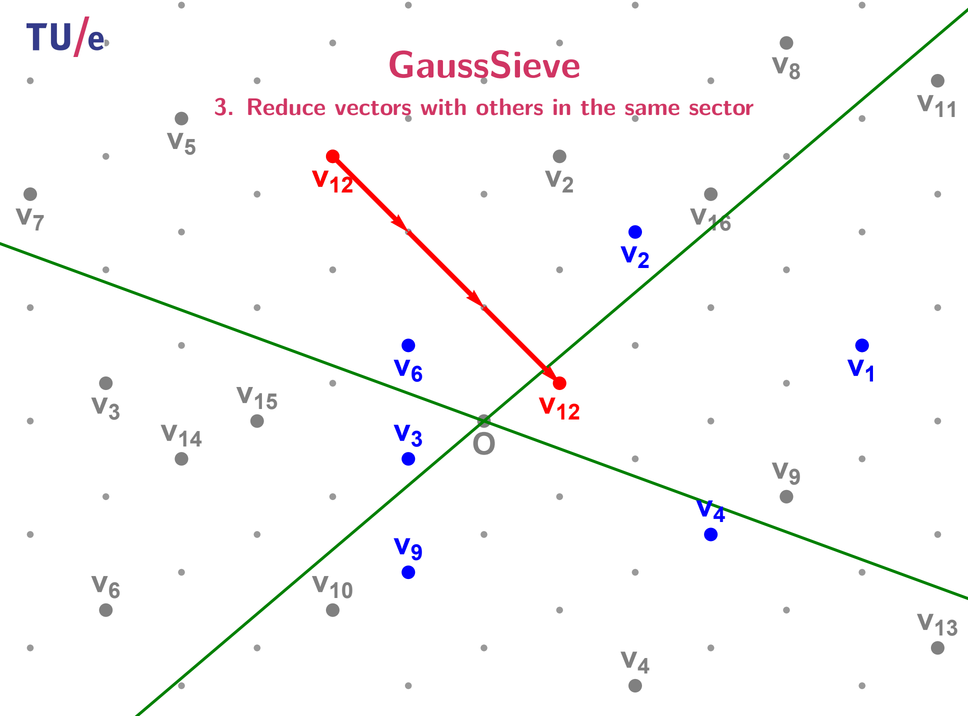

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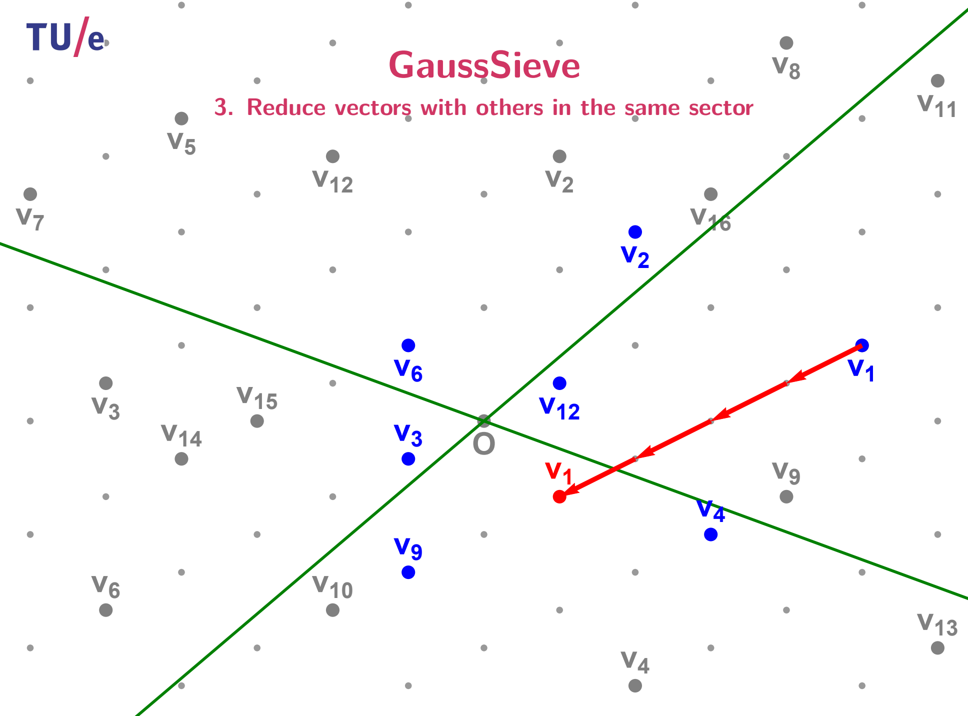

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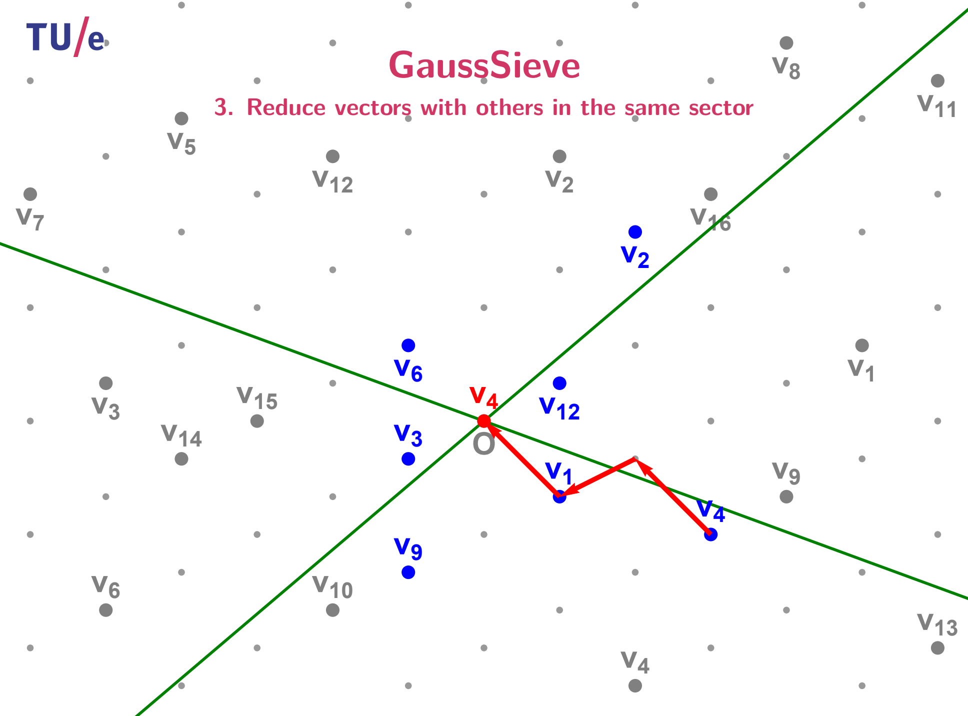

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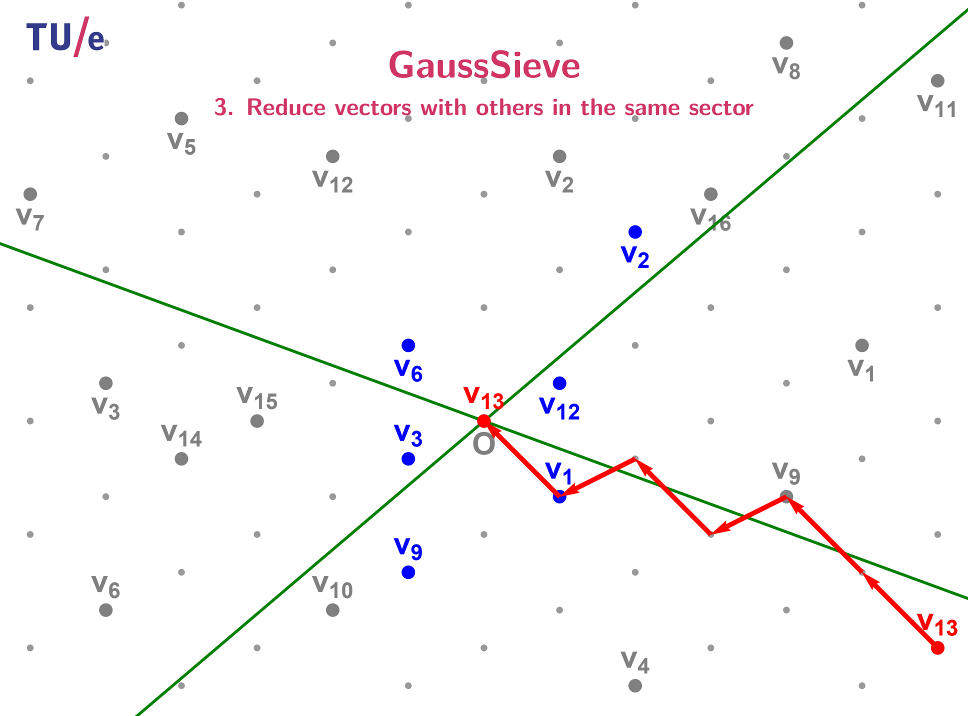

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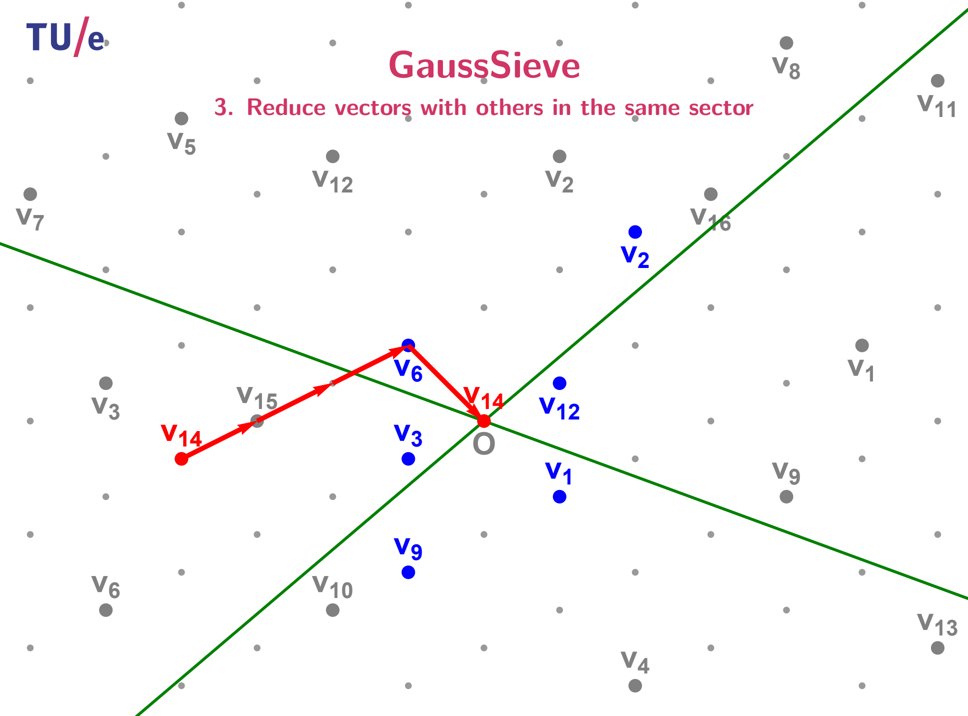

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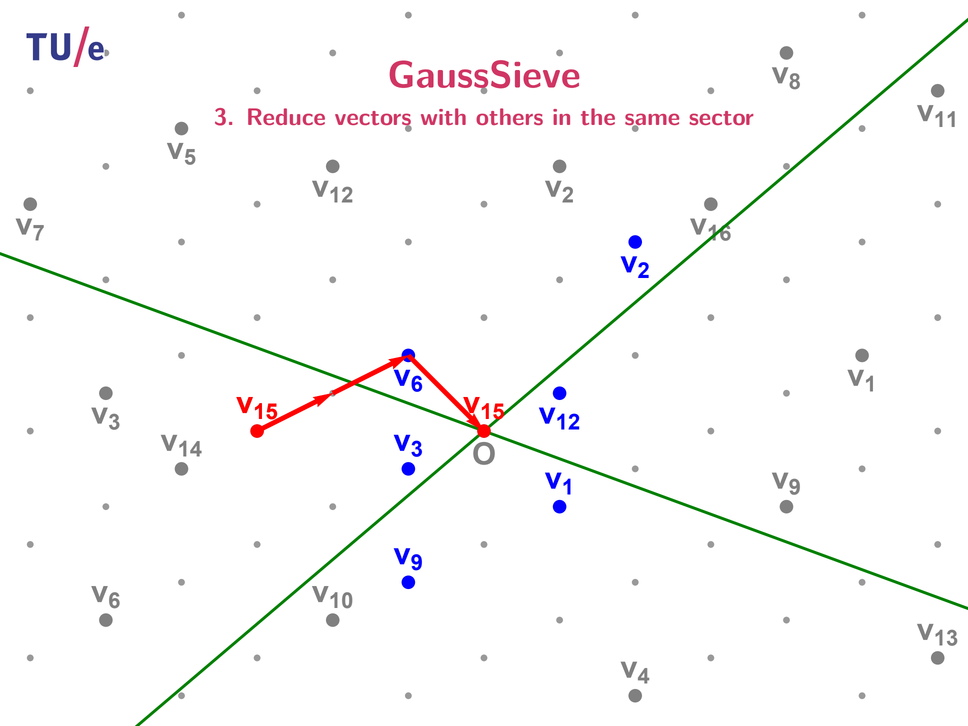

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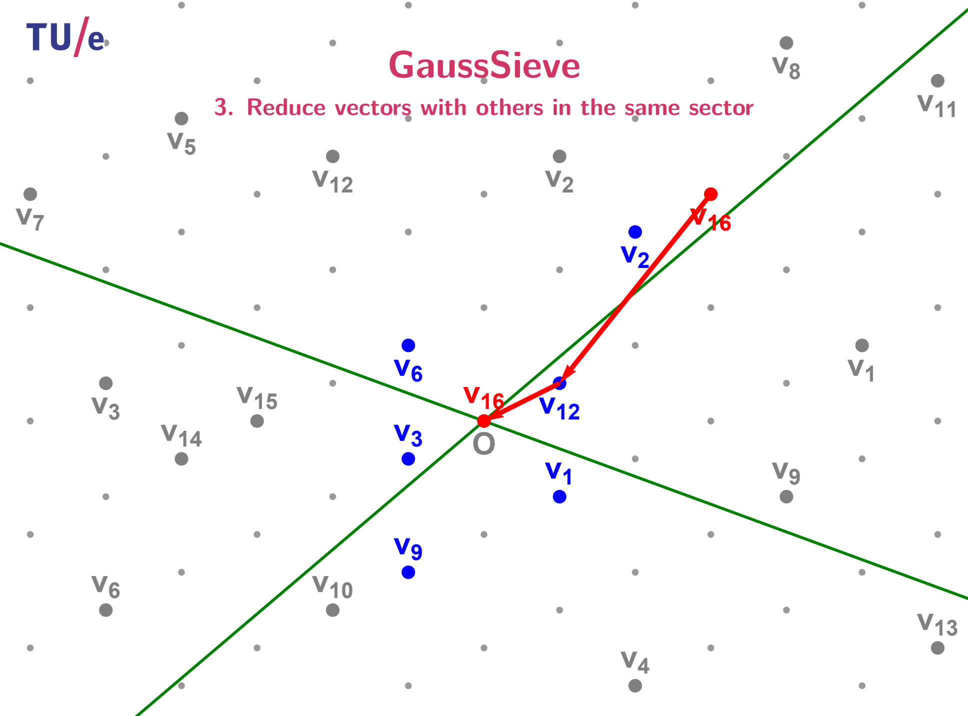

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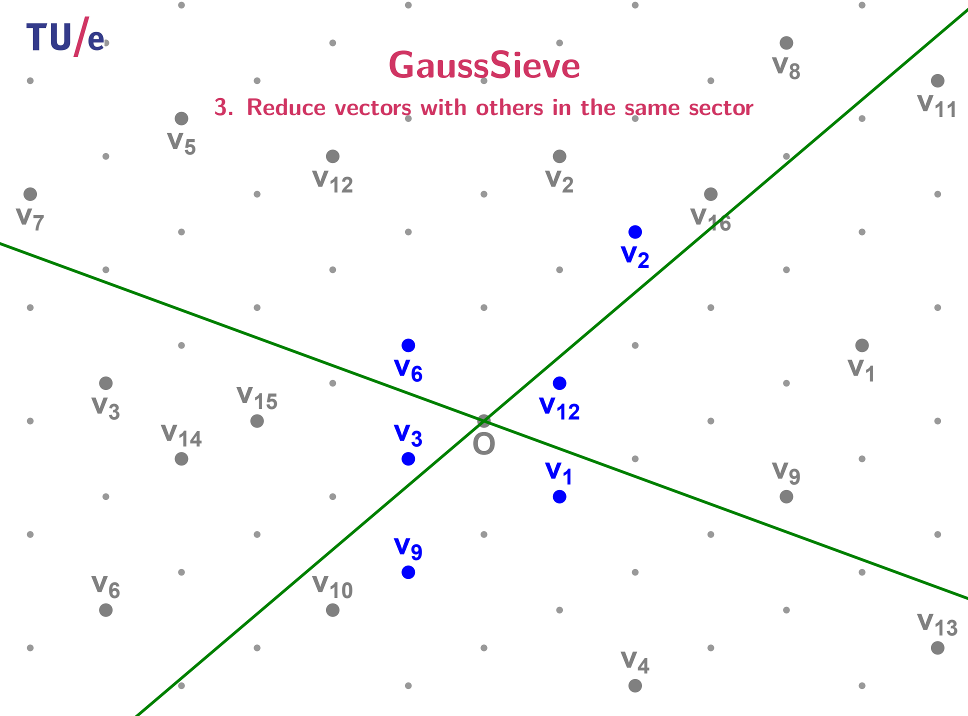
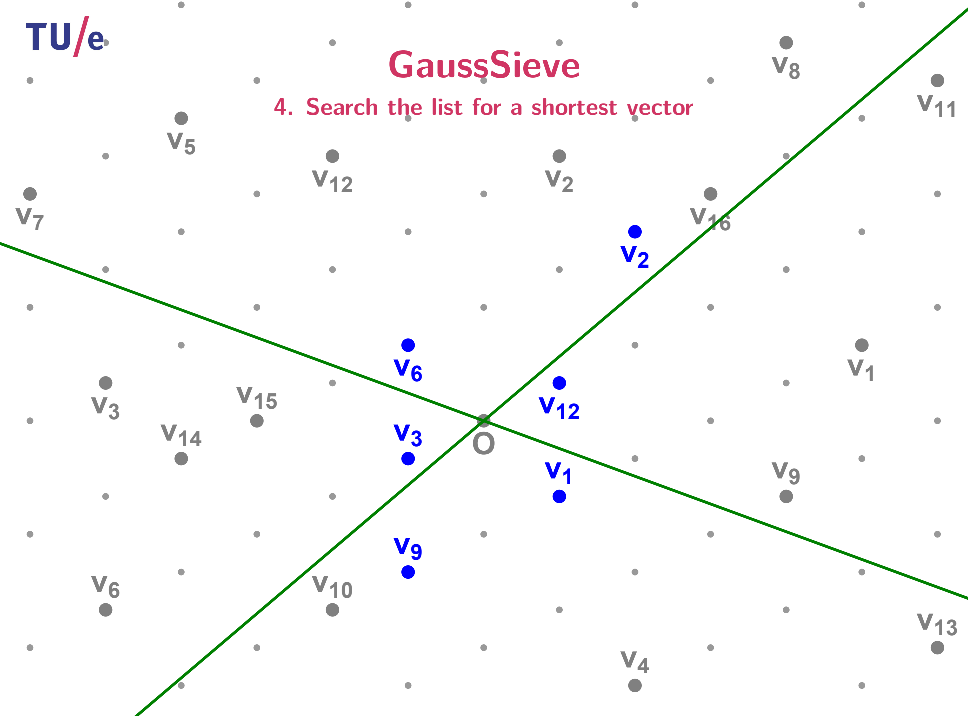


GaussSieve

4. Search the list for a shortest vector



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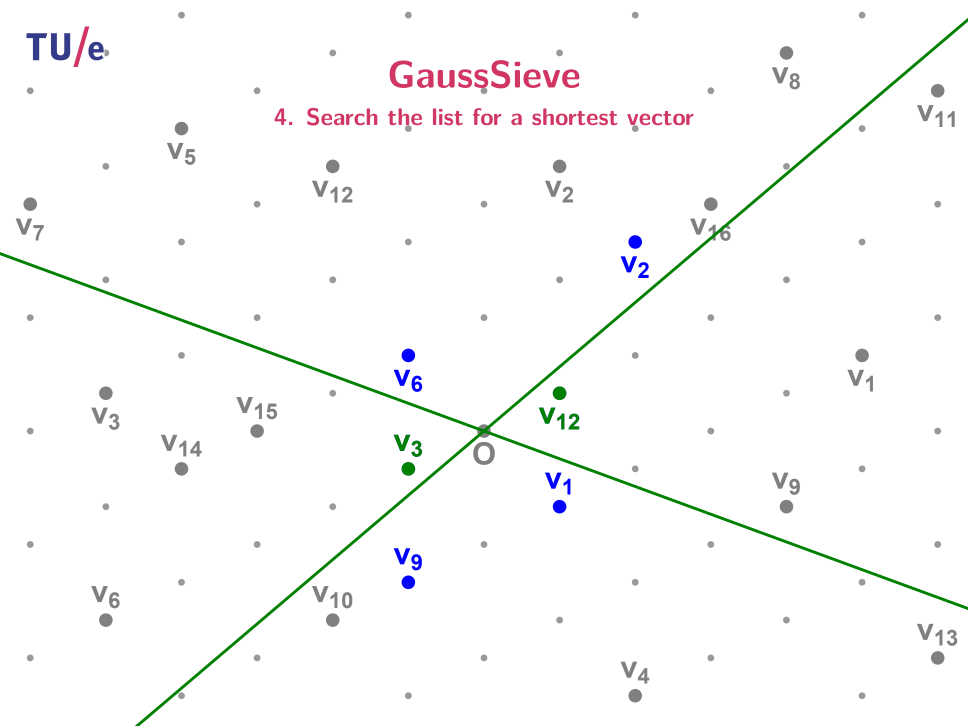

Sieving with angular LSH

Overview

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Sieving with angular LSH

Overview

- Two parameters to tune
  - $k = O(n)$: Number of hyperplanes, leading to 2^k regions
  - $t = 2^{O(n)}$: Number of different, independent "hash tables"

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Sieving with LSH runs in time $2^{0.34n+o(n)}$ and space $2^{0.34n+o(n)}$.

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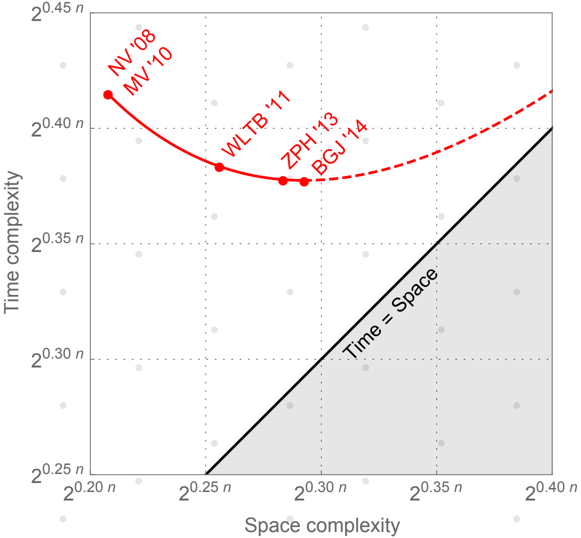
Full details online at `http://eprint.iacr.org/2014/744`

Results

Space/time trade-off



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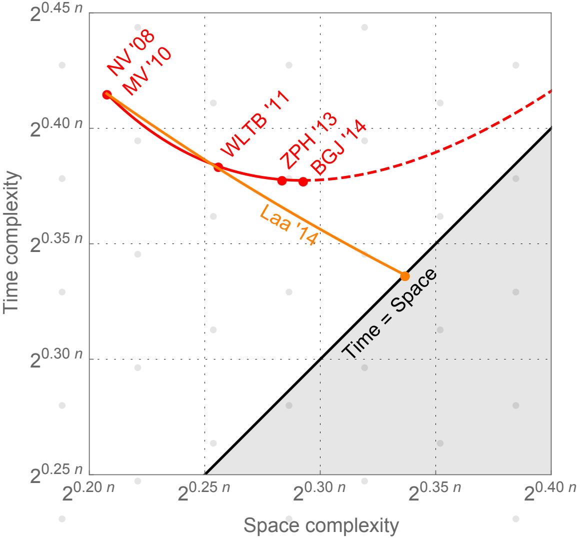

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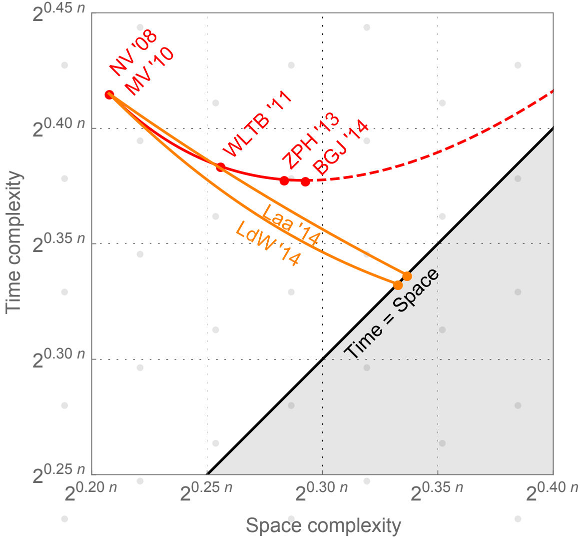

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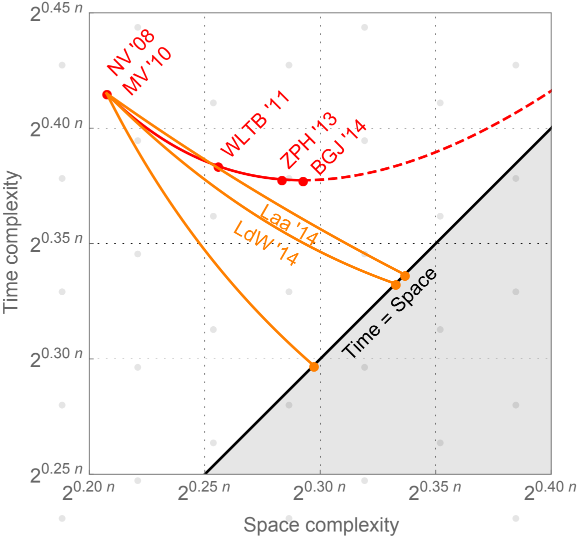

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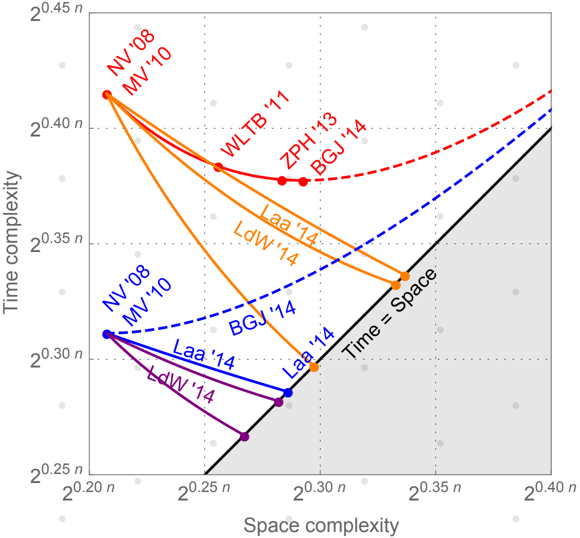

Questions