

Sieving for shortest lattice vectors using nearest neighbor techniques

Thijs Laarhoven

mail@thijs.com
<http://www.thijs.com/>

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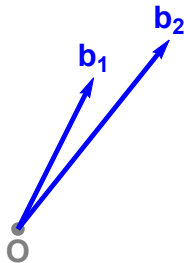
Lattices

What is a lattice?



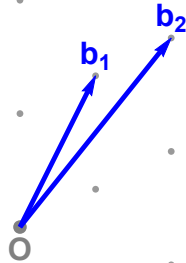
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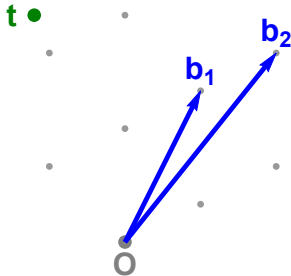
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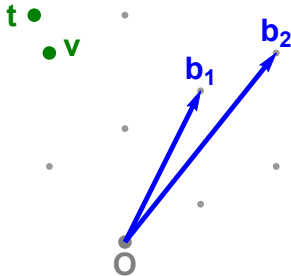
Lattices

Closest Vector Problem (CVP)



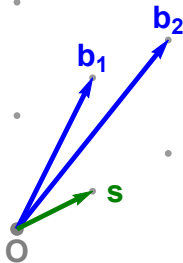
Lattices

Closest Vector Problem (CVP)



Lattices

Shortest Vector Problem (SVP)



Lattices

Applications

- “Constructive cryptography”: Lattice-based cryptosystems
 - ▶ Based on hard lattice problems (SVP, CVP, LWE, SIS)
 - ▶ NTRU cryptosystem [HPS98, . . . , HPSSWZ15]
 - ▶ HIMMO key pre-distribution scheme [GMGPGRST14]
 - ▶ Fully Homomorphic Encryption [Gen09, . . . , CM15]
 - ▶ Candidate for “post-quantum cryptography”

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- “Destructive cryptography”: Lattice cryptanalysis
 - ▶ Attack knapsack-based cryptosystems [Sha82, LO85, ...]
 - ▶ Attack RSA with Coppersmith’s method [Cop97, ...]
 - ▶ Attack lattice-based cryptosystems [Ngu99, JJ00, ...]

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How hard are hard lattice problems such as SVP?

Lattices

Exact SVP algorithms

	Algorithm	$\log_2(\text{Time})$	$\log_2(\text{Space})$
Provable SVP	Enumeration [Poh81, Kan83, . . . , MW15]	$\Omega(n \log n)$	$O(\log n)$
	AKS-sieve [AKS01, NV08, MV10, HPS11]	$3.398n$	$1.985n$
	ListSieve [MV10, MDB14]	$3.199n$	$1.327n$
	AKS-sieve-birthday [PS09, HPS11]	$2.648n$	$1.324n$
	ListSieve-birthday [PS09]	$2.465n$	$1.233n$
	Voronoi cell algorithm [AEVZ02, MV10b]	$2.000n$	$1.000n$
	Discrete Gaussians [ADRS15, ADS15, Ste16]	$1.000n$	$1.000n$
Heuristic SVP	Nguyen-Vidick sieve [NV08]	$0.415n$	$0.208n$
	GaussSieve [MV10, . . . , IKMT14, BNvdP14]	$0.415n?$	$0.208n$
	Two-level sieve [WLTB11]	$0.384n$	$0.256n$
	Three-level sieve [ZPH13]	$0.3778n$	$0.283n$
	Overlattice sieve [BGJ14]	$0.3774n$	$0.293n$
	Hyperplane LSH [Laa15, MLB15]	$0.337n$	$0.208n$
	May and Ozerov's NNS method [BGJ15]	$0.311n$	$0.208n$
	Spherical LSH [LdW15]	$0.298n$	$0.208n$
	Cross-polytope LSH [BL15]	$0.298n$	$0.208n$
	Spherical filtering [BDGL16]	$0.293n$	$0.208n$

Nguyen-Vidick sieve

0

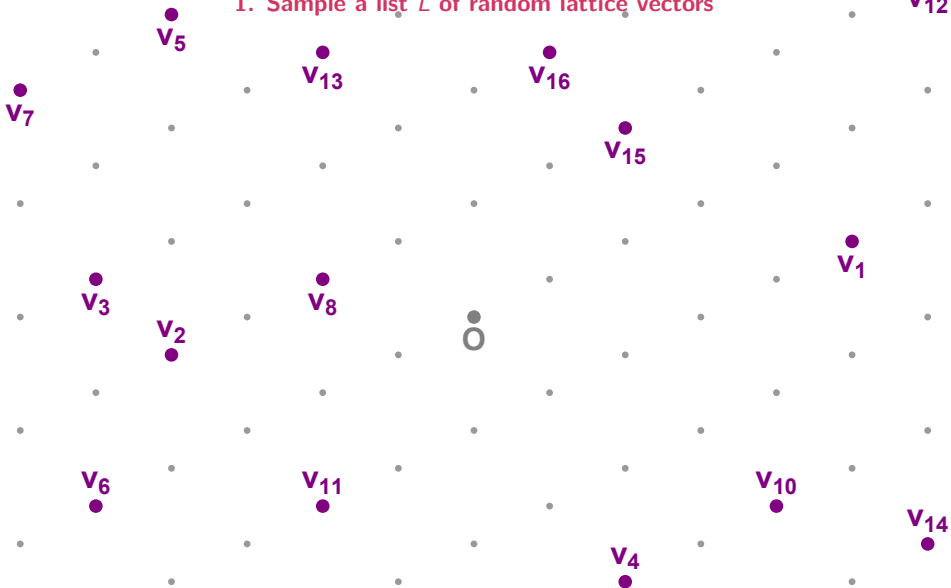
Nguyen-Vidick sieve

1. Sample a list L of random lattice vectors



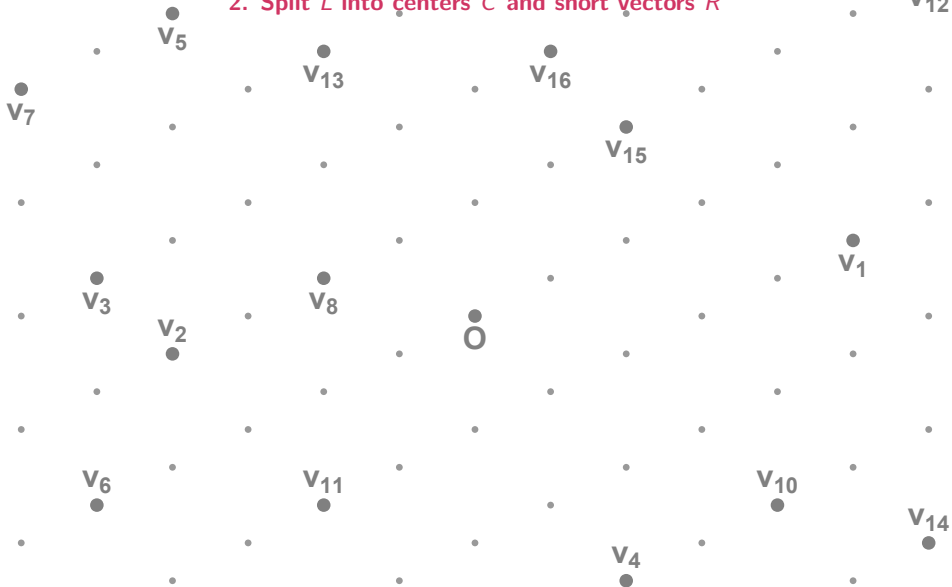
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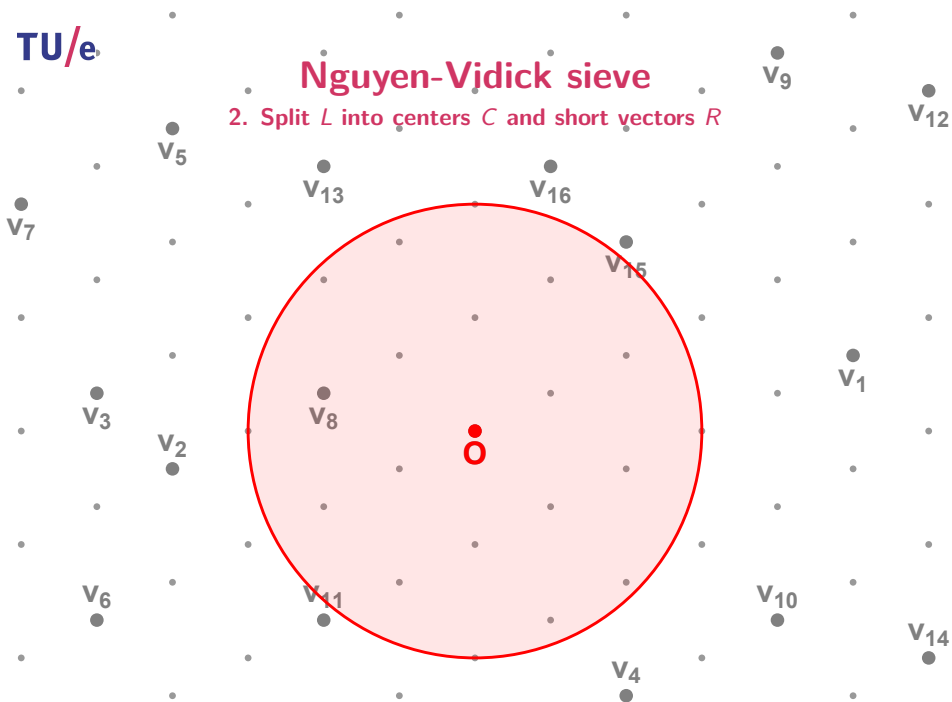
Nguyen-Vidick sieve

2. Split L into centers C and short vectors R



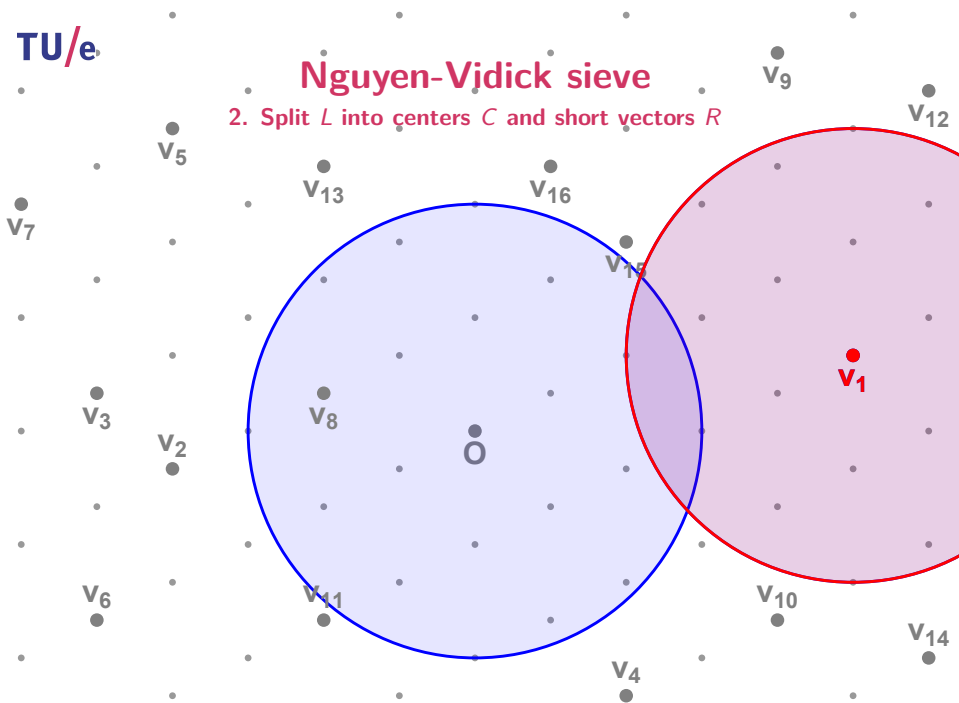
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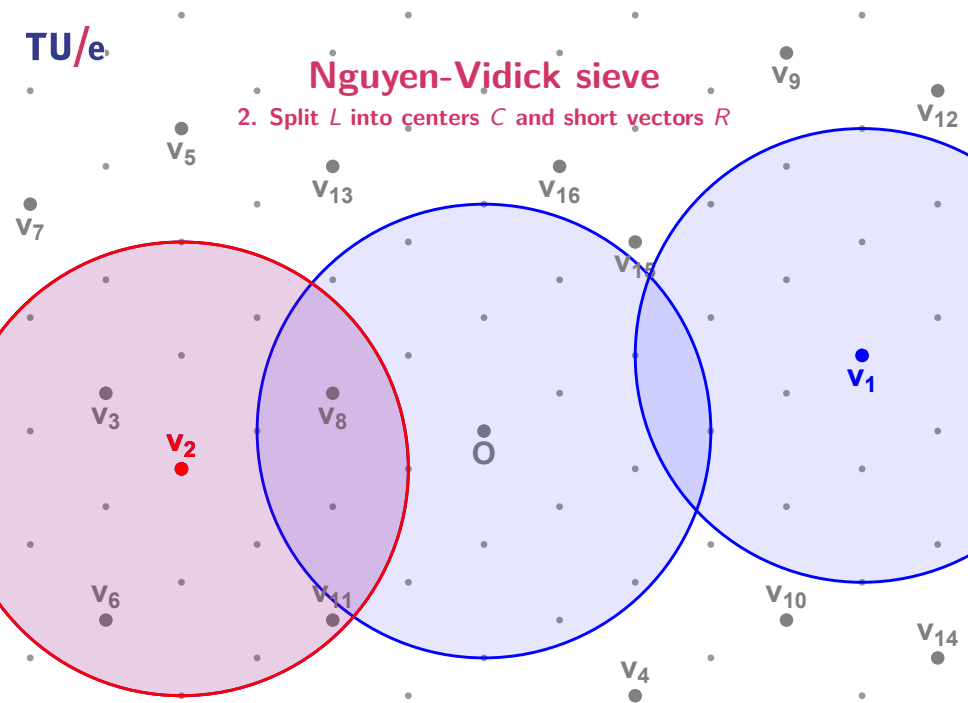
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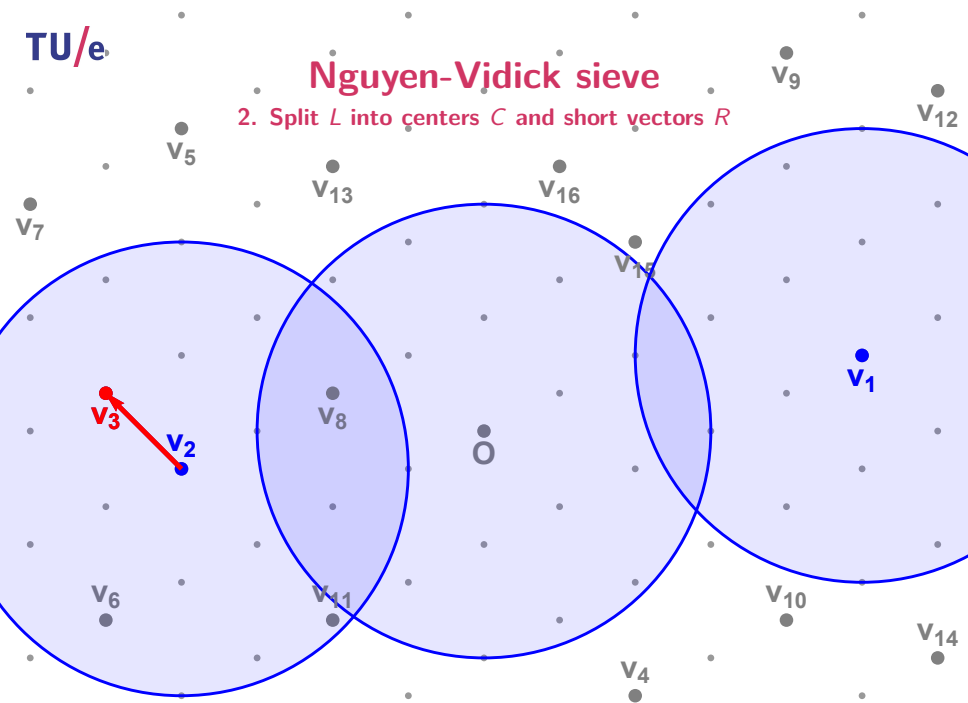
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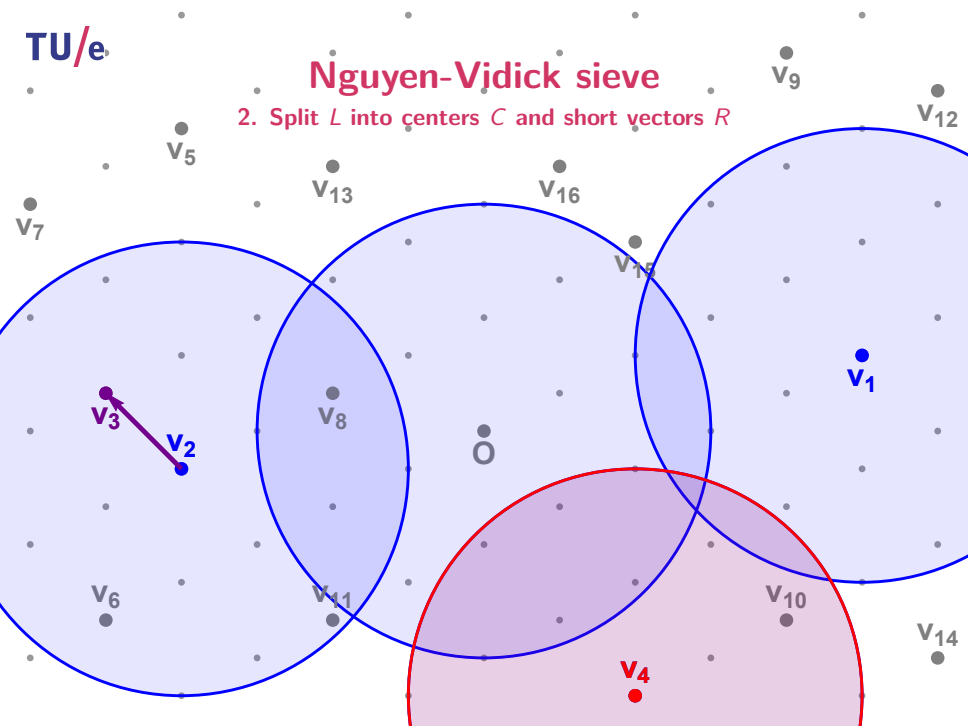
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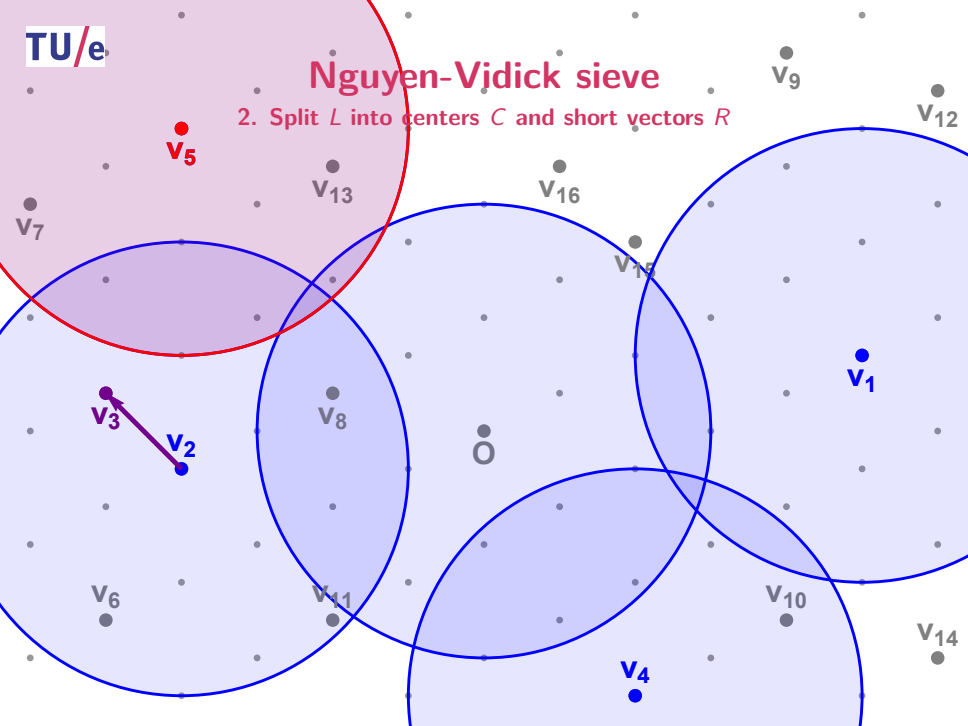
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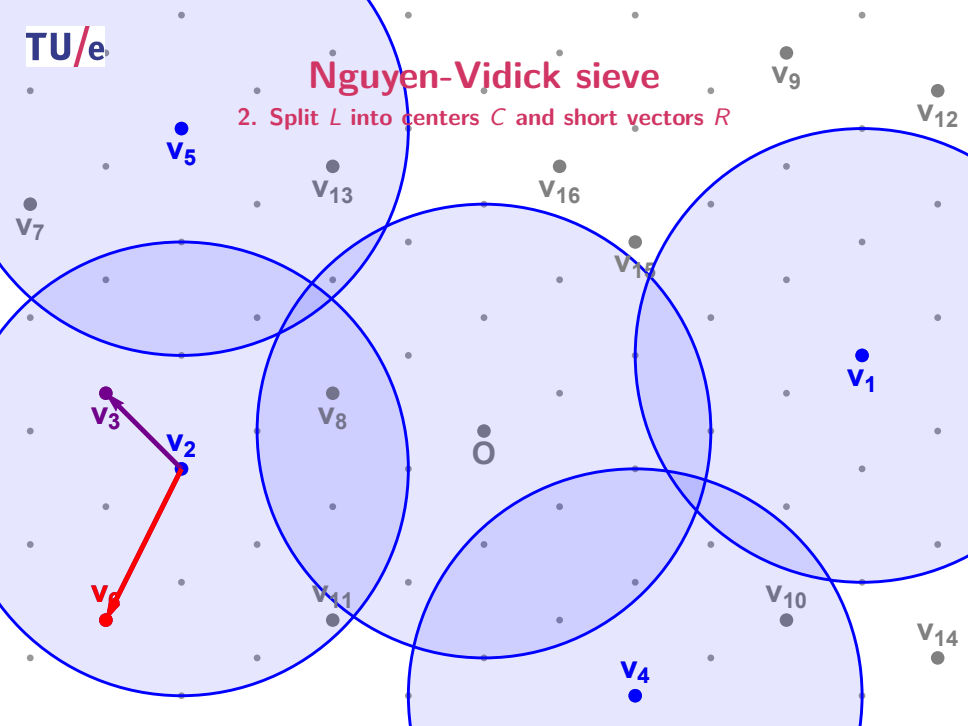
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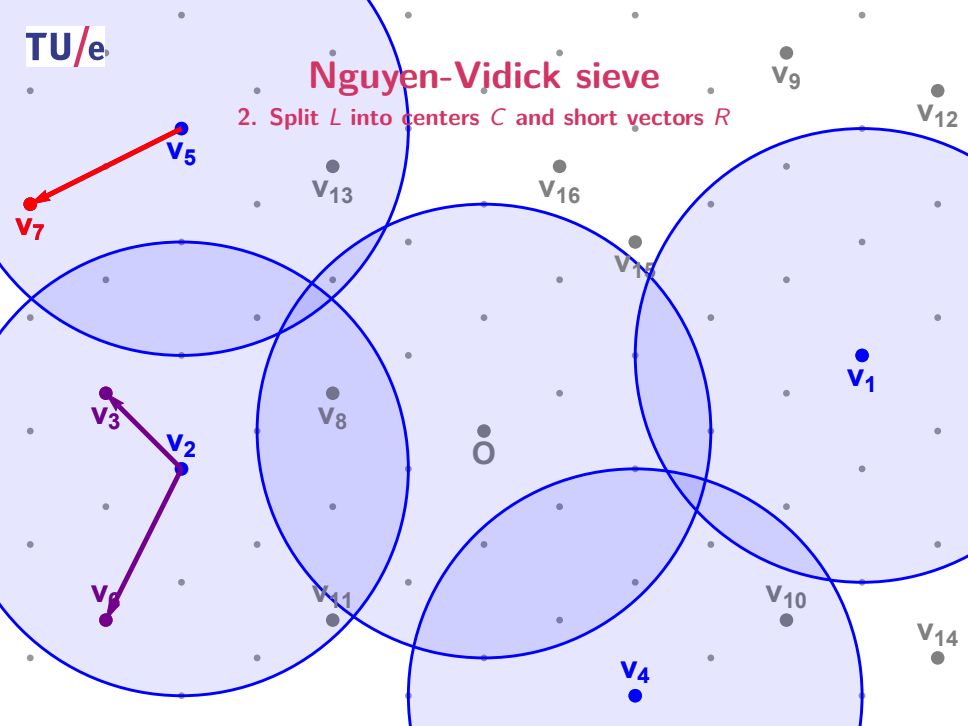
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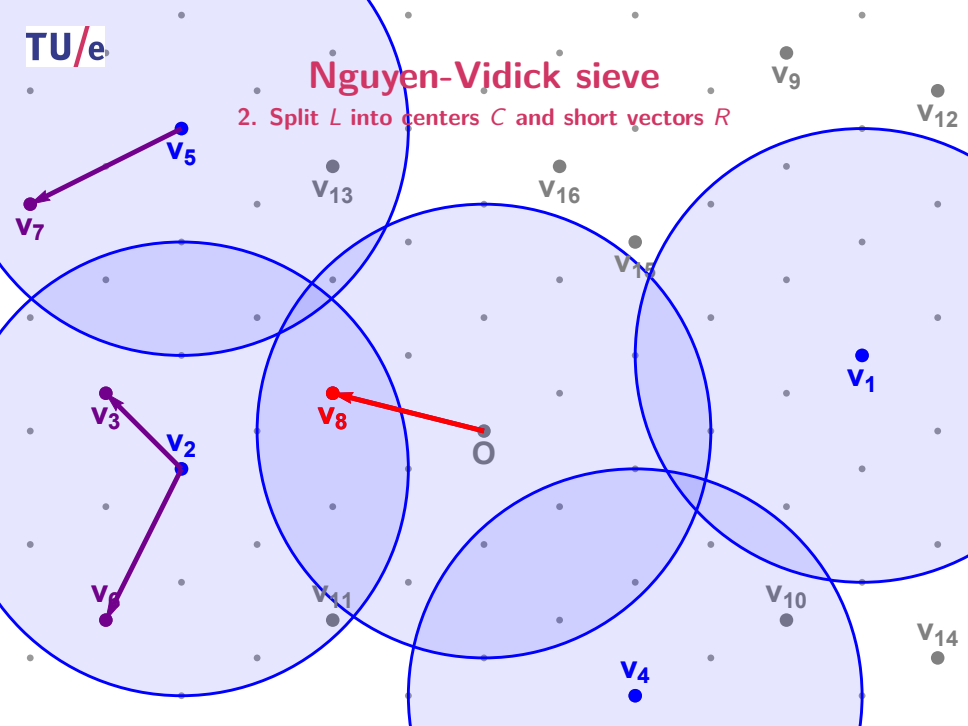
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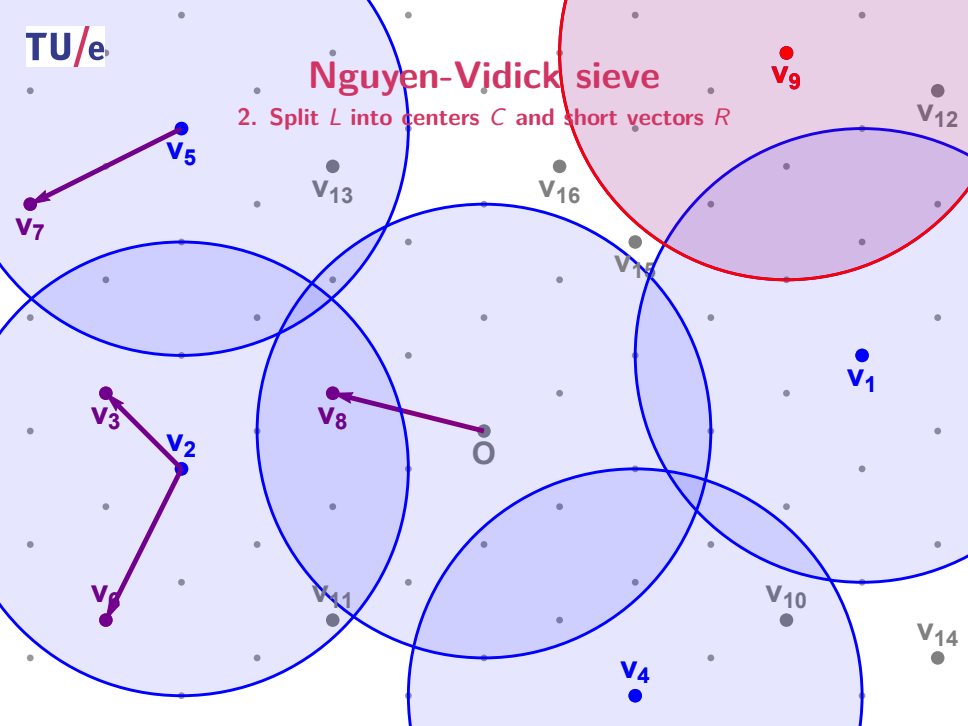
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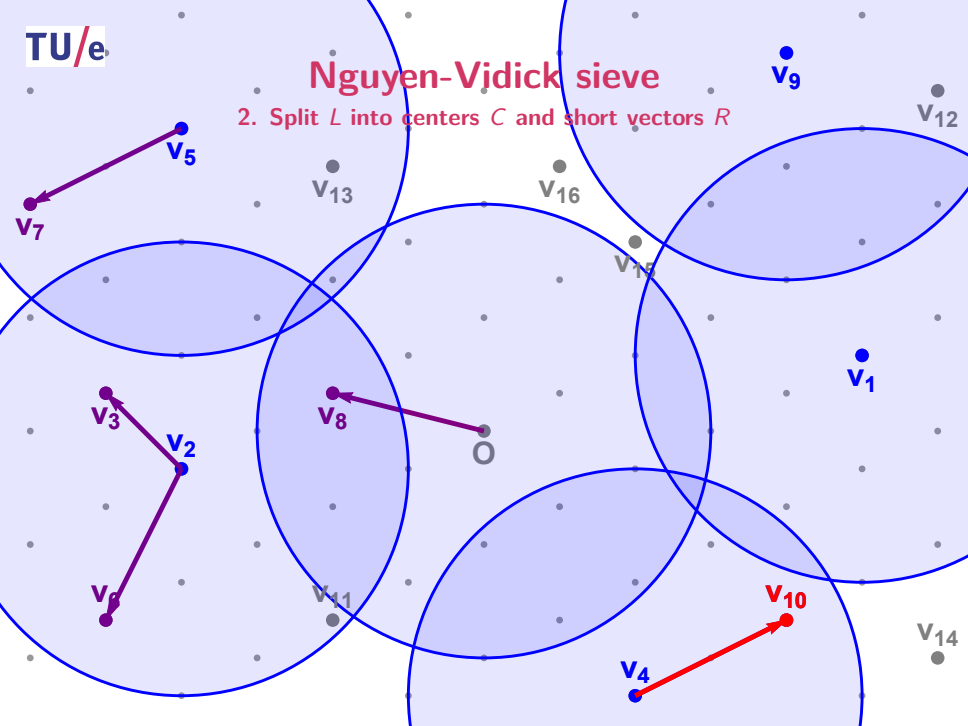
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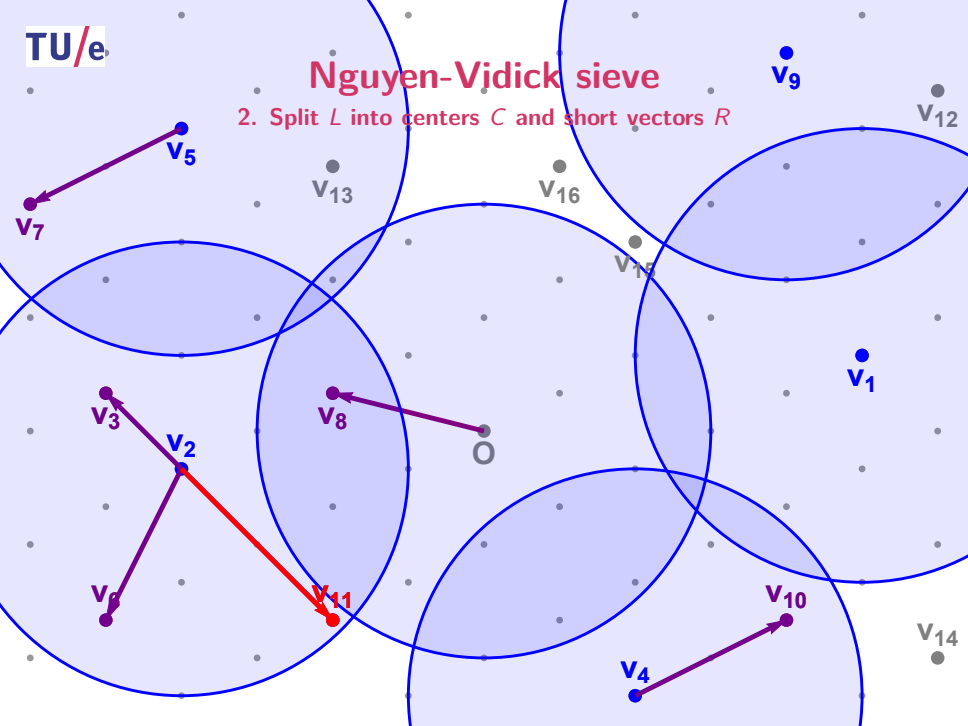
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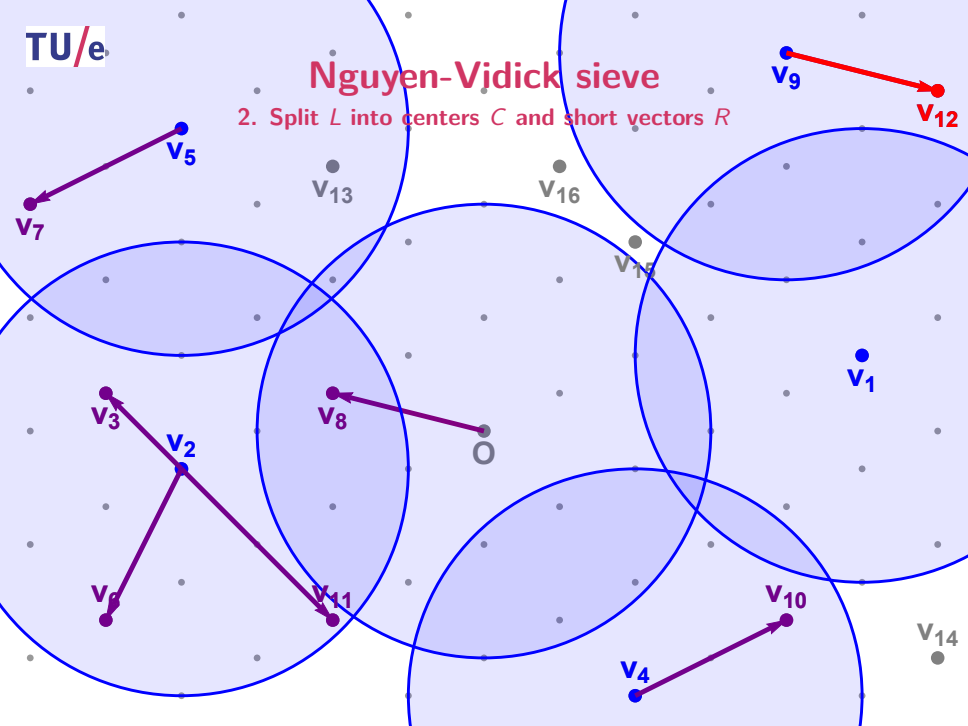
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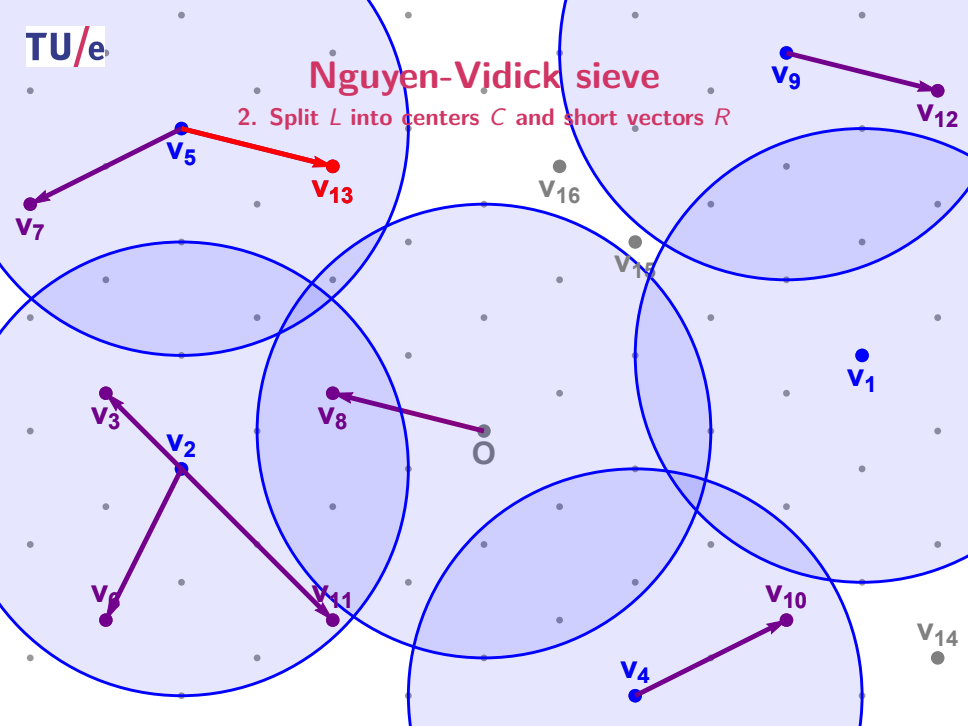
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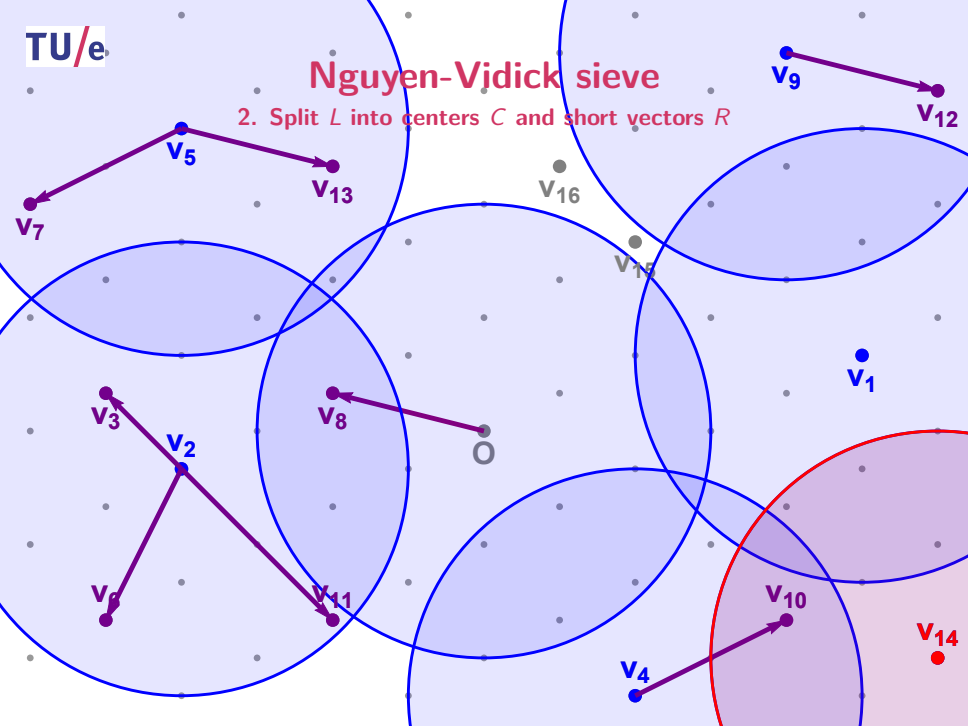
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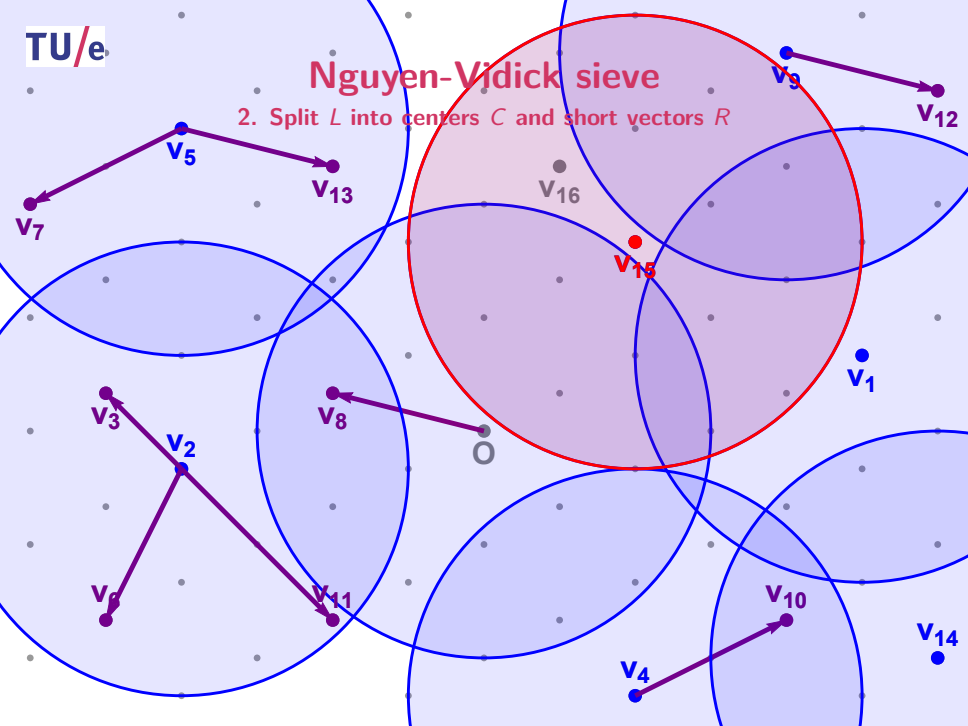
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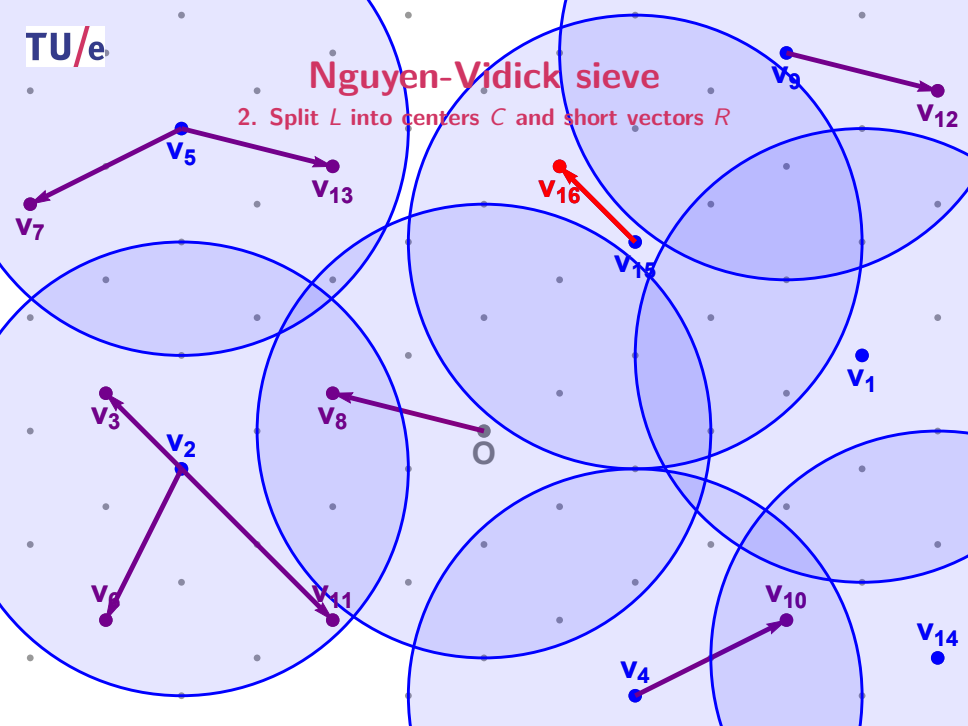
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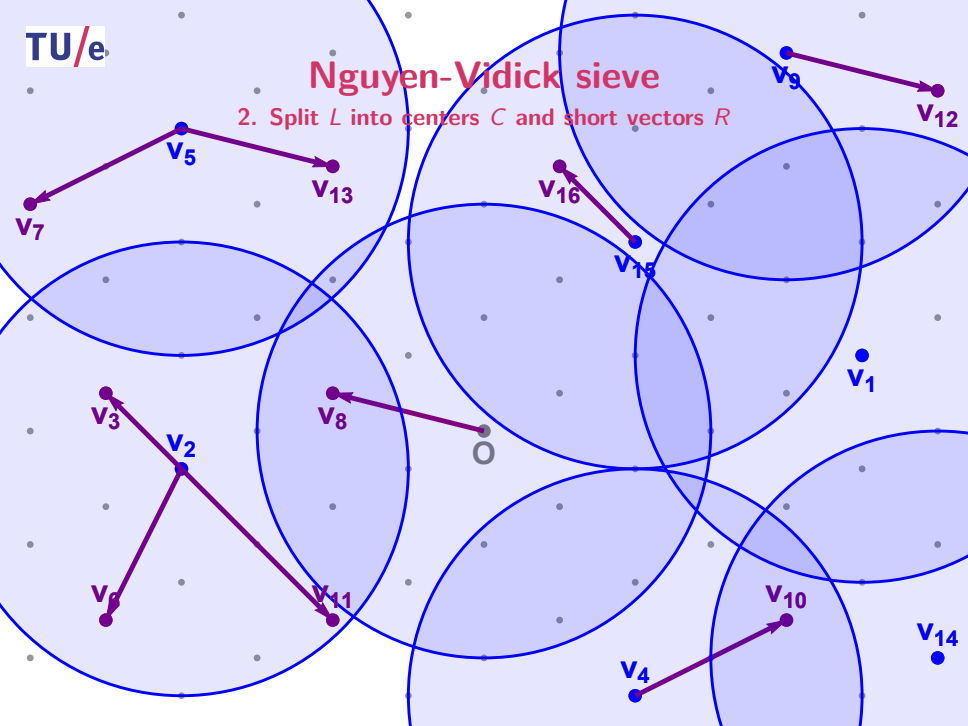
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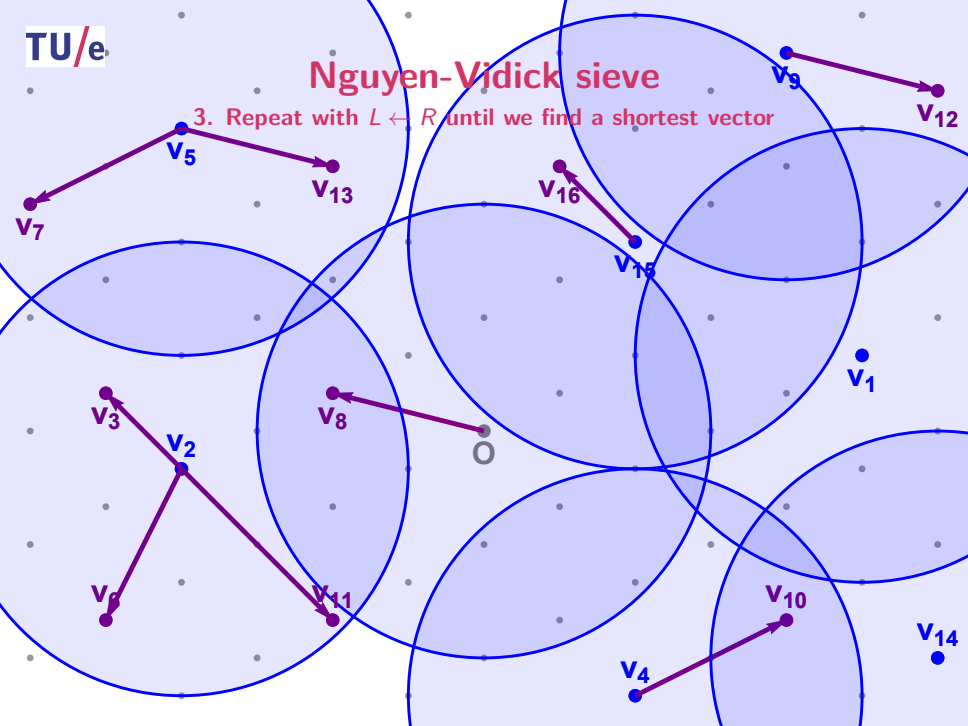
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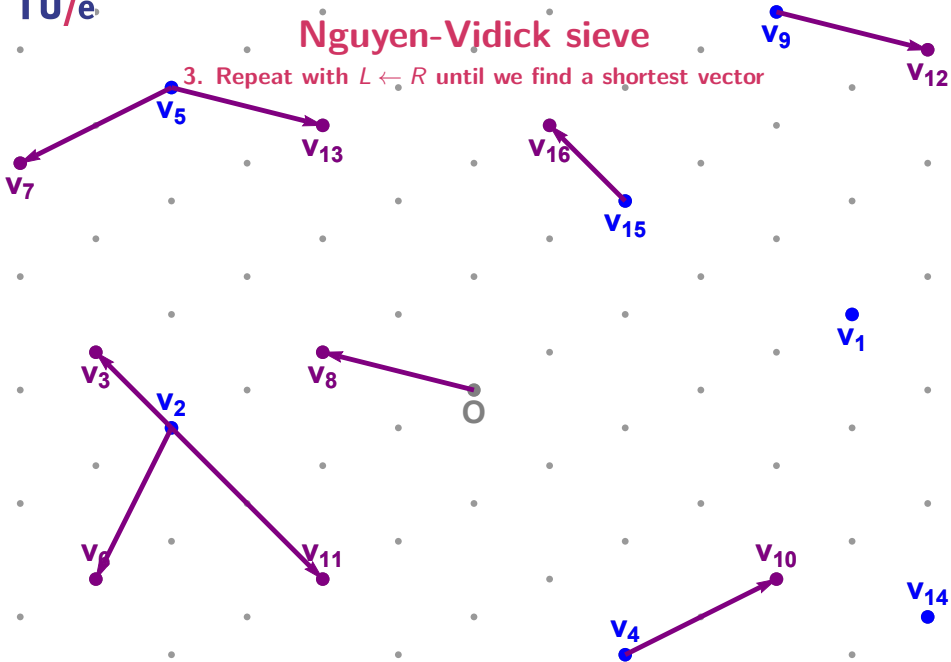
Nguyen-Vidick sieve

3. Repeat with $L \leftarrow R$ until we find a shortest vector



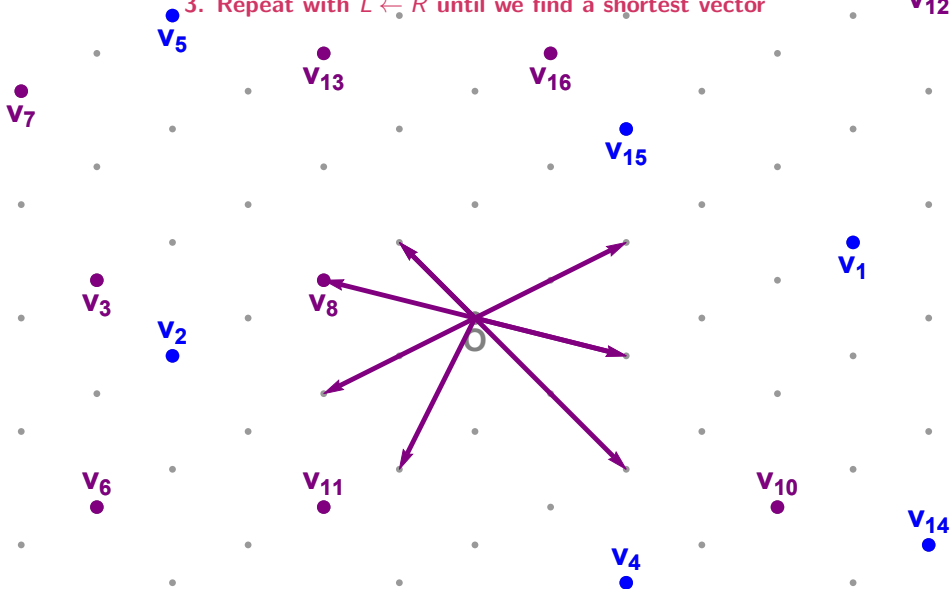
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Nguyen-Vidick sieve

Overview



Nguyen-Vidick sieve

Overview

- Space complexity: $\sqrt{4/3}^n \approx 2^{0.21n+o(n)}$ vectors
 - ▶ Need $\sqrt{4/3}^n$ vectors to cover all corners of $\mathbb{R}^n$



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- Space complexity: $\sqrt{4/3}^n \approx 2^{0.21n+o(n)}$ vectors
 - ▶ Need $\sqrt{4/3}^n$ vectors to cover all corners of $\mathbb{R}^n$
- Time complexity: $(4/3)^n \approx 2^{0.42n+o(n)}$
 - ▶ Comparing a target vector to all centers: $2^{0.21n+o(n)}$
 - ▶ Repeating this for each list vector: $2^{0.21n+o(n)}$
 - ▶ Repeating the whole sieving procedure: $\text{poly}(n)$

Nguyen-Vidick sieve

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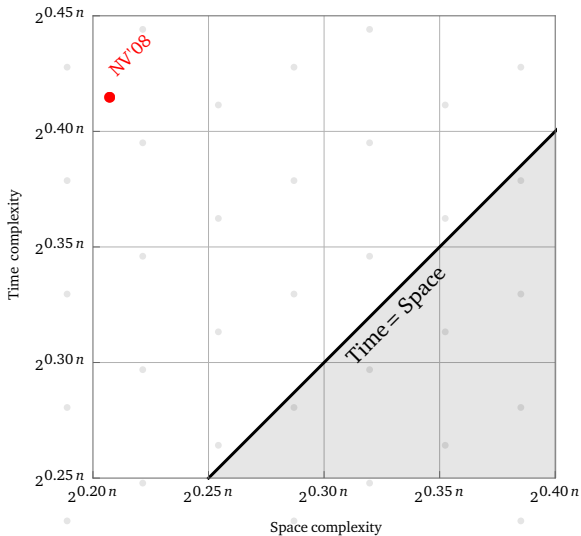
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Heuristic (Nguyen and Vidick, J. Math. Crypt. '08)

The NV-sieve runs in time $2^{0.42n+o(n)}$ and space $2^{0.21n+o(n)}$.

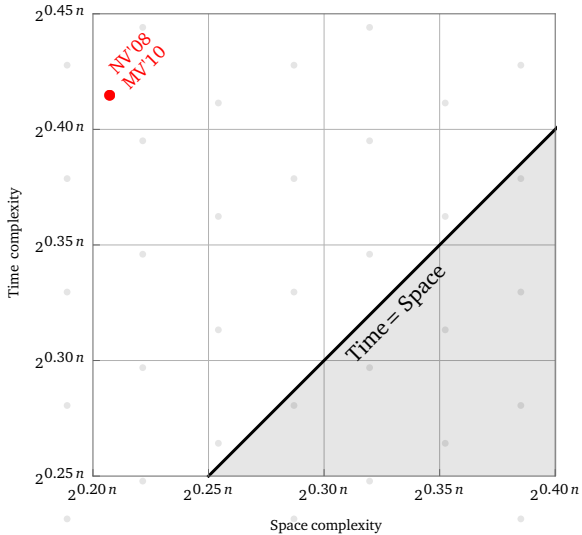
Nguyen-Vidick sieve

Space/time trade-off



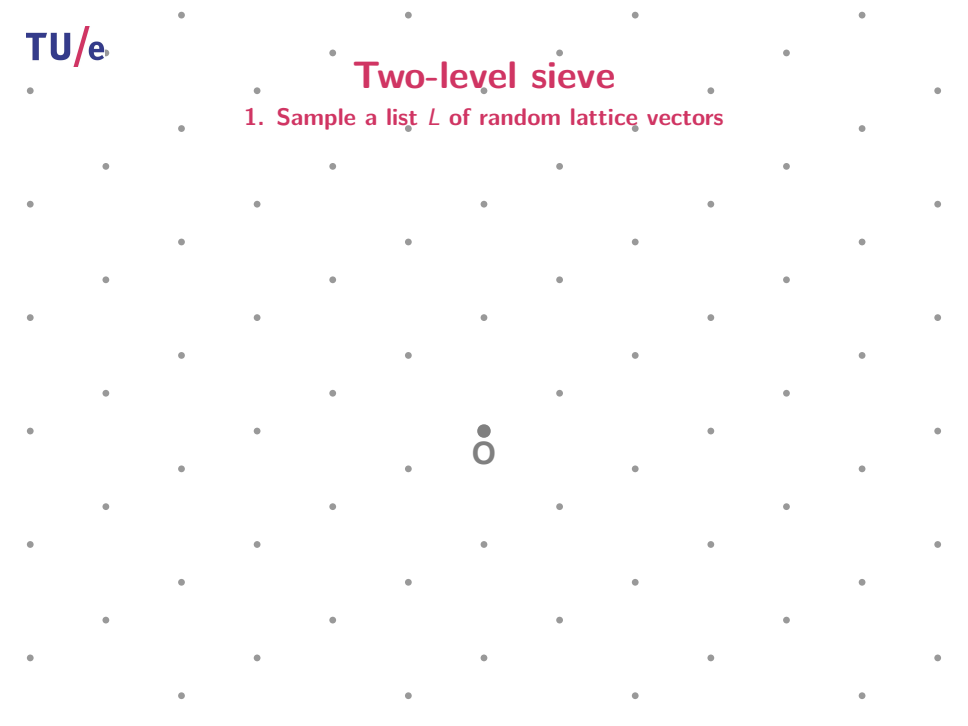
GaussSieve

Space/time trade-off



Two-level sieve

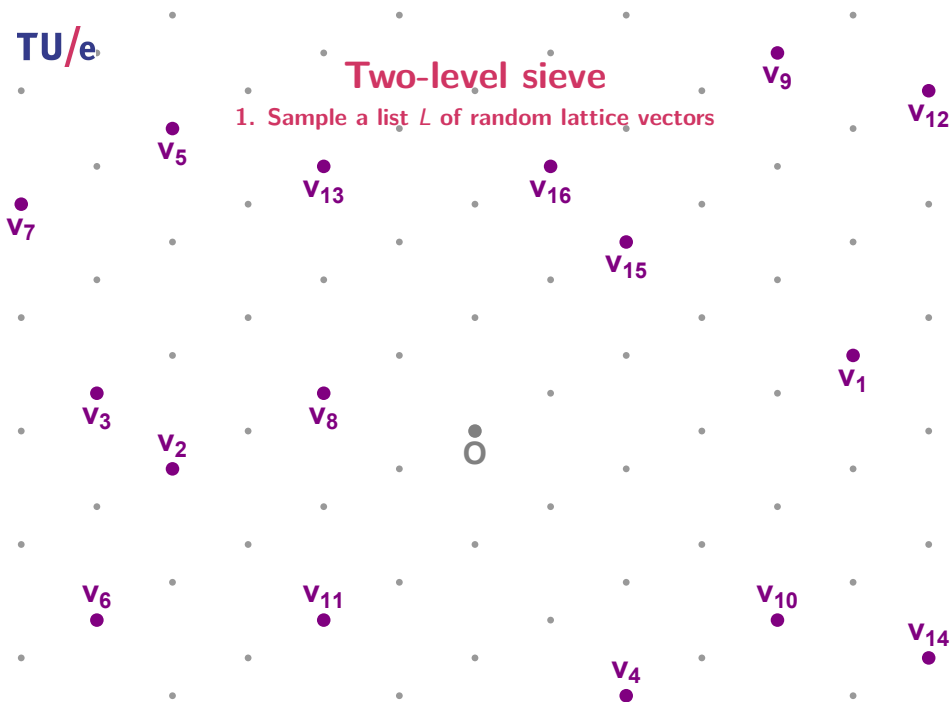
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O

Two-level sieve

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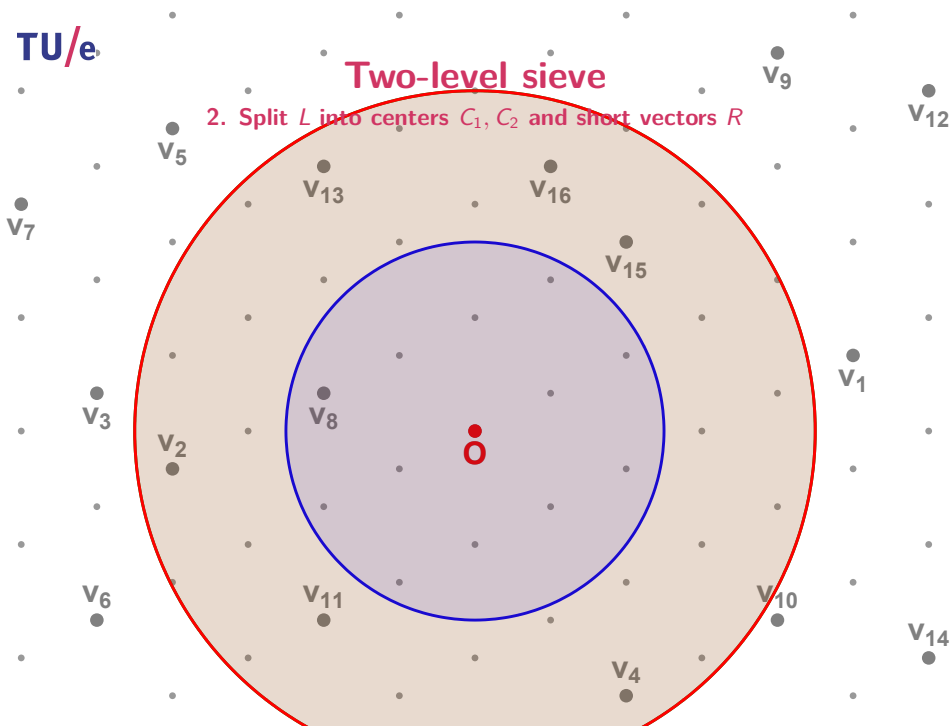
Two-level sieve

2. Split L into centers C_1, C_2 and short vectors R



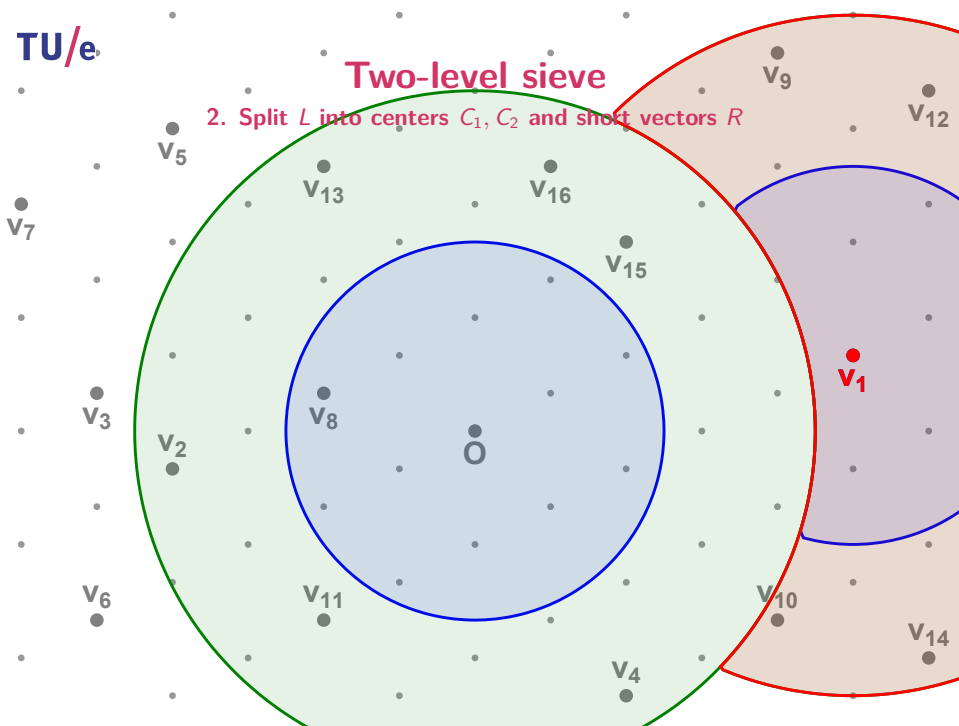
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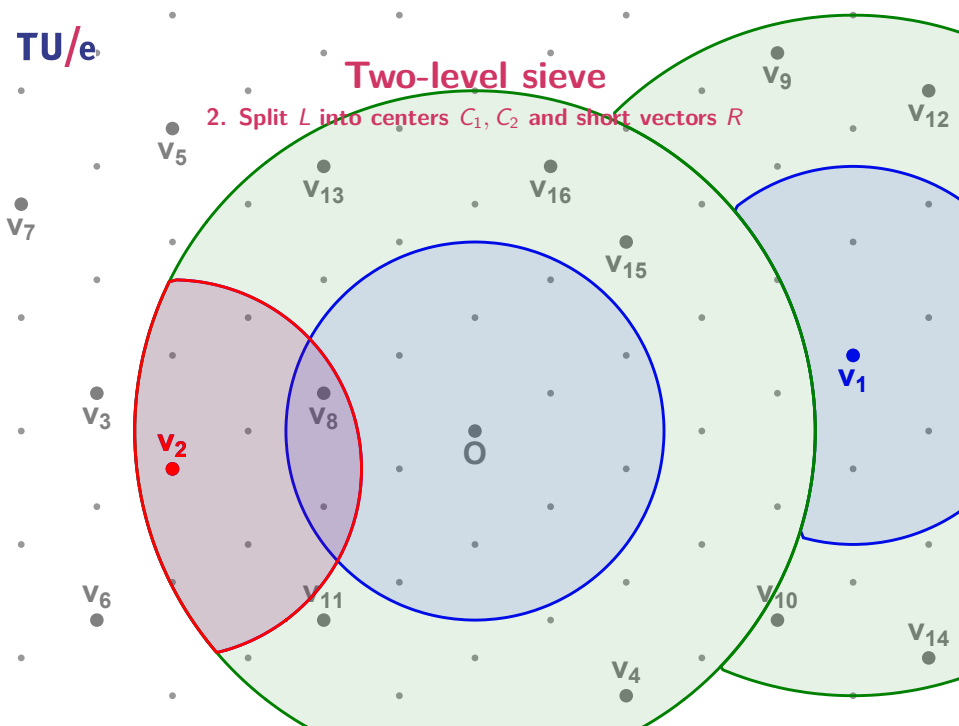
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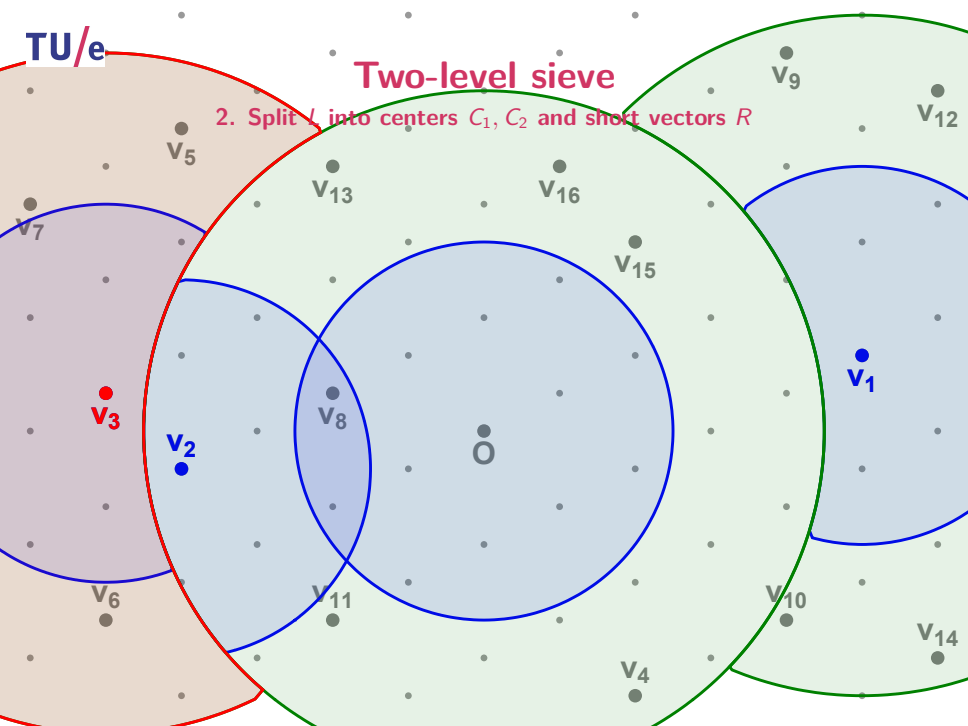
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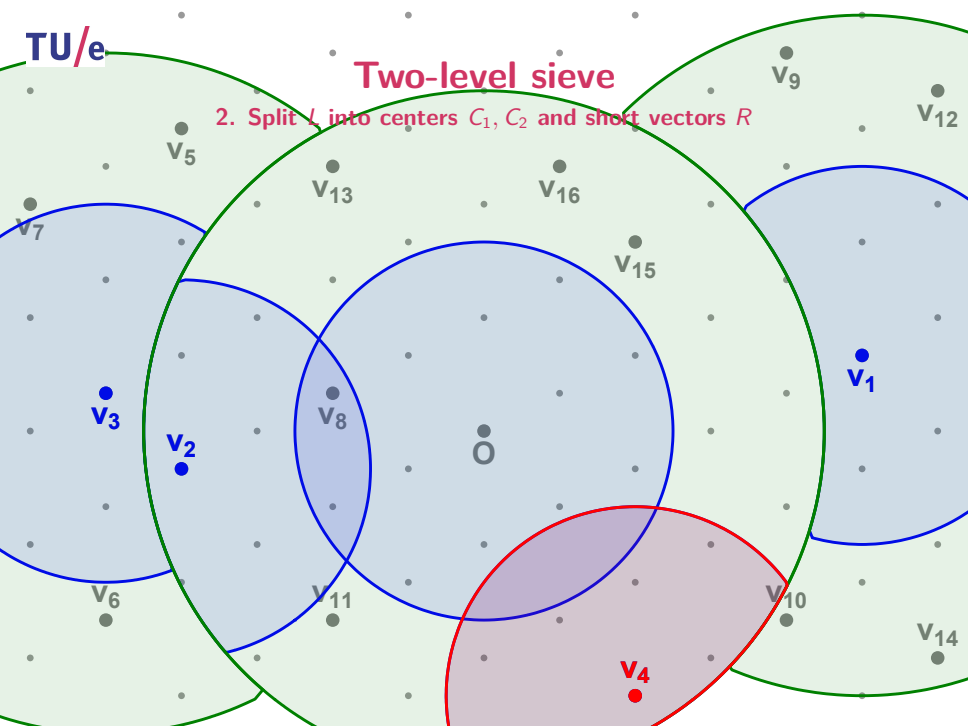
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TU/e

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The diagram shows a central point O surrounded by several overlapping circles. A red circle labeled v_5 is highlighted, and a green circle labeled v_9 is also highlighted. Other circles are labeled $v_1, v_2, v_3, v_4, v_6, v_7, v_8, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}$. The circles are colored in shades of blue and green.

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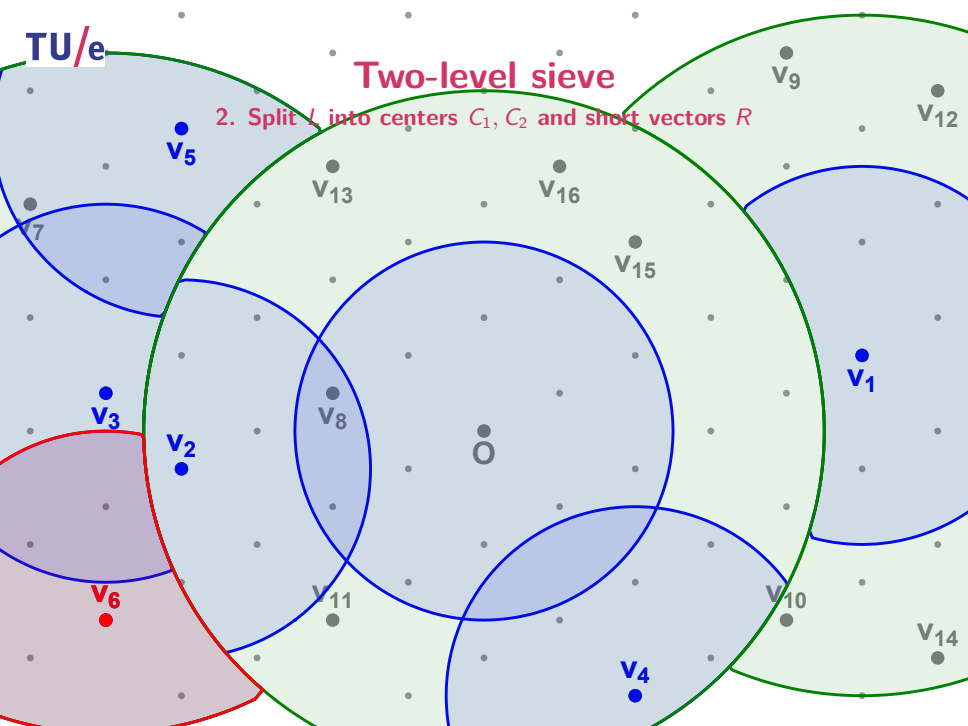
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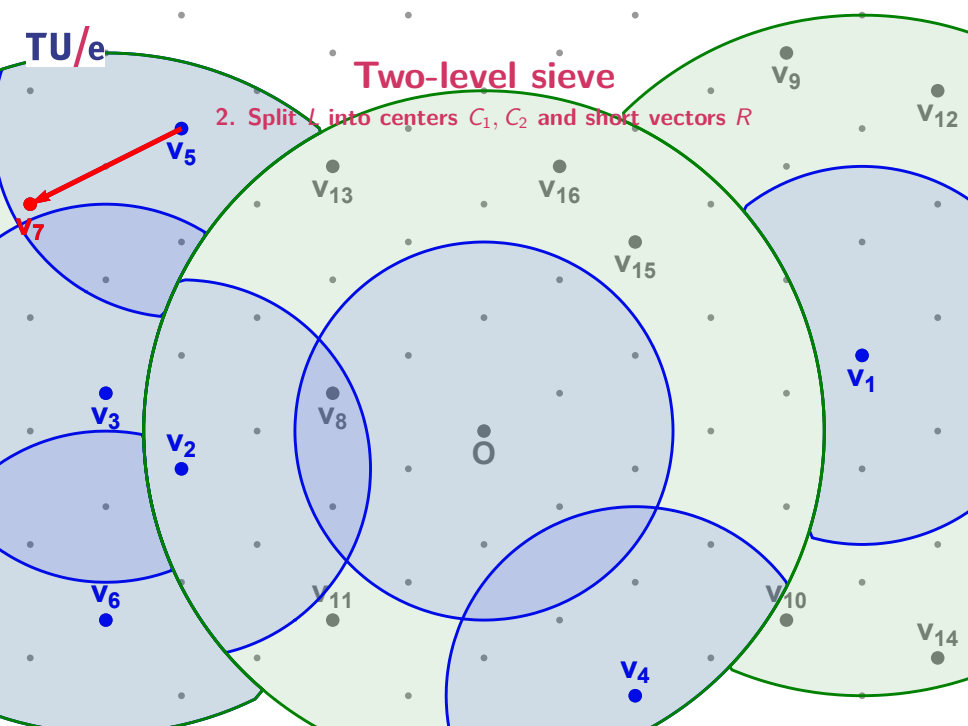
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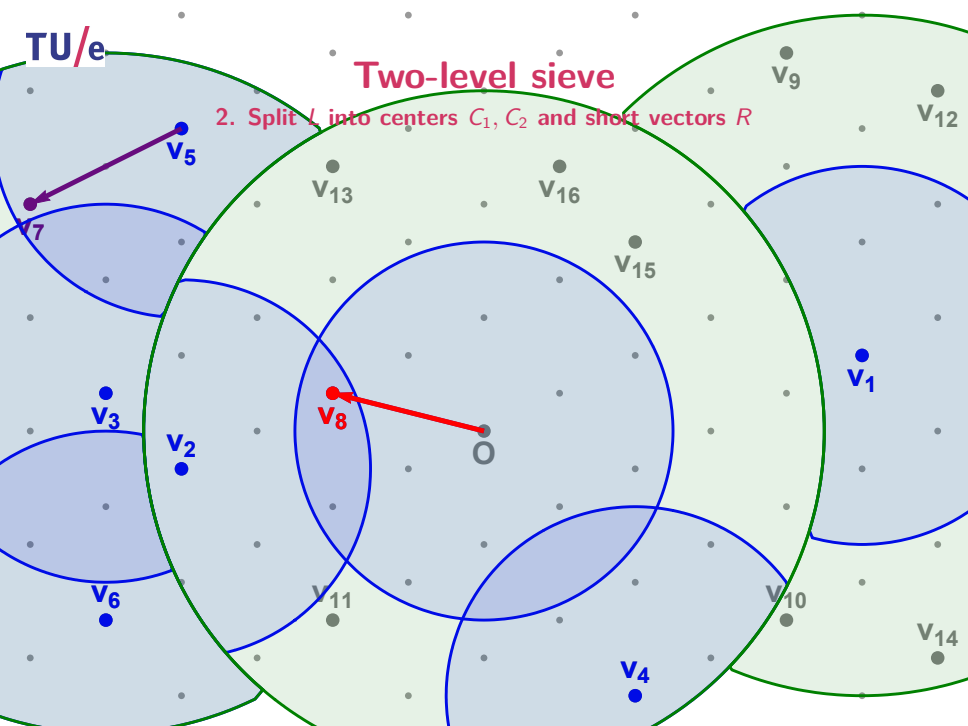
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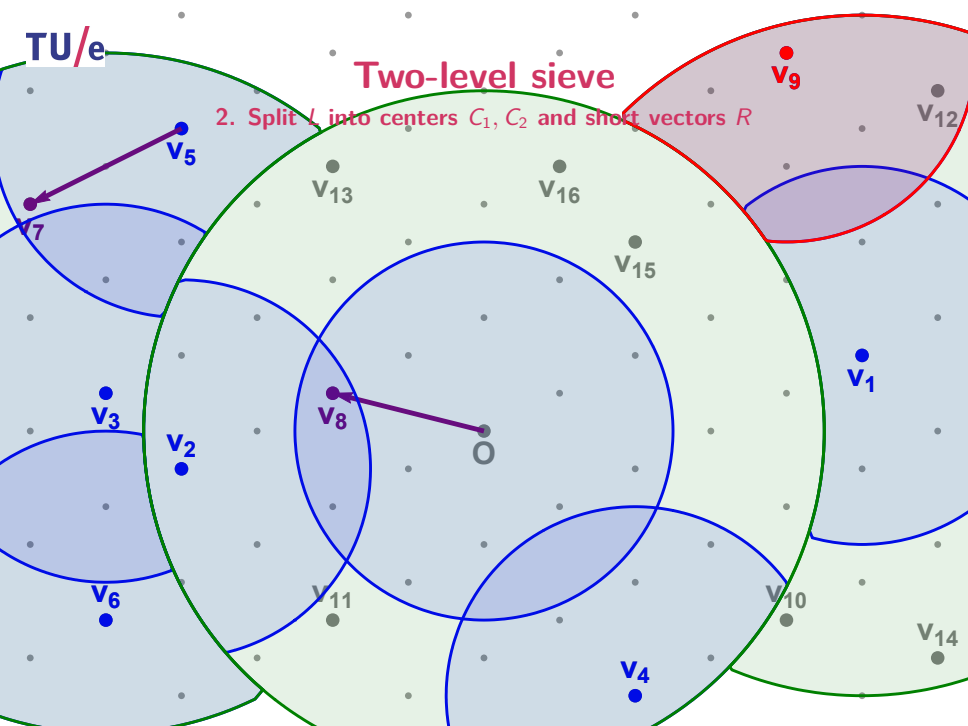
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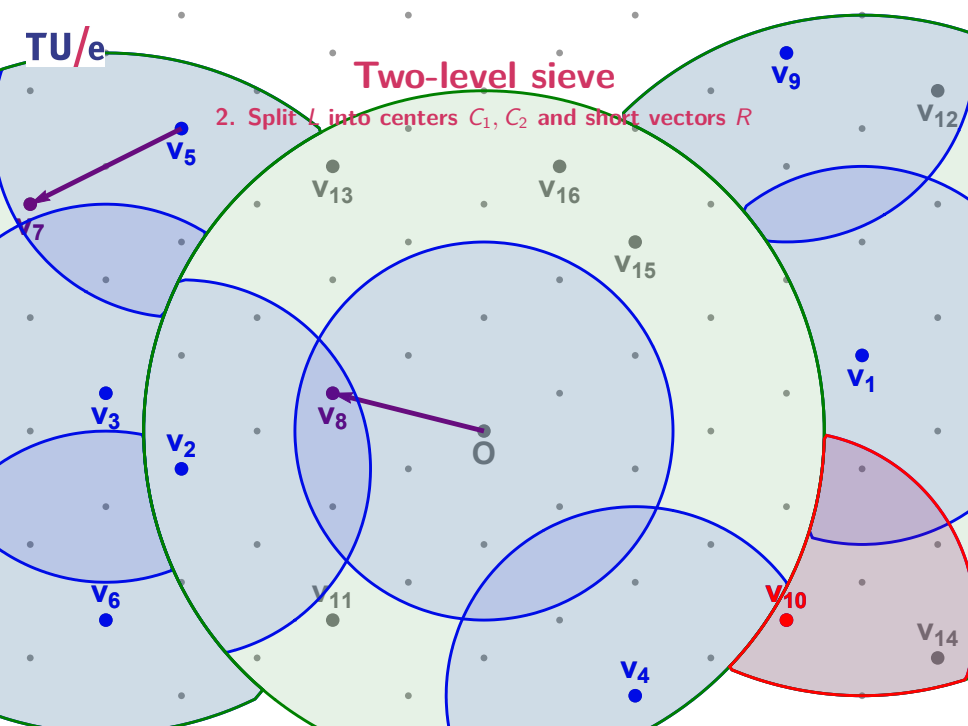
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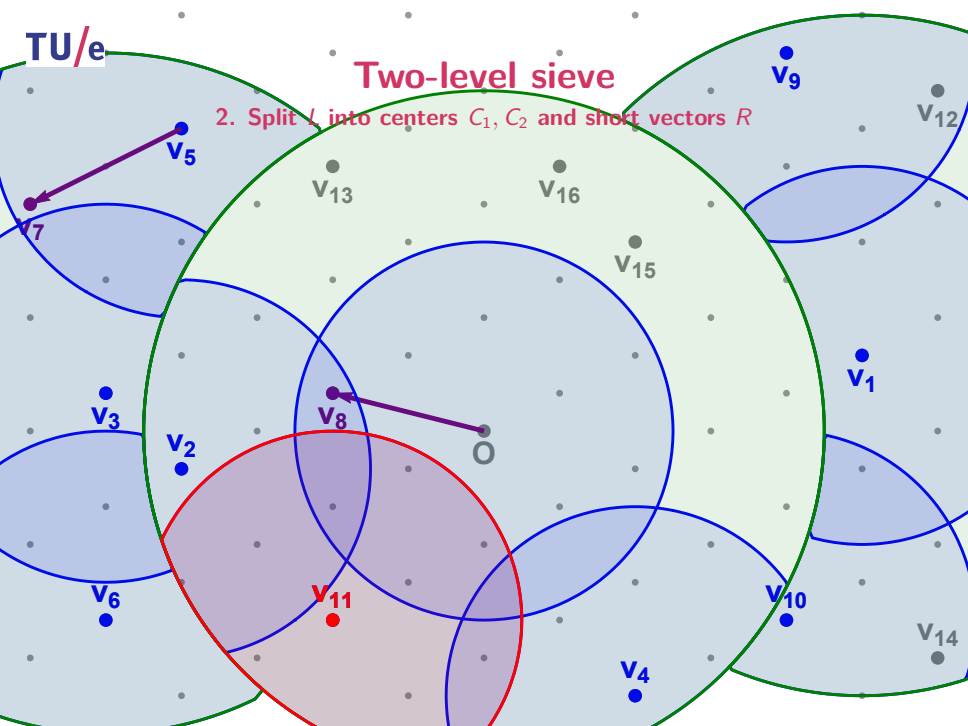
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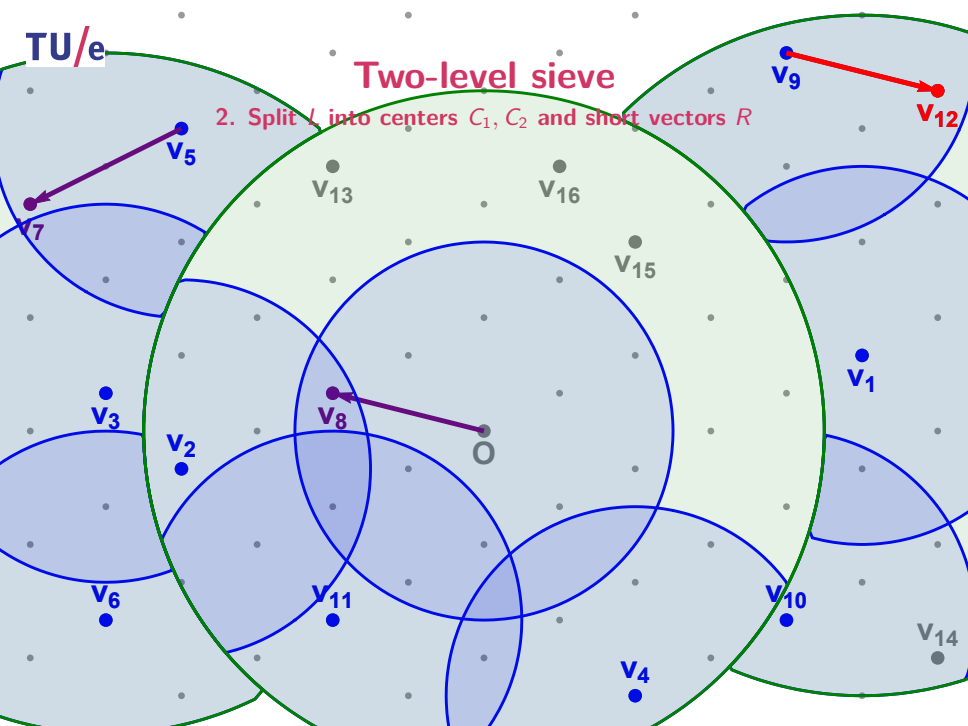
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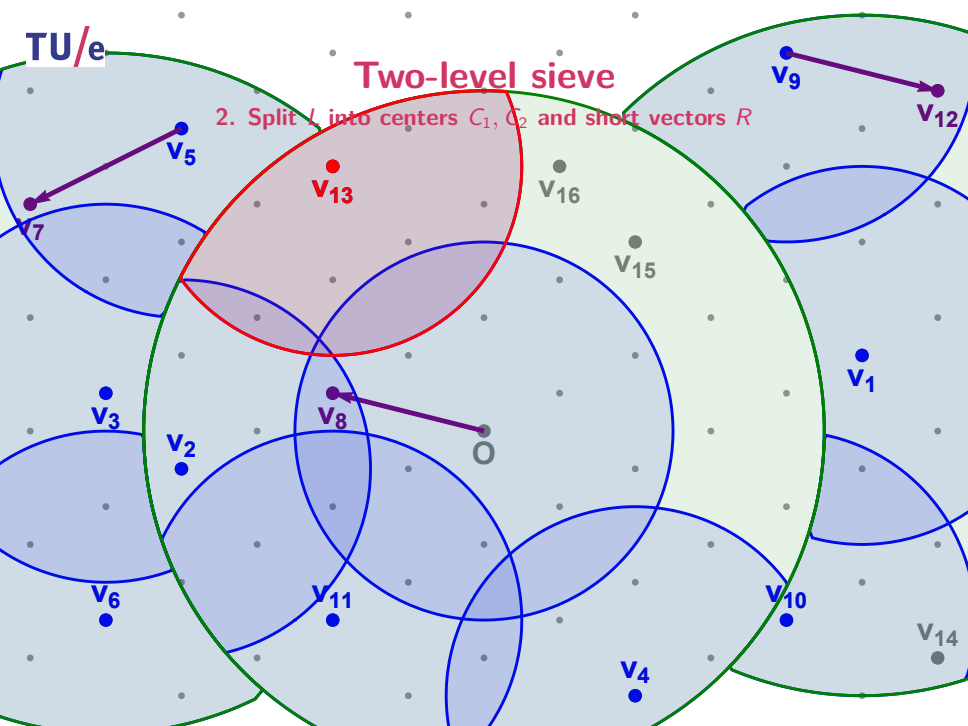
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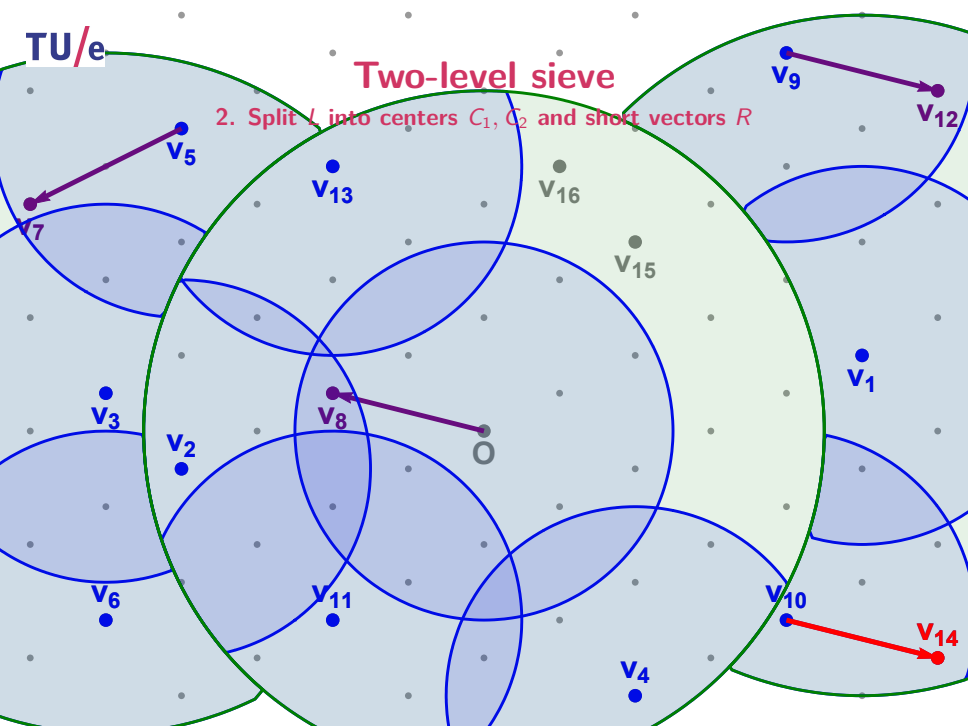
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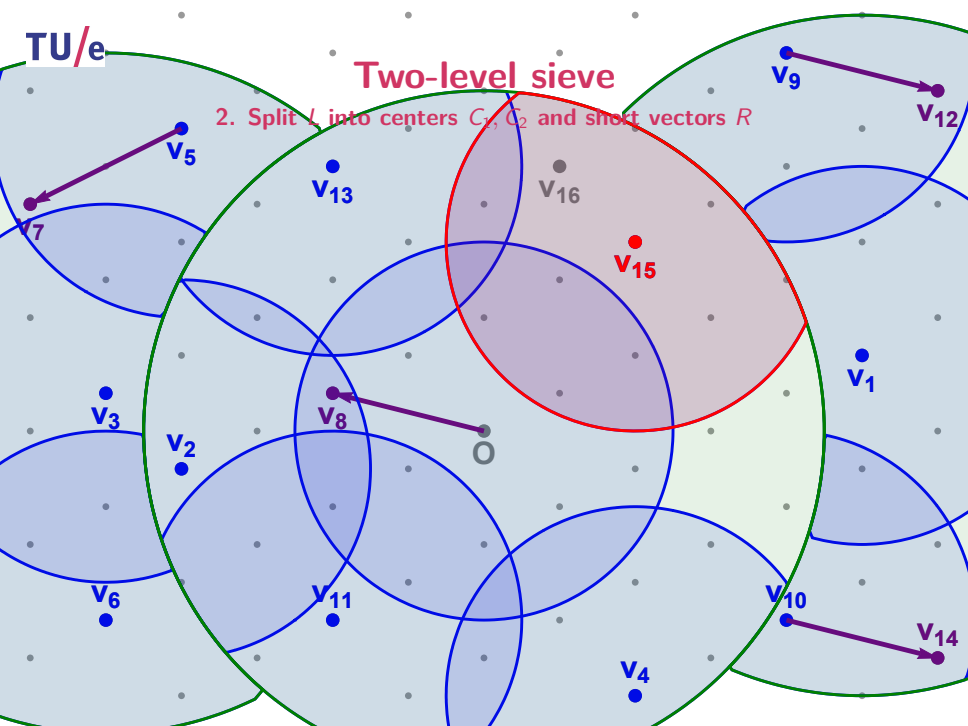
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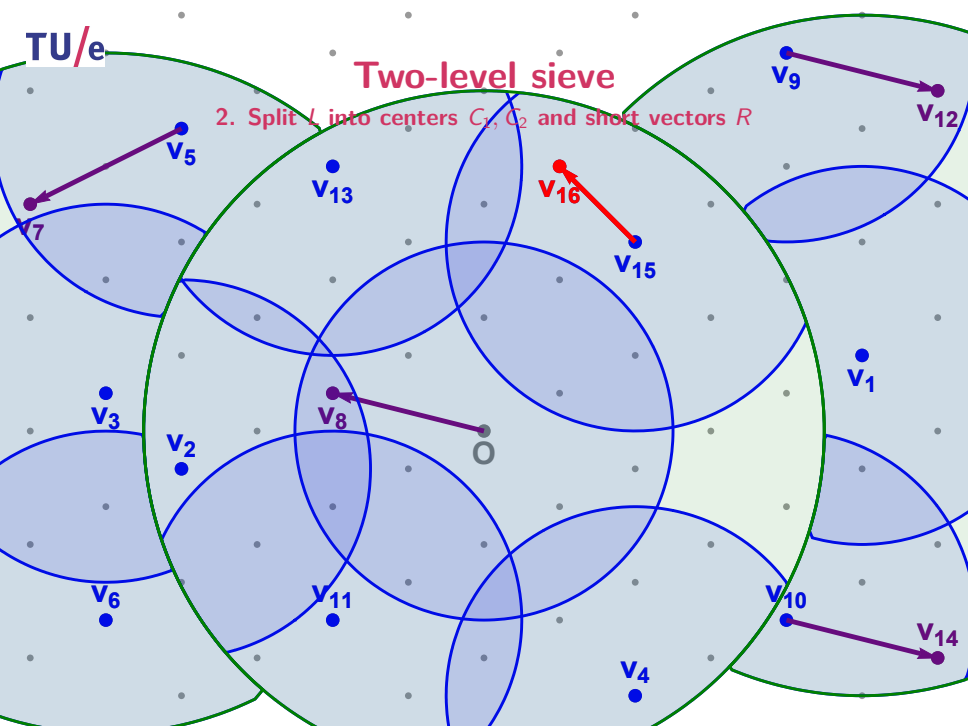
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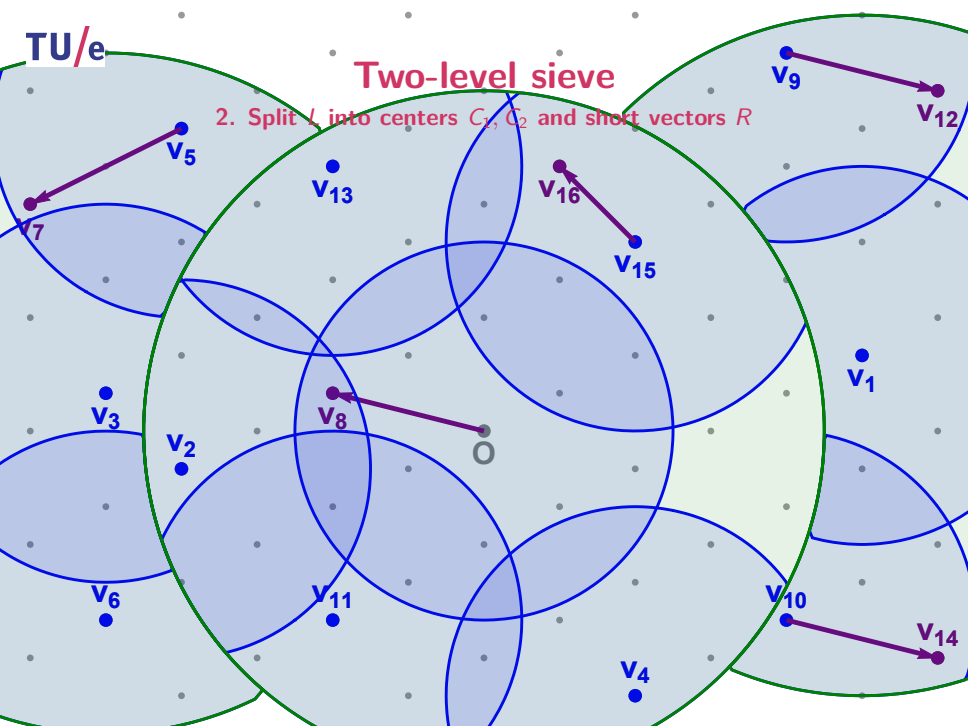
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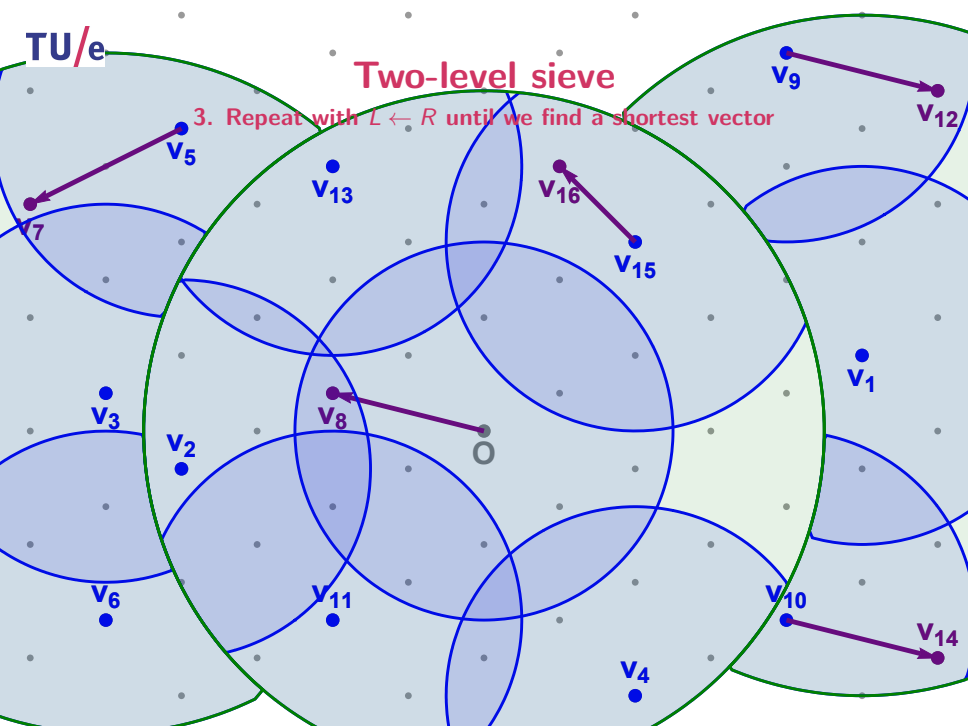
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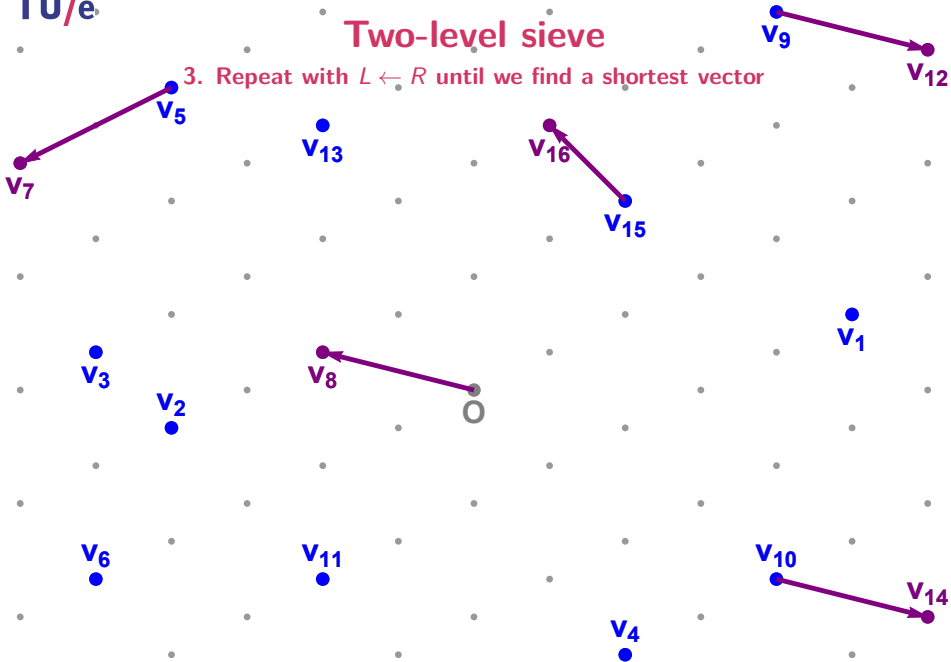
Two-level sieve

3. Repeat with $L \leftarrow R$ until we find a shortest vector



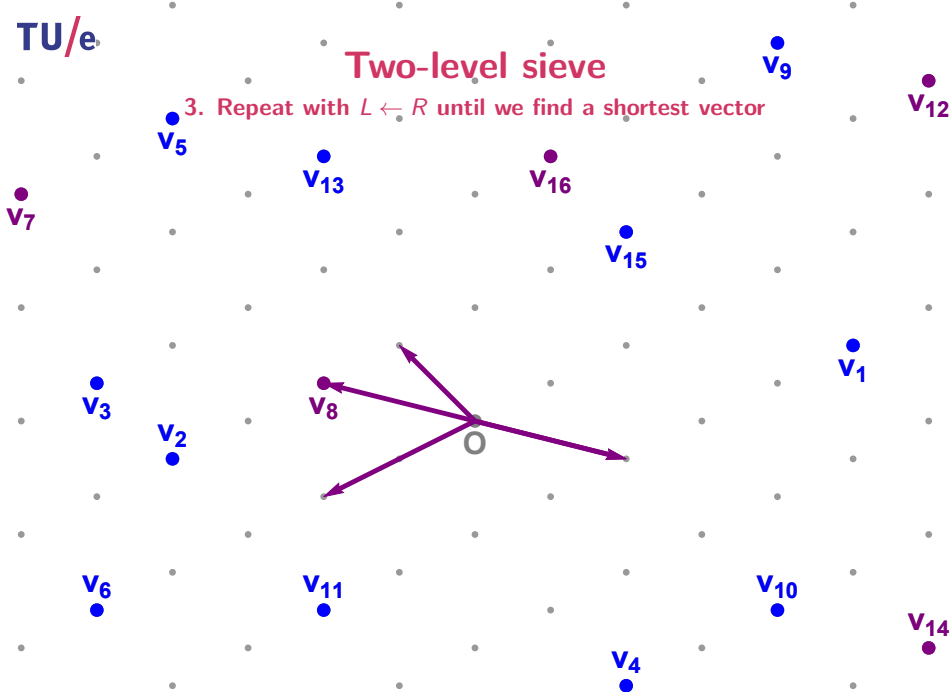
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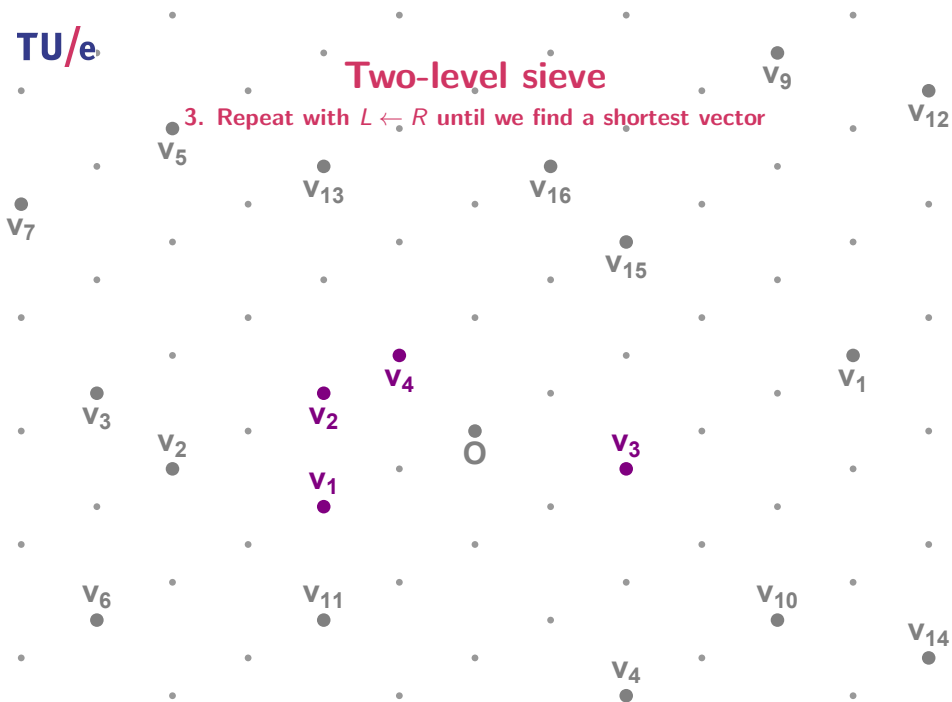
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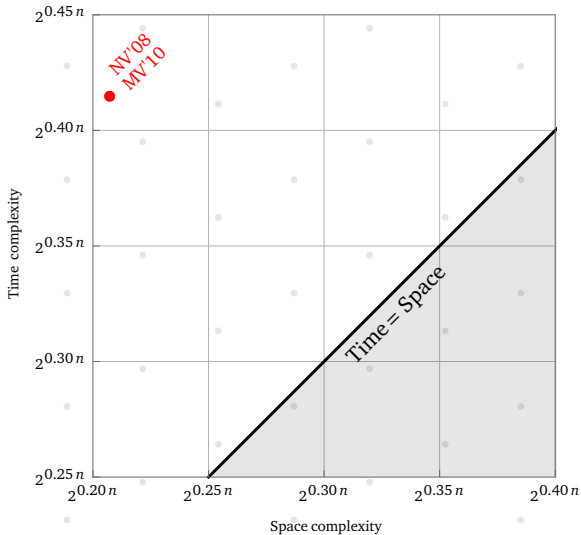
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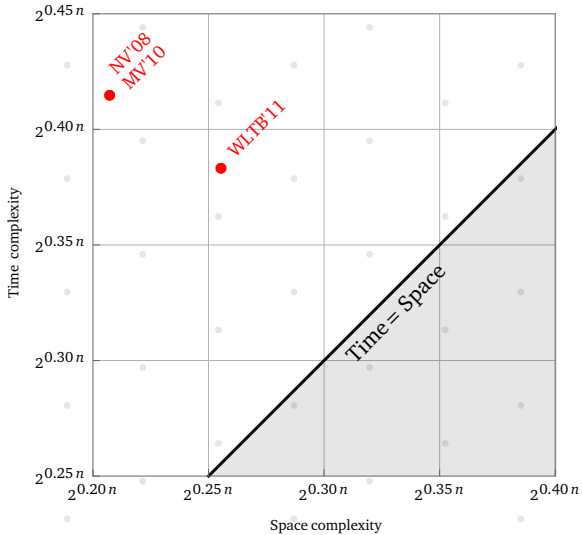
Two-level sieve

Space/time trade-off



Two-level sieve

Space/time trade-off



Three-level sieve

Overview

Heuristic (Nguyen and Vidick, J. Math. Crypt. '08)

The one-level sieve runs in time $2^{0.4150n}$ and space $2^{0.2075n}$.

Three-level sieve

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Heuristic (Nguyen and Vidick, J. Math. Crypt. '08)

The one-level sieve runs in time $2^{0.4150n}$ and space $2^{0.2075n}$.

Heuristic (Wang et al., ASIACCS'11)

The two-level sieve runs in time $2^{0.3836n}$ and space $2^{0.2557n}$.

Three-level sieve

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Heuristic (Nguyen and Vidick, J. Math. Crypt. '08)

The one-level sieve runs in time $2^{0.4150n}$ and space $2^{0.2075n}$.

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The two-level sieve runs in time $2^{0.3836n}$ and space $2^{0.2557n}$.

Heuristic (Zhang et al., SAC'13)

The three-level sieve runs in time $2^{0.3778n}$ and space $2^{0.2833n}$.

Three-level sieve

Overview

Heuristic (Nguyen and Vidick, J. Math. Crypt. '08)

The one-level sieve runs in time $2^{0.4150n}$ and space $2^{0.2075n}$.

Heuristic (Wang et al., ASIACCS'11)

The two-level sieve runs in time $2^{0.3836n}$ and space $2^{0.2557n}$.

Heuristic (Zhang et al., SAC'13)

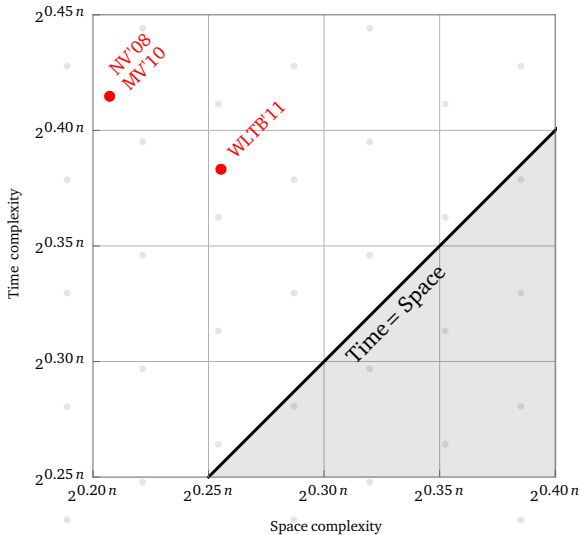
The three-level sieve runs in time $2^{0.3778n}$ and space $2^{0.2833n}$.

Conjecture

The four-level sieve runs in time $2^{0.3774n}$ and space $2^{0.2925n}$, and higher-level sieves are not faster than this.

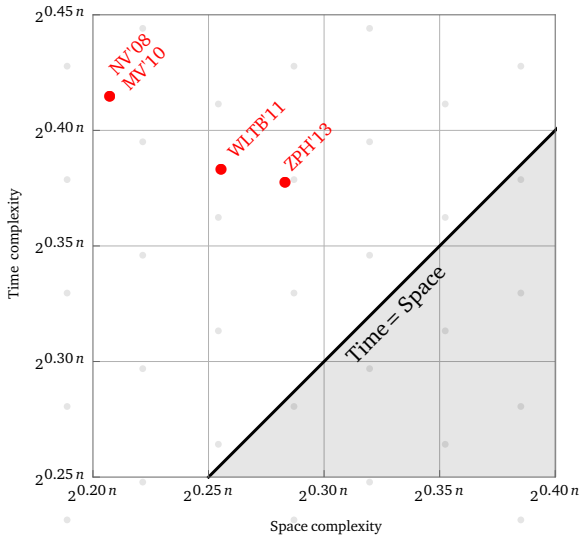
Three-level sieve

Space/time trade-off



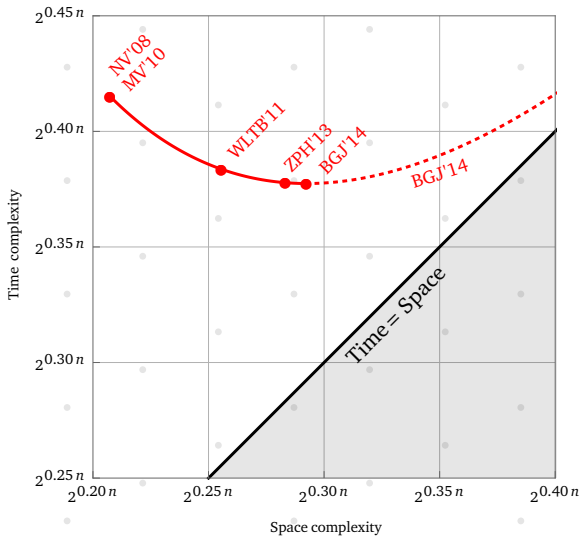
Three-level sieve

Space/time trade-off



Decomposition approach

Space/time trade-off



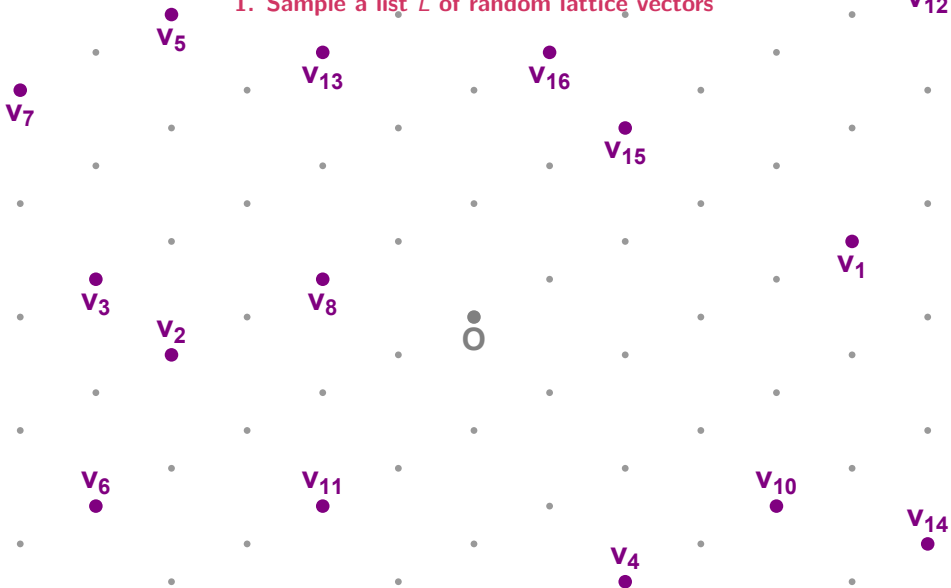
Hyperplane LSH

1. Sample a list L of random lattice vectors



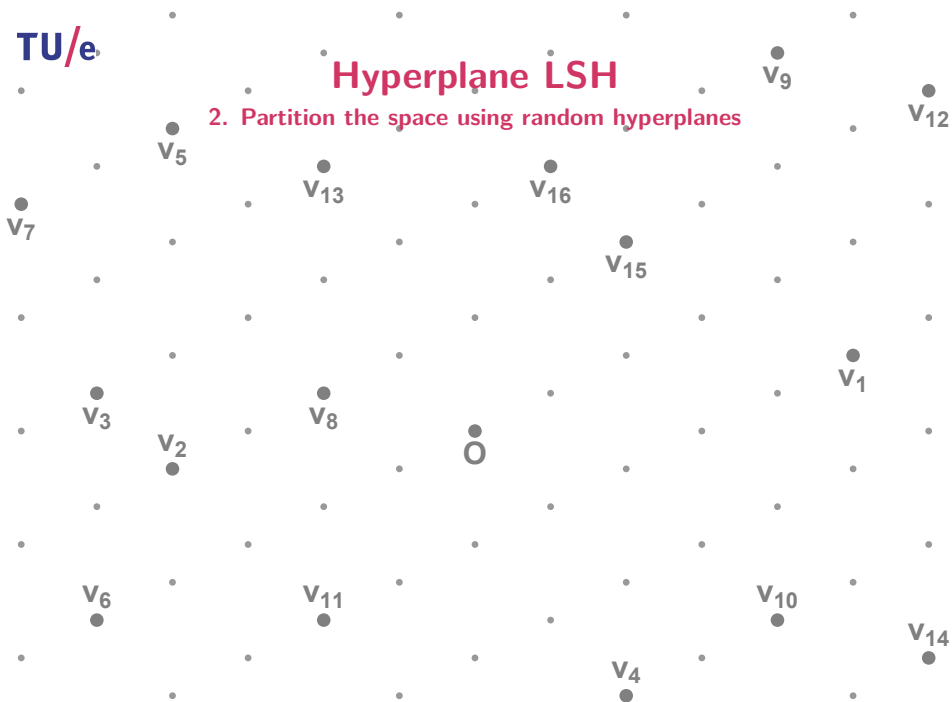
Hyperplane LSH

1. Sample a list L of random lattice vectors



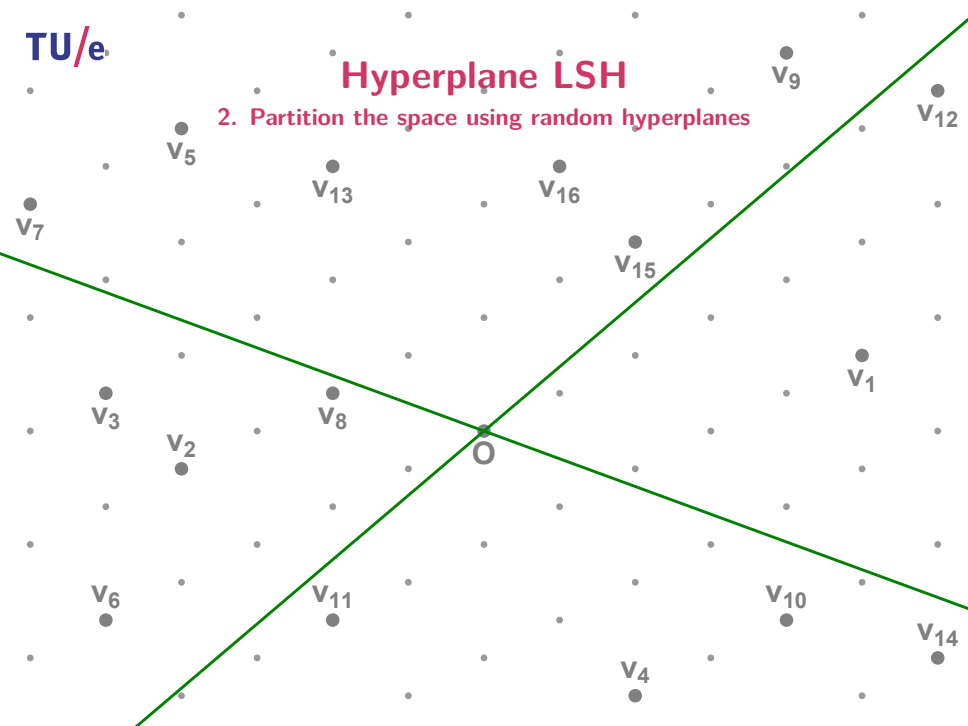
Hyperplane LSH

2. Partition the space using random hyperplanes



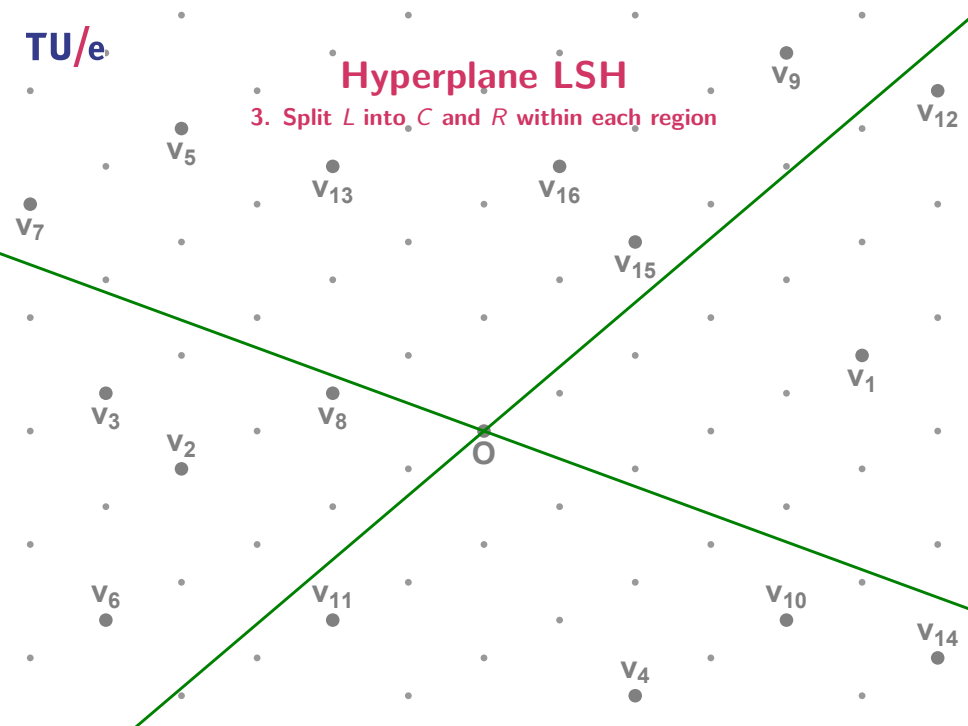
Hyperplane LSH

2. Partition the space using random hyperplanes



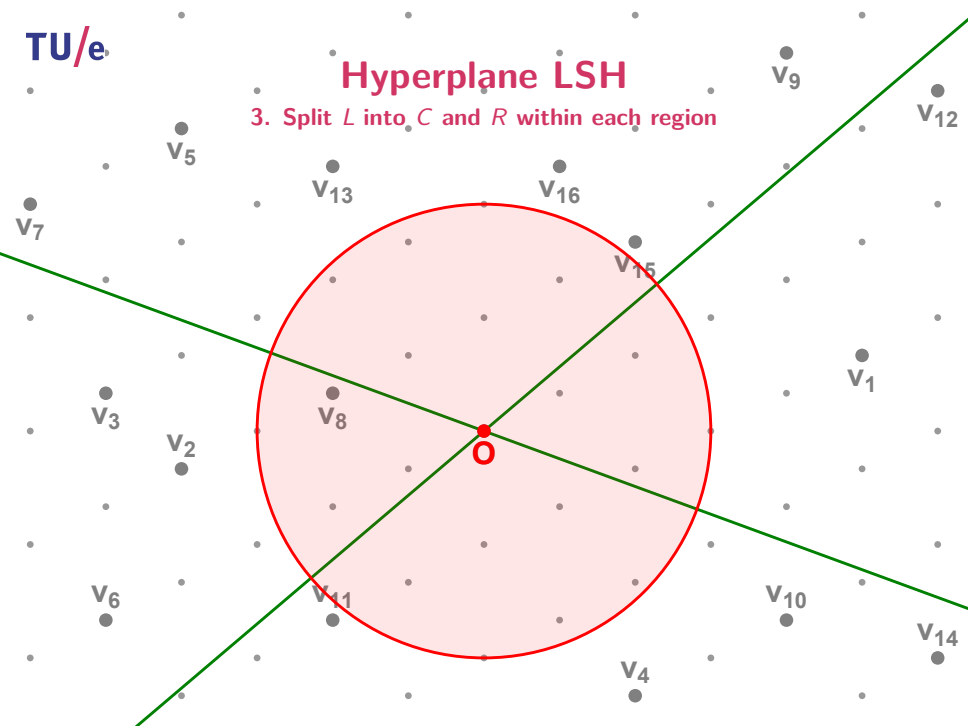
Hyperplane LSH

3. Split L into C and R within each region



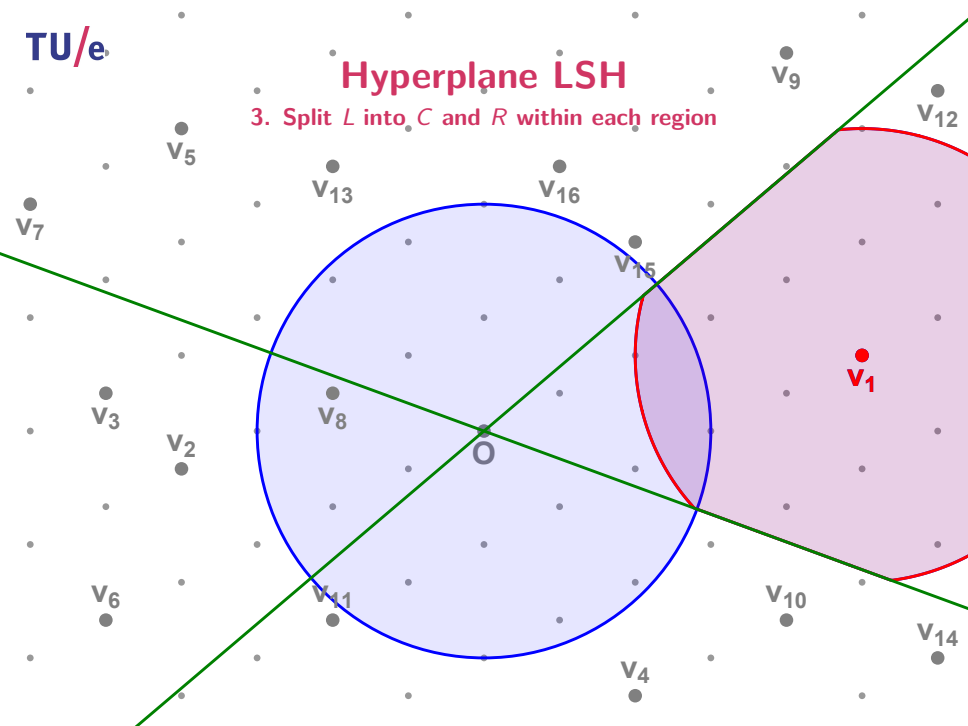
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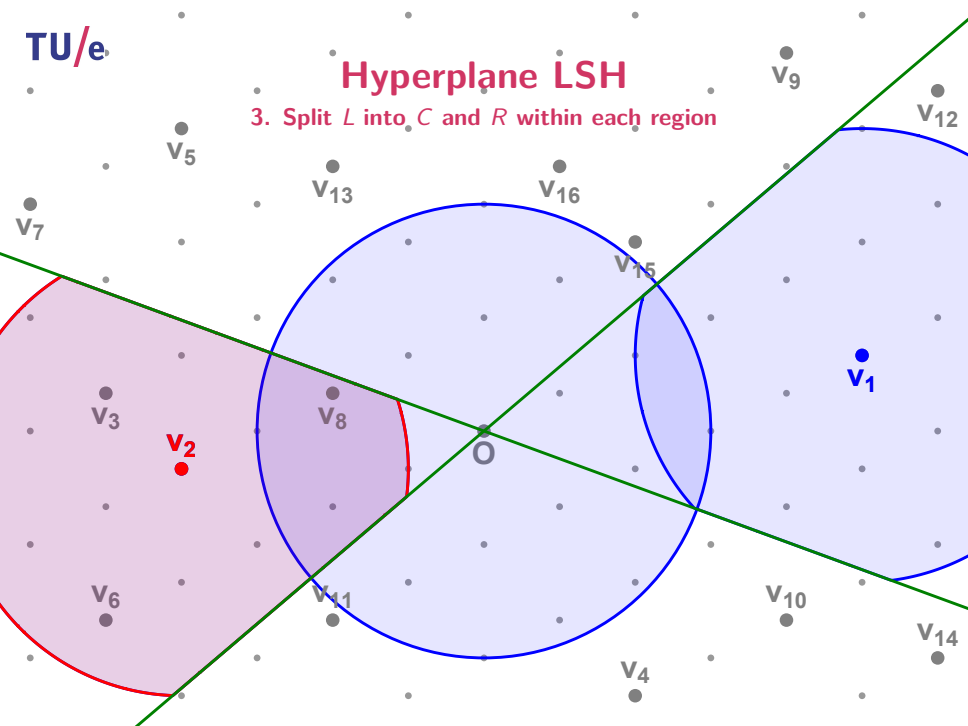
Hyperplane LSH

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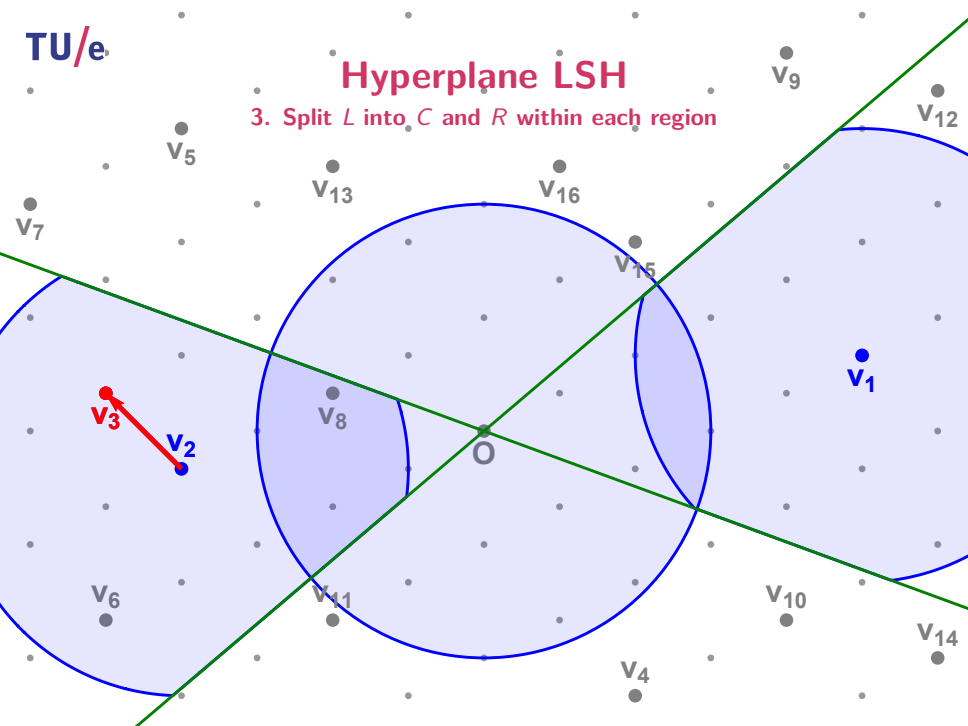
Hyperplane LSH

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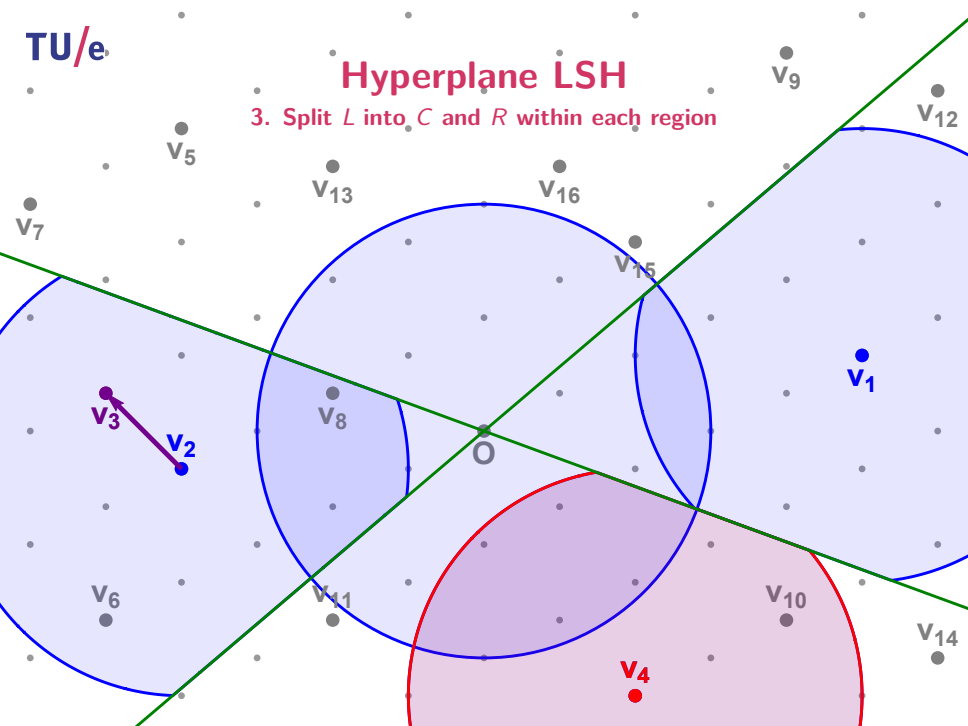
Hyperplane LSH

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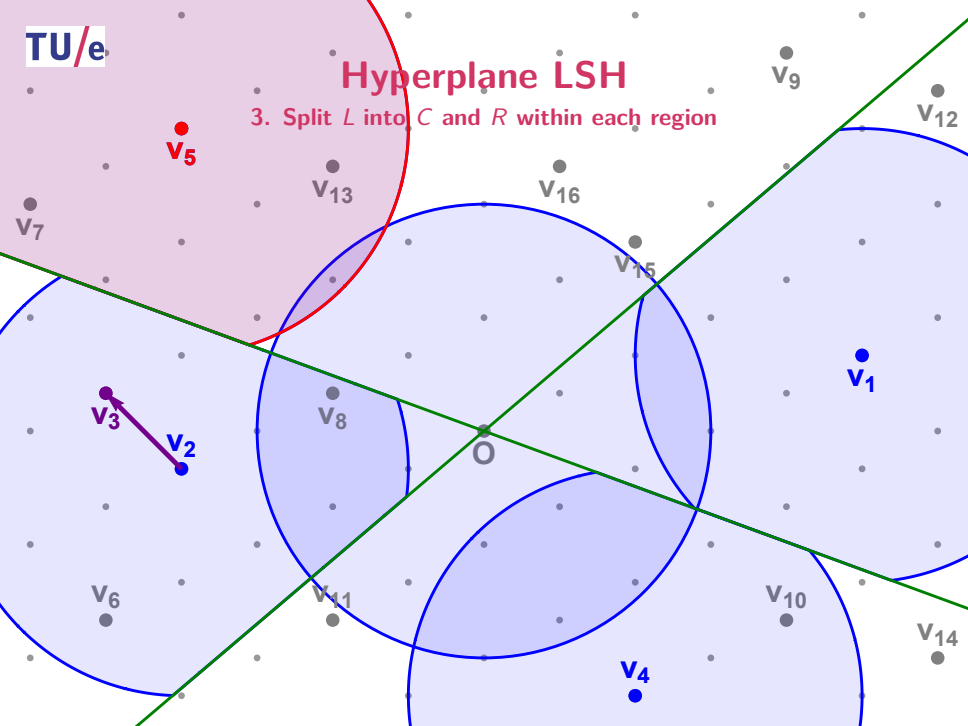
Hyperplane LSH

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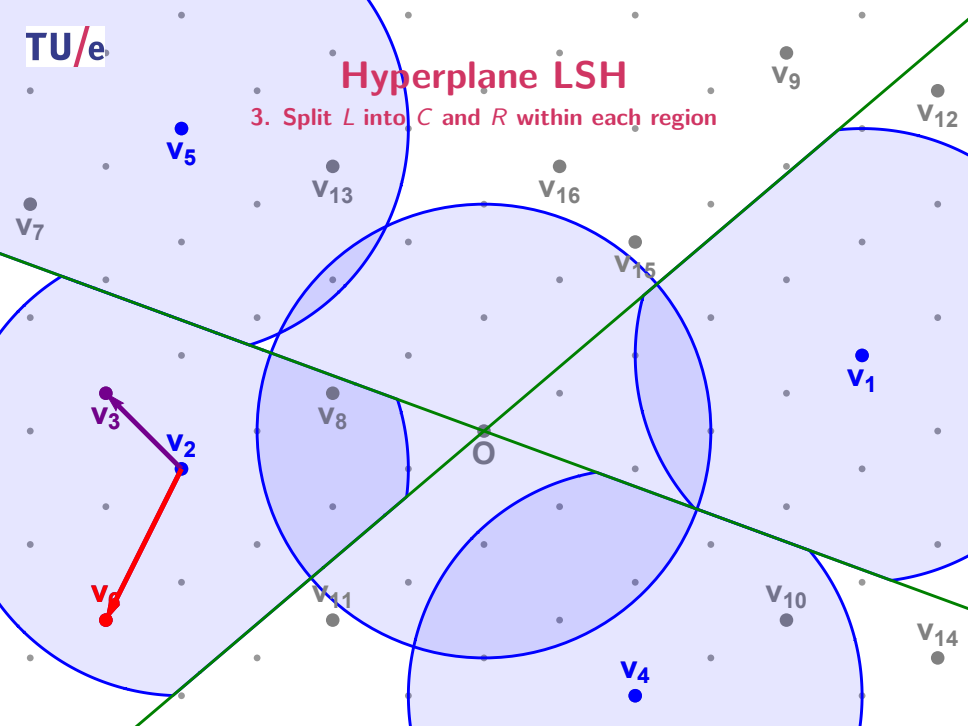
Hyperplane LSH

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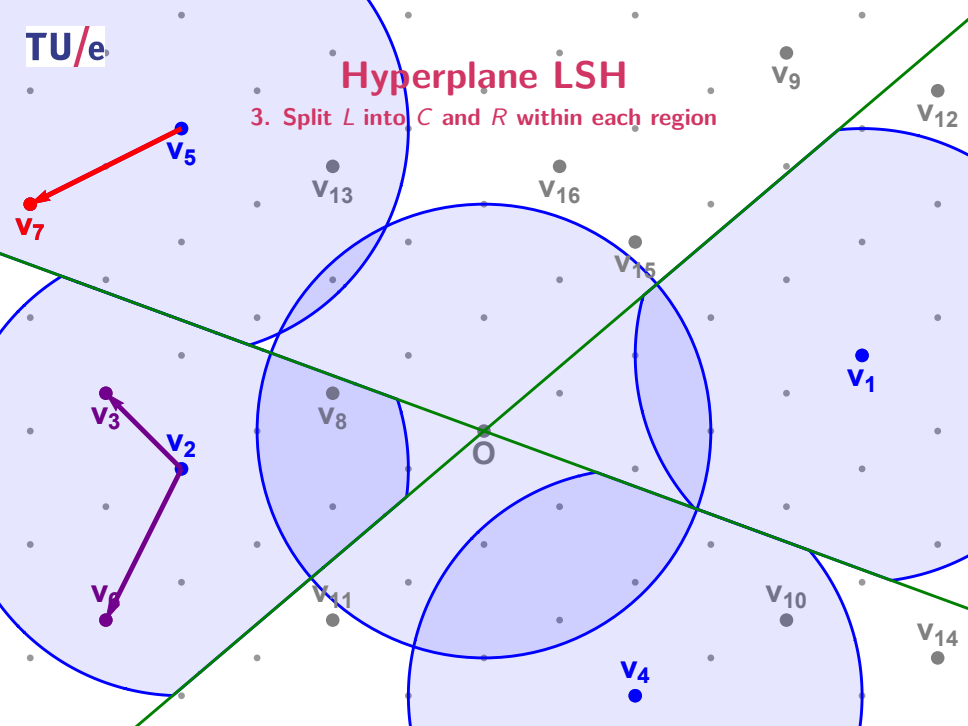
Hyperplane LSH

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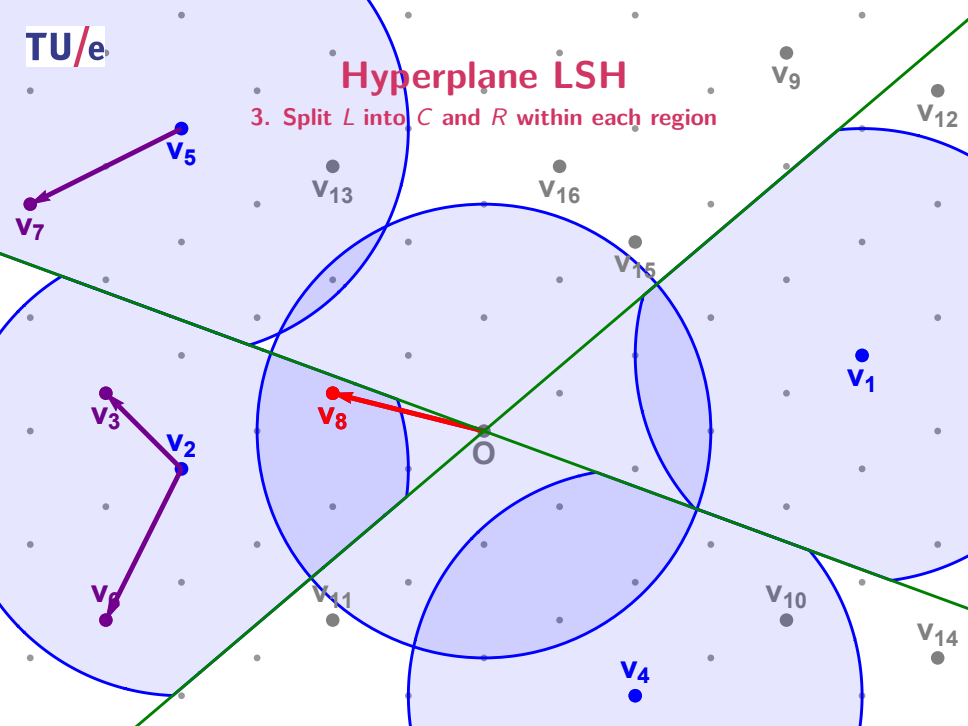
Hyperplane LSH

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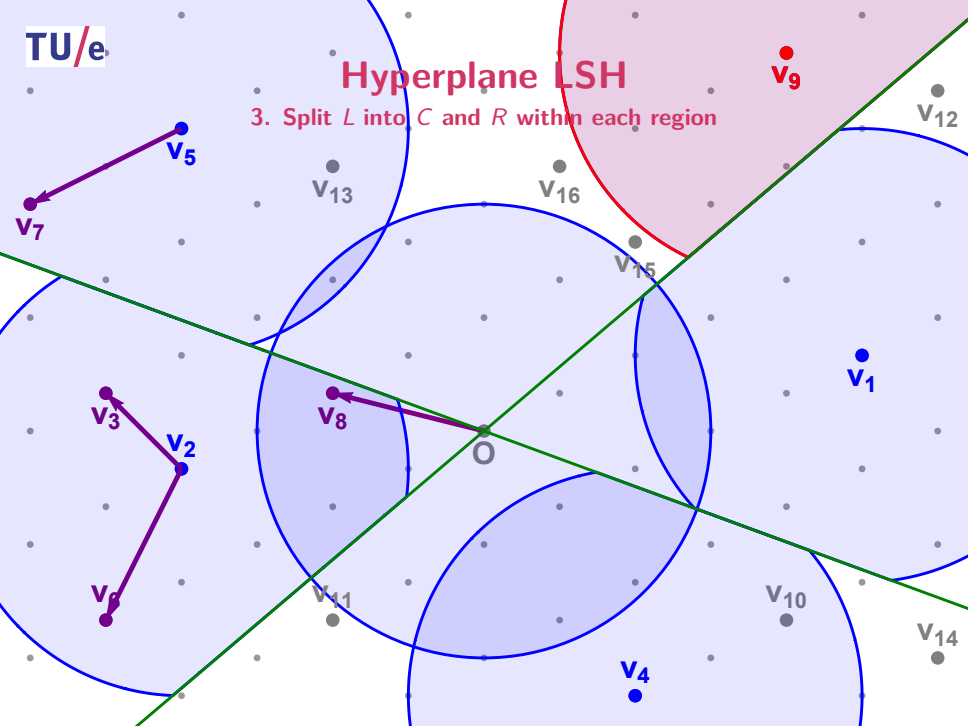
Hyperplane LSH

3. Split L into C and R within each region



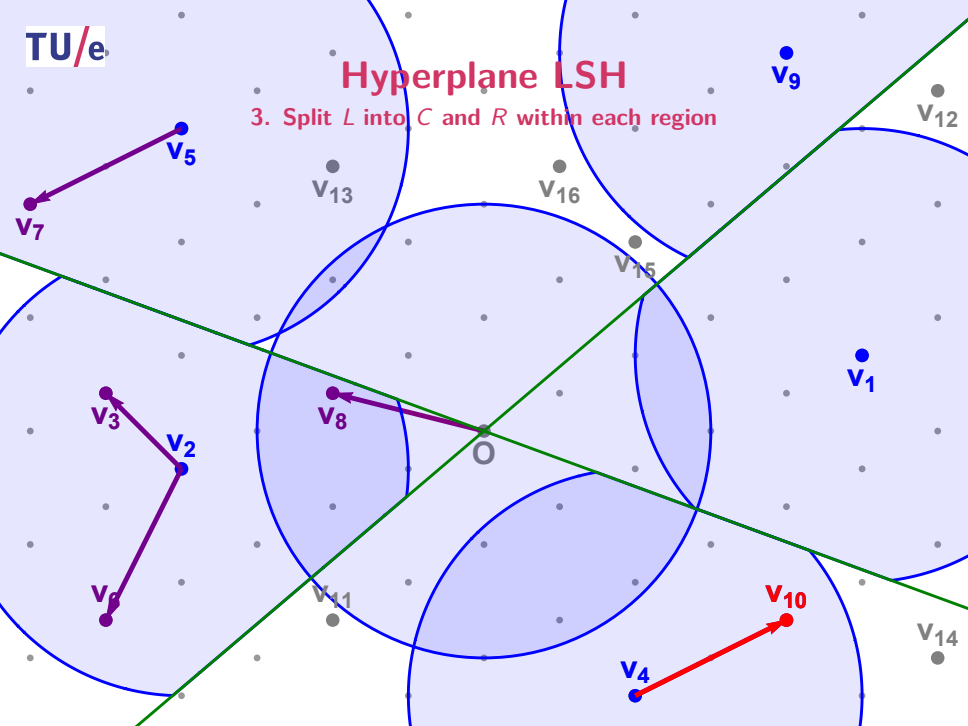
Hyperplane LSH

3. Split L into C and R within each region



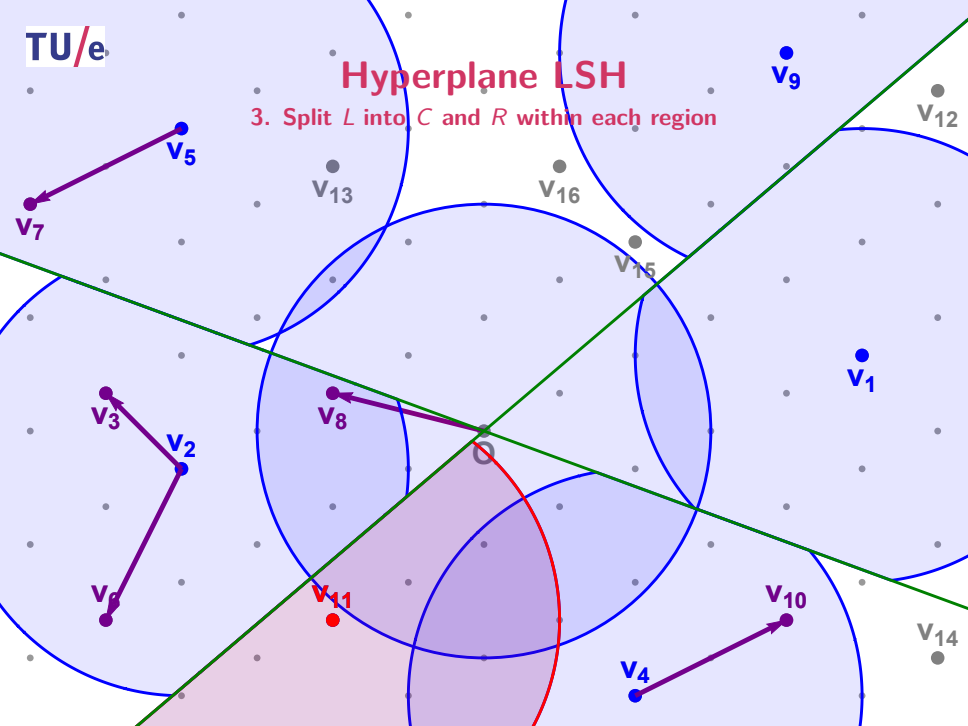
Hyperplane LSH

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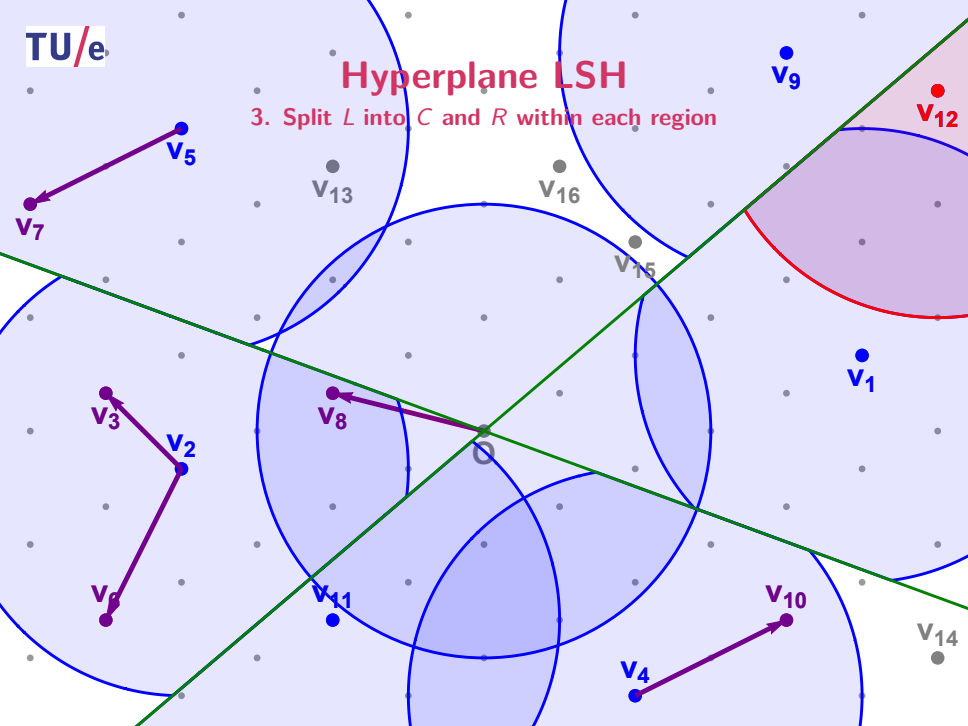
Hyperplane LSH

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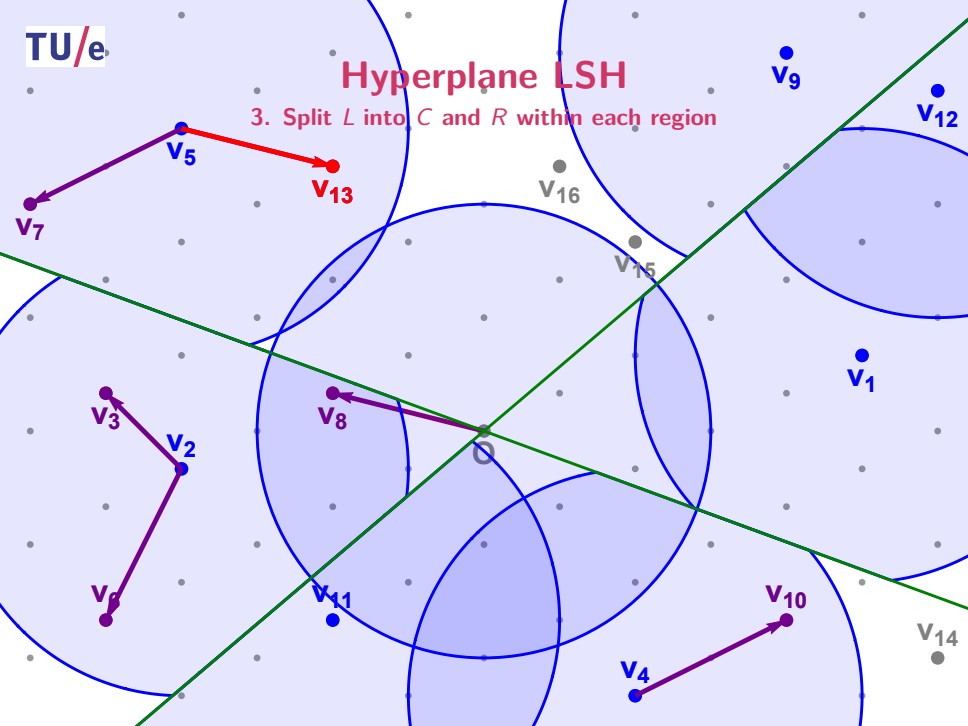
Hyperplane LSH

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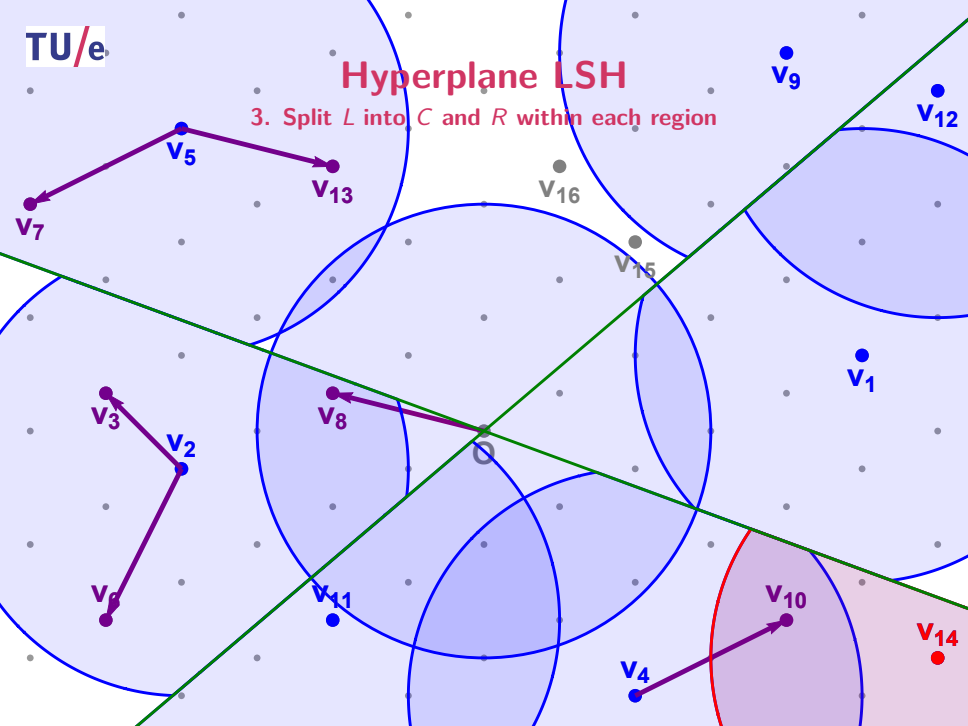
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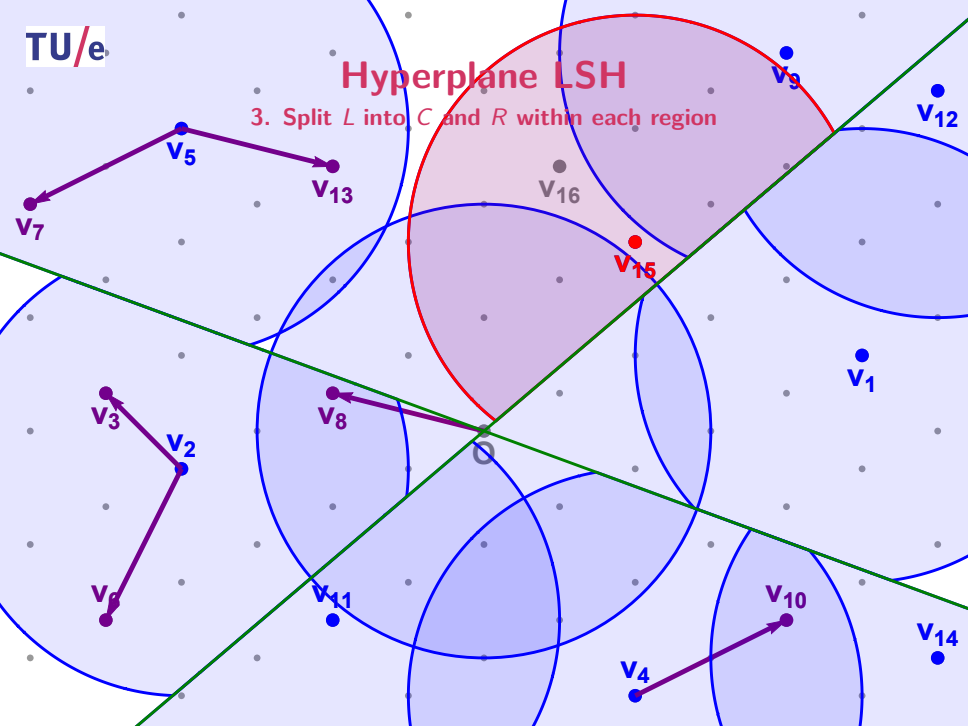
Hyperplane LSH

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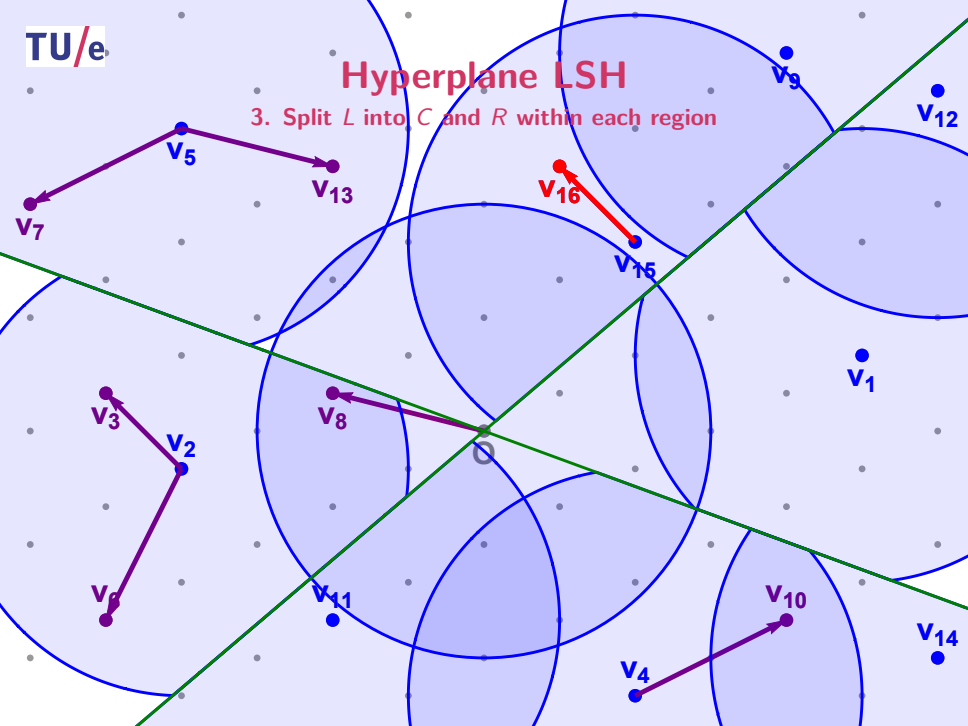
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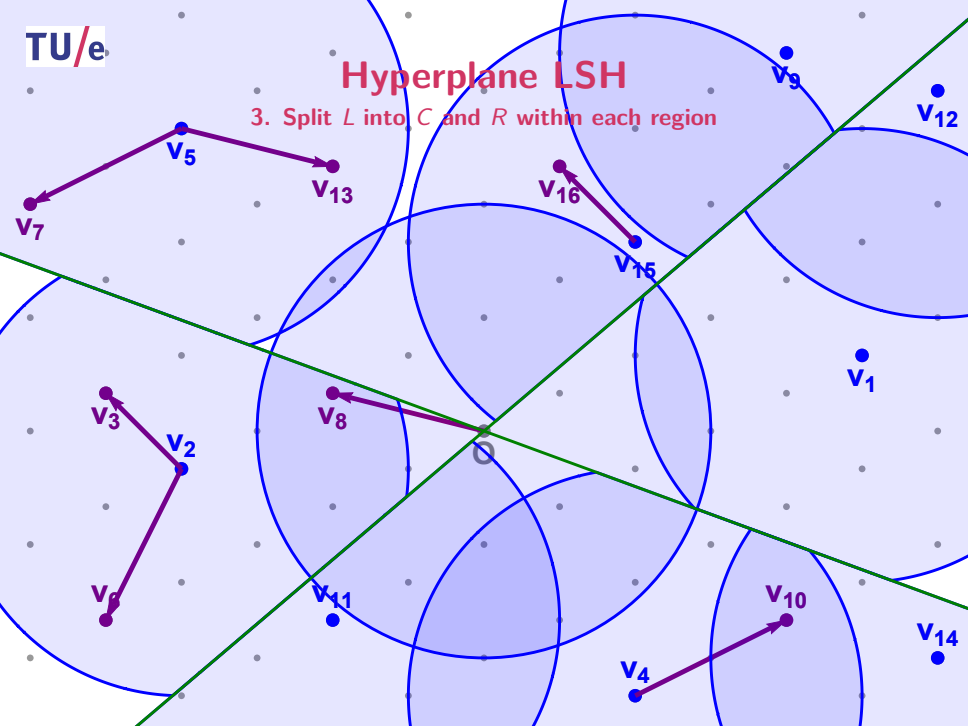
Hyperplane LSH

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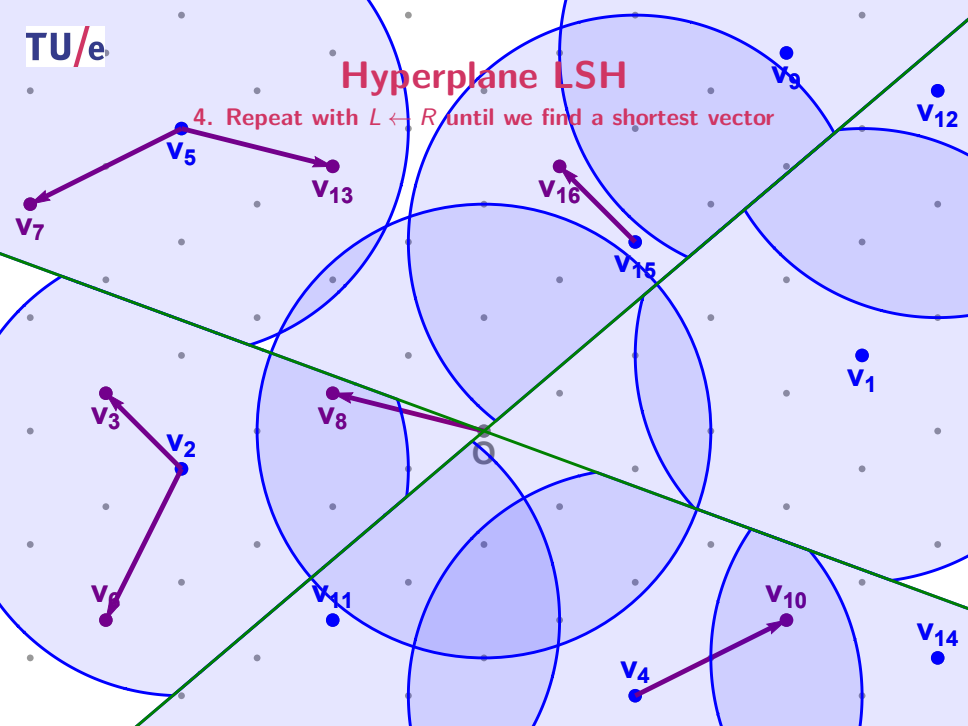
Hyperplane LSH

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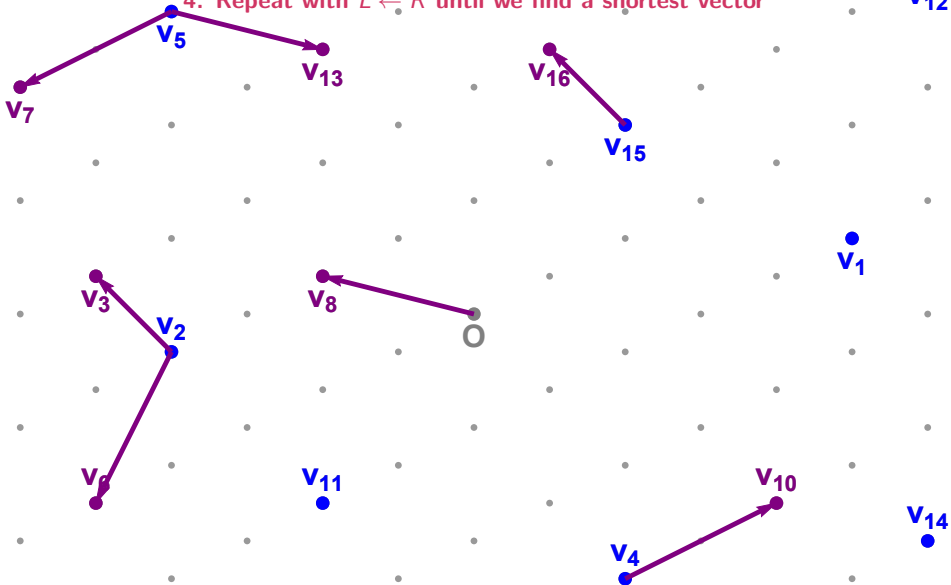
Hyperplane LSH

4. Repeat with $L \leftarrow R$ until we find a shortest vector



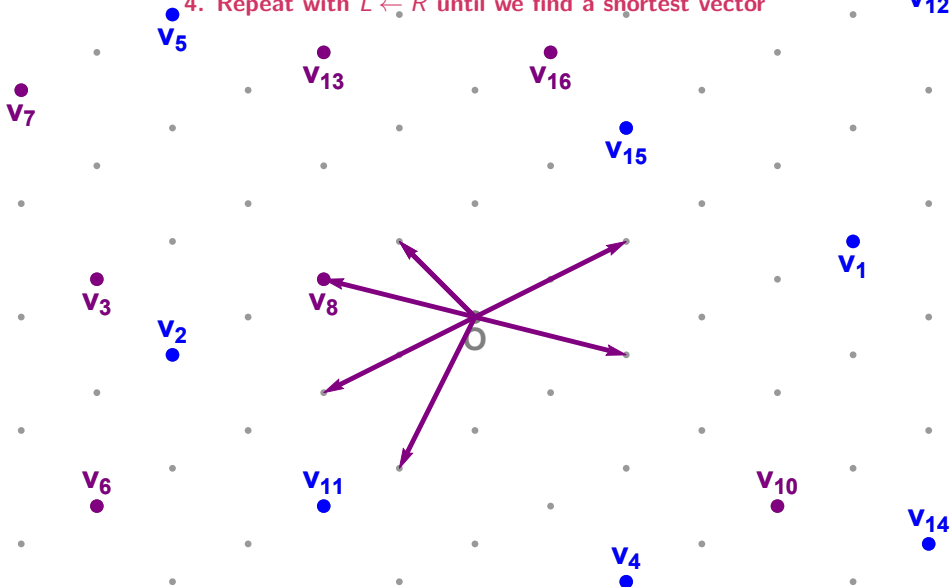
Hyperplane LSH

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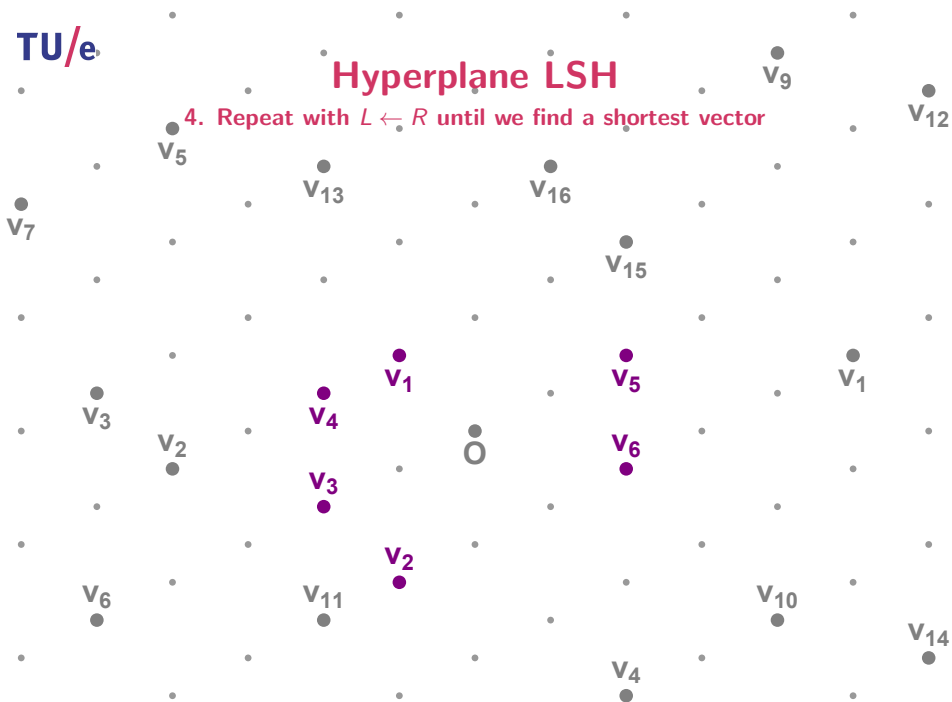
Hyperplane LSH

4. Repeat with $L \leftarrow R$ until we find a shortest vector



Hyperplane LSH

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Hyperplane LSH

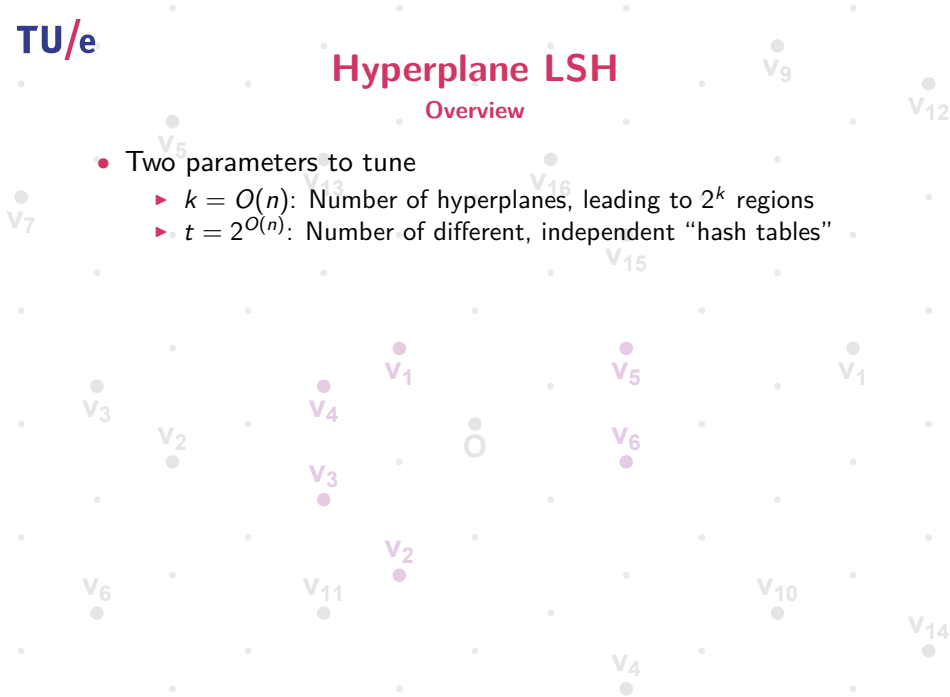
Overview



Hyperplane LSH

Overview

- Two parameters to tune
 - ▶ $k = O(n)$: Number of hyperplanes, leading to 2^k regions
 - ▶ $t = 2^{O(n)}$: Number of different, independent “hash tables”



Hyperplane LSH

Overview

- Two parameters to tune
 - ▶ $k = O(n)$: Number of hyperplanes, leading to 2^k regions
 - ▶ $t = 2^{O(n)}$: Number of different, independent “hash tables”
- Space complexity: $2^{0.337n+o(n)}$
 - ▶ Number of vectors: $2^{0.208n+o(n)}$
 - ▶ Number of hash tables: $2^{0.129n+o(n)}$
 - ▶ Each hash table contains all vectors

Hyperplane LSH

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 - ▶ Cost of computing hashes: $2^{0.129n+o(n)}$
 - ▶ Candidate nearest vectors: $2^{0.129n+o(n)}$
 - ▶ Repeat this for each list vector: $2^{0.208n+o(n)}$

Hyperplane LSH

Overview

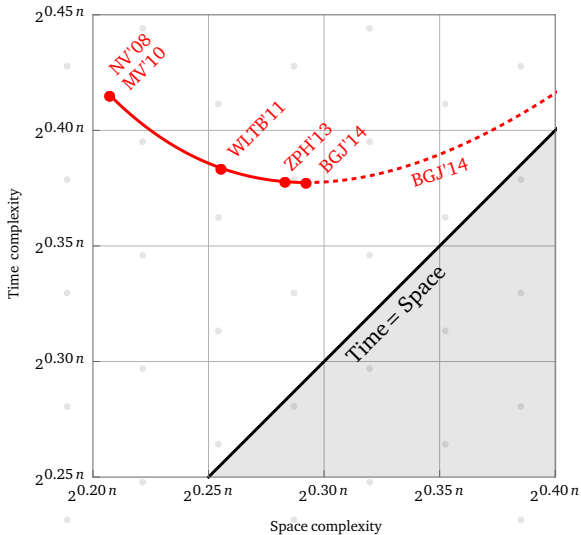
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Theorem

Sieving with hyperplane LSH heuristically solves SVP in time and space $2^{0.337n+o(n)}$.

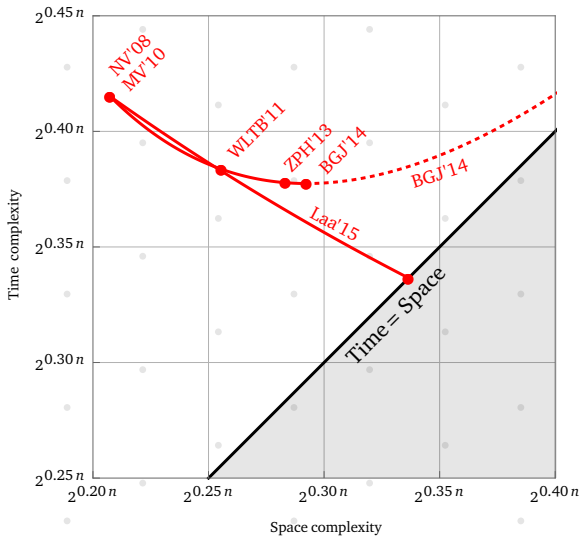
Hyperplane LSH

Space/time trade-off



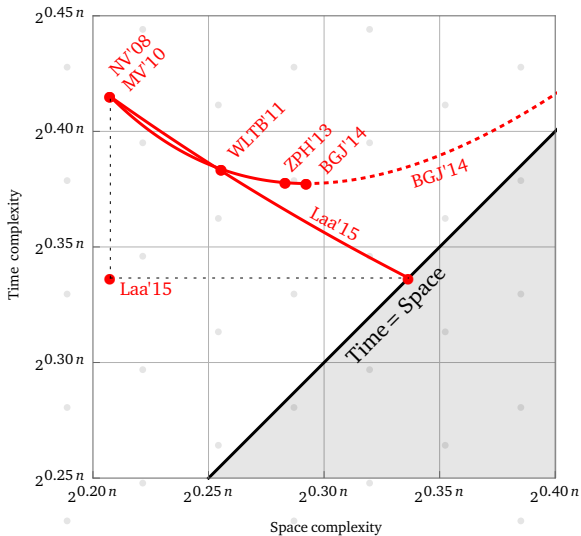
Hyperplane LSH

Space/time trade-off



Hyperplane LSH

Space/time trade-off



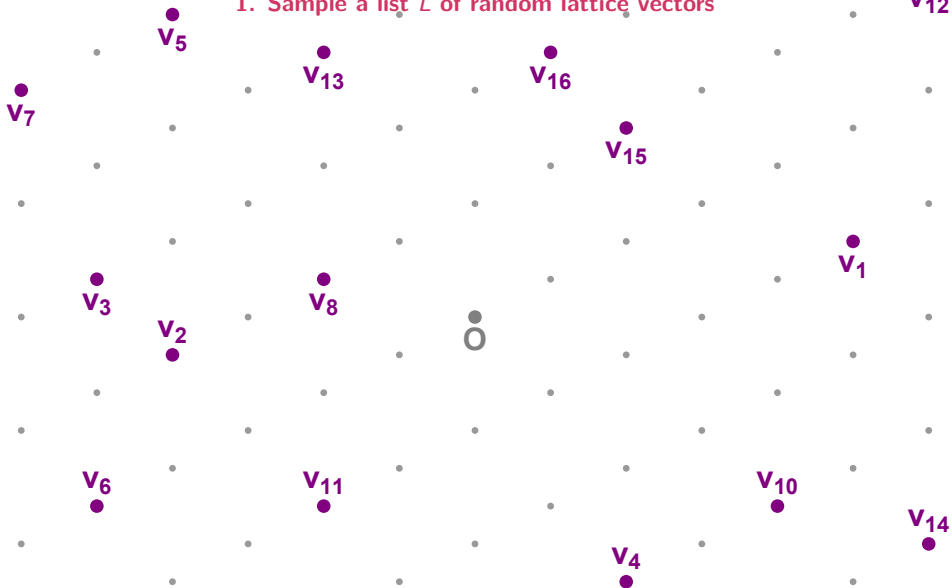
Spherical LSH

1. Sample a list L of random lattice vectors



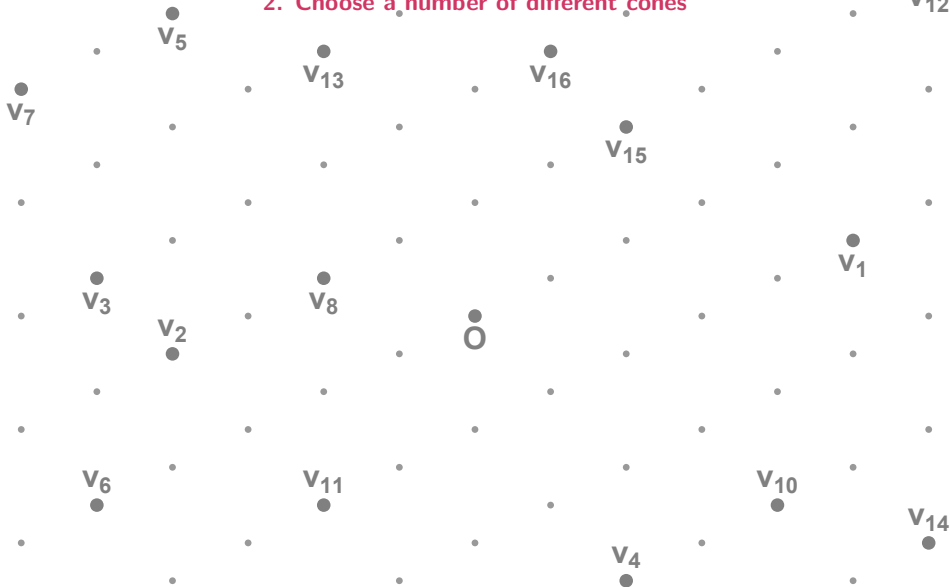
Spherical LSH

1. Sample a list L of random lattice vectors



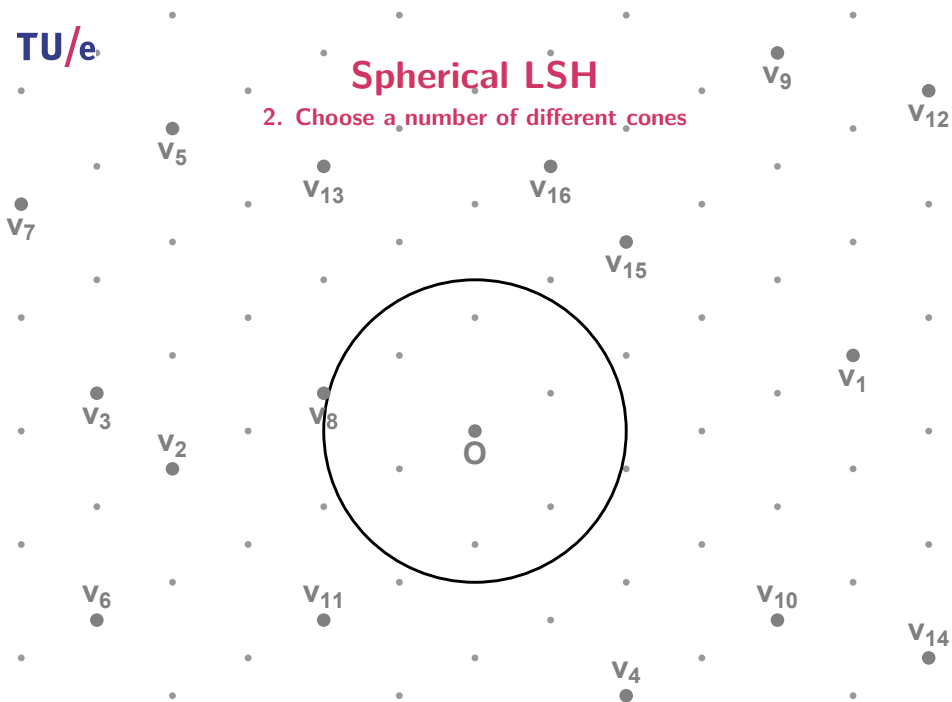
Spherical LSH

2. Choose a number of different cones



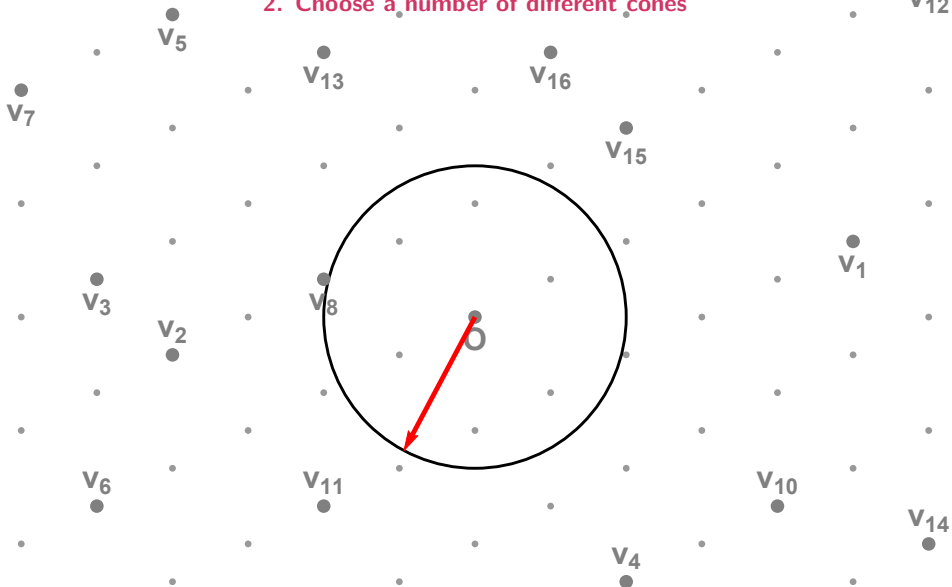
Spherical LSH

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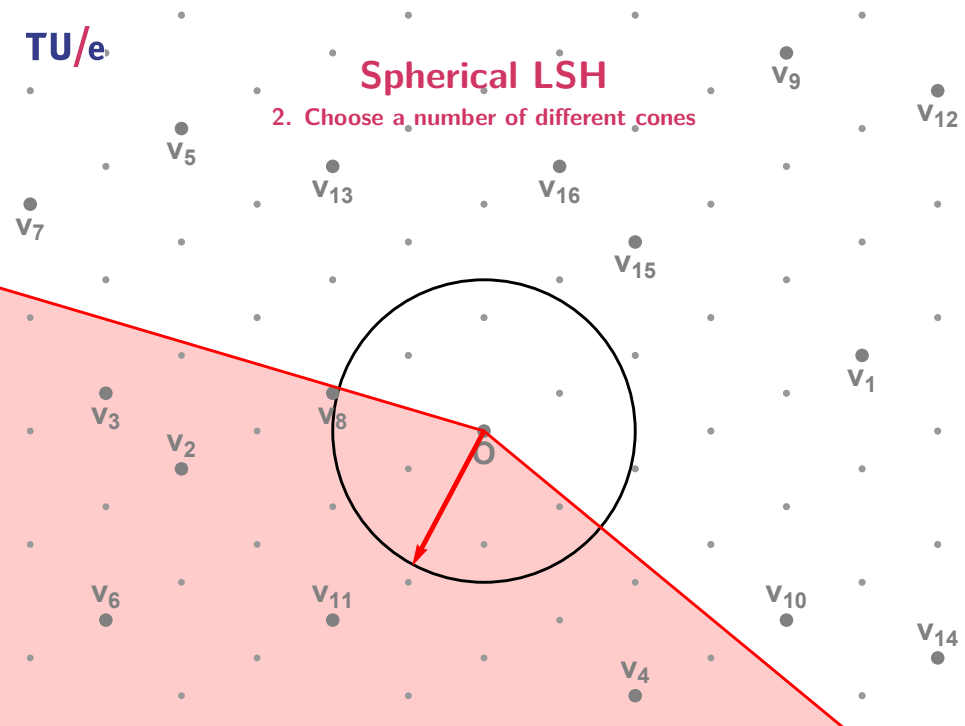
Spherical LSH

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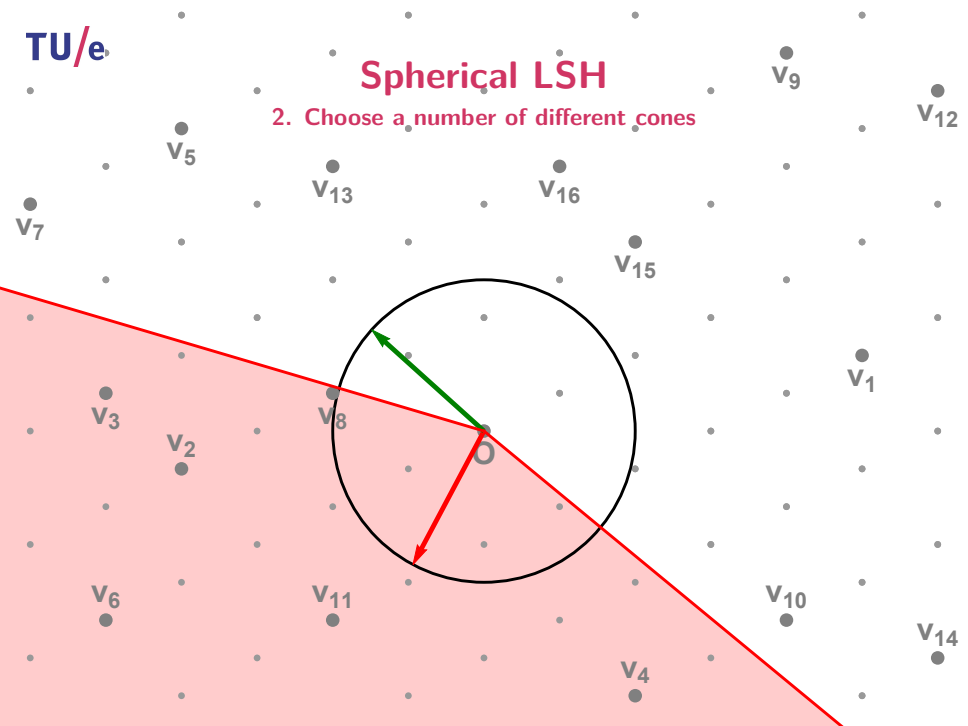
Spherical LSH

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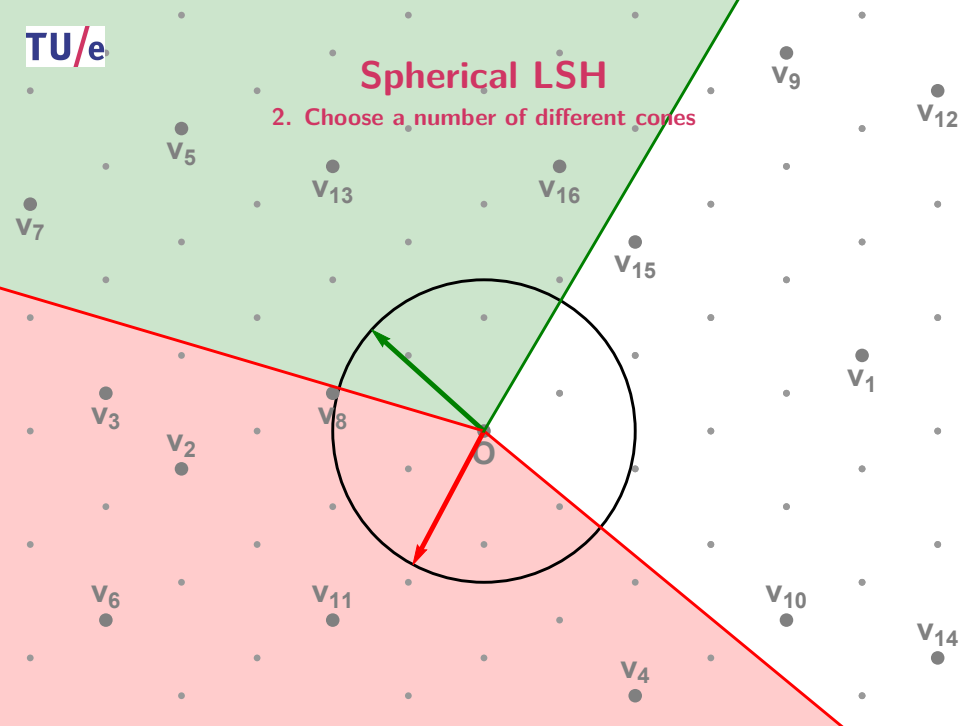
Spherical LSH

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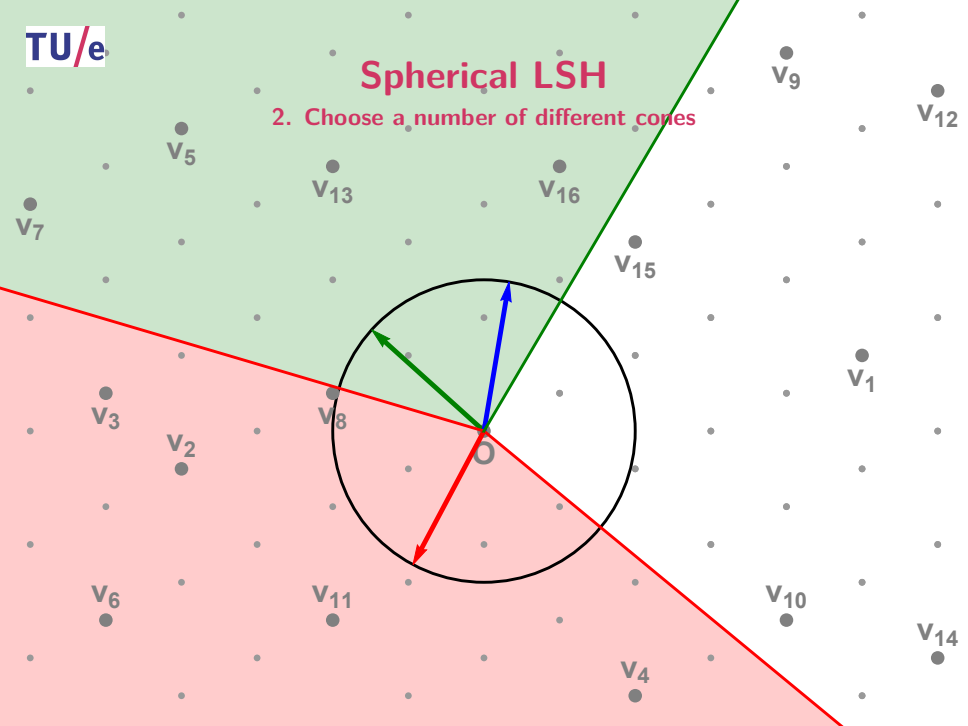
Spherical LSH

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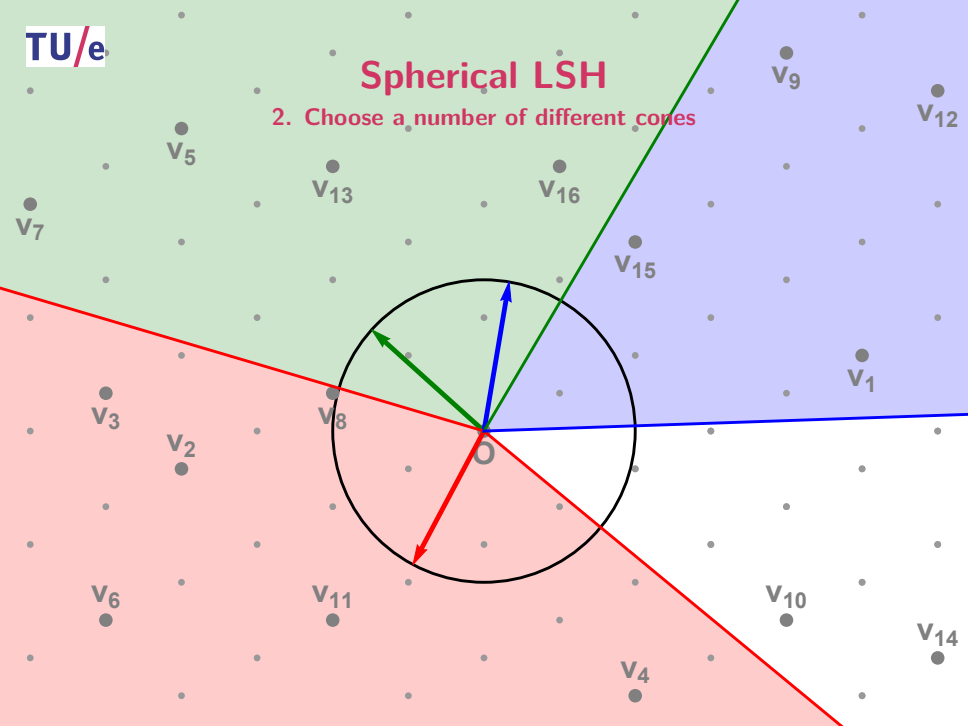
Spherical LSH

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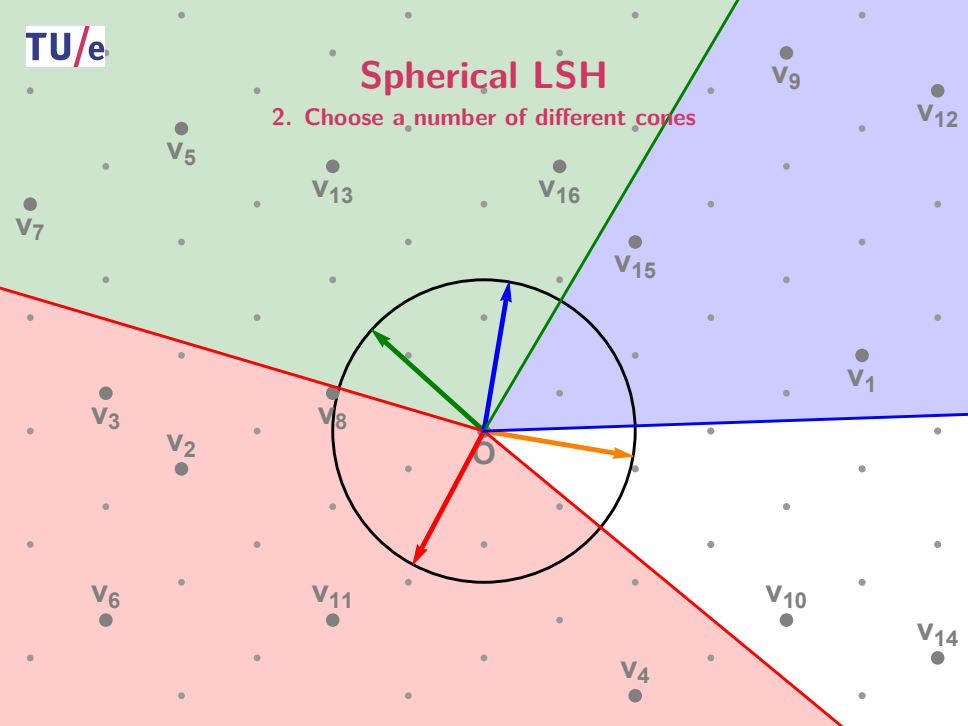
Spherical LSH

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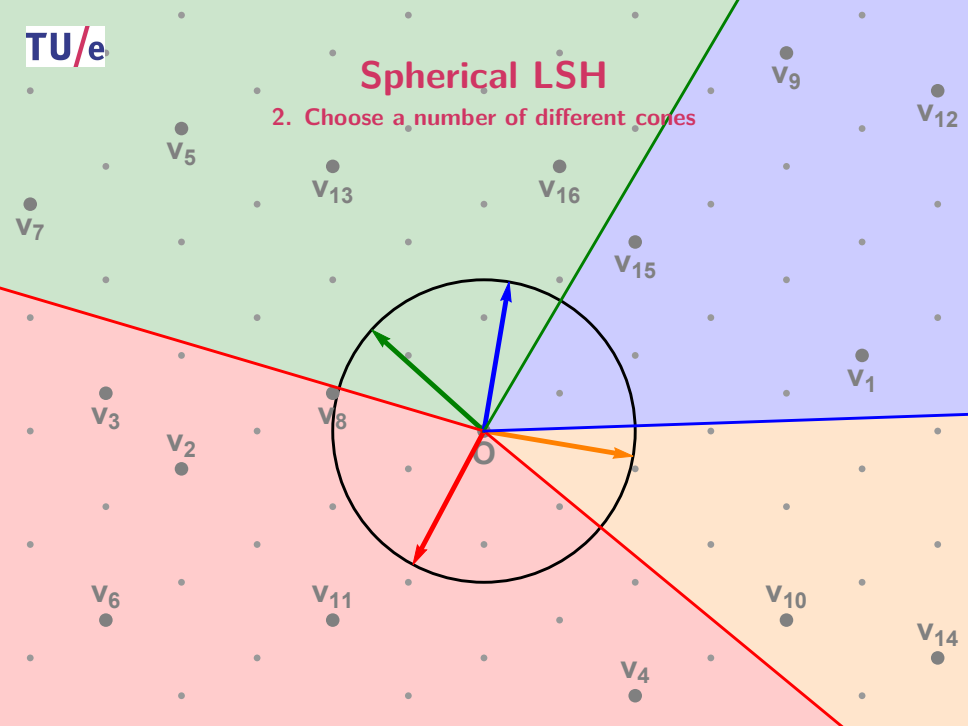
Spherical LSH

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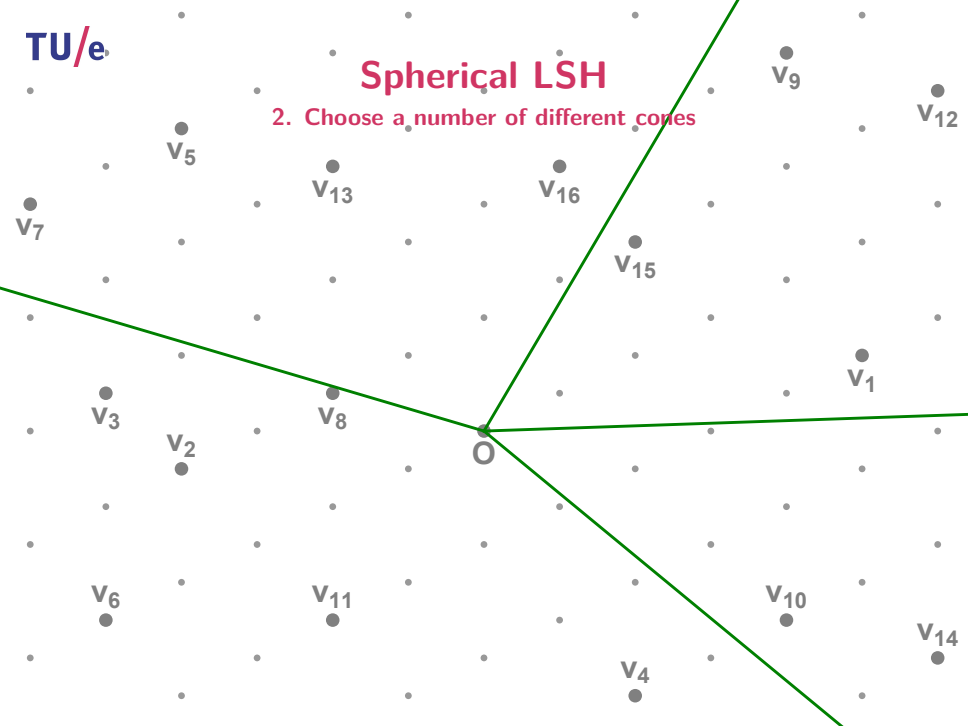
Spherical LSH

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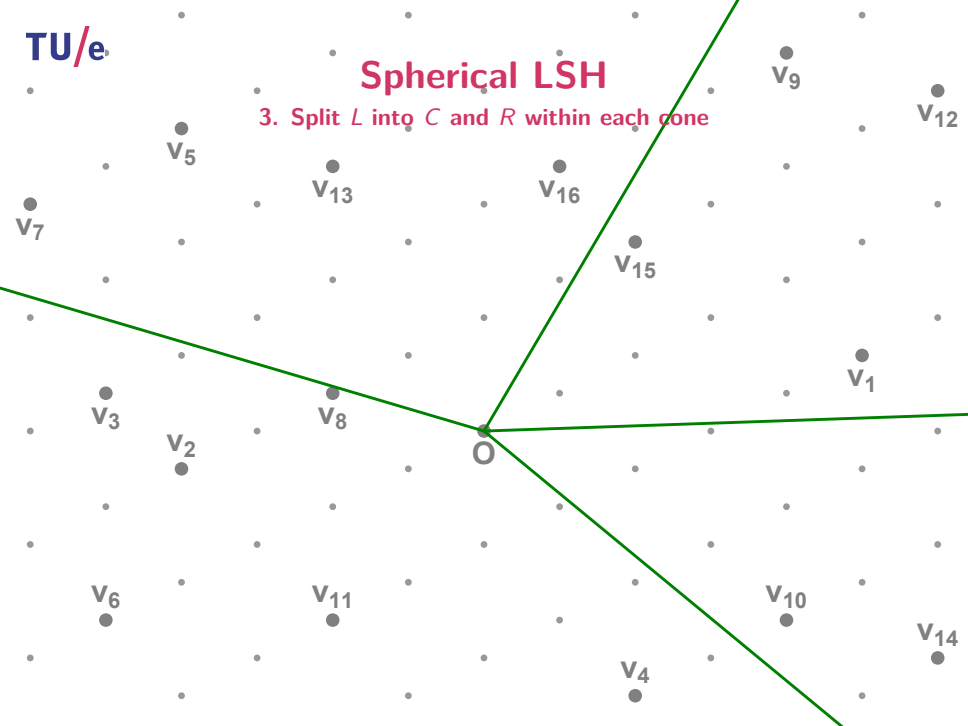
Spherical LSH

2. Choose a number of different cones



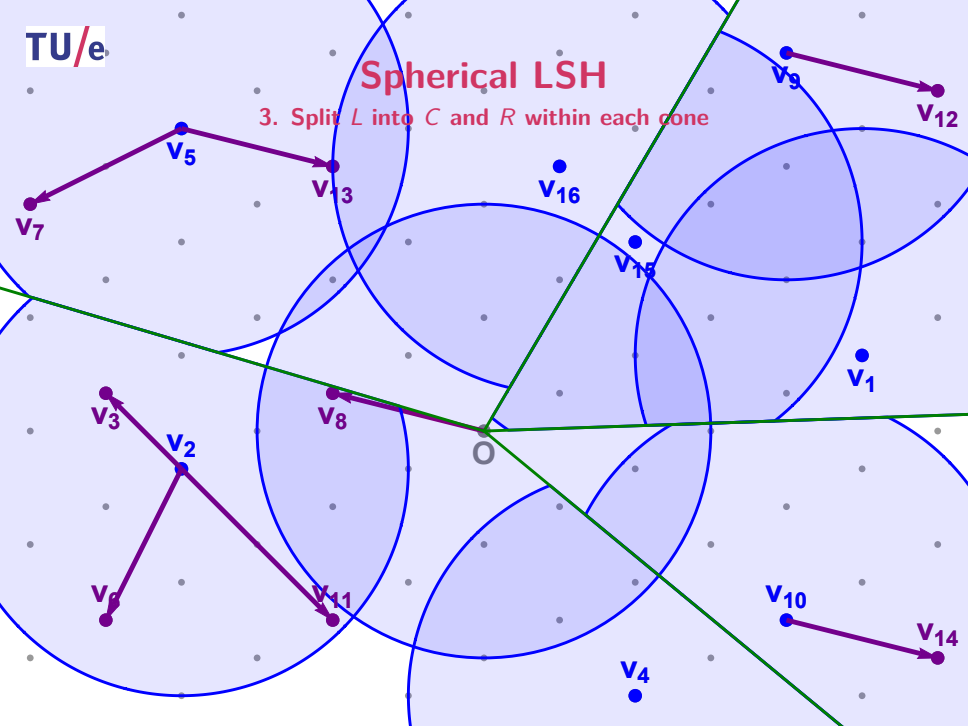
Spherical LSH

3. Split L into C and R within each cone



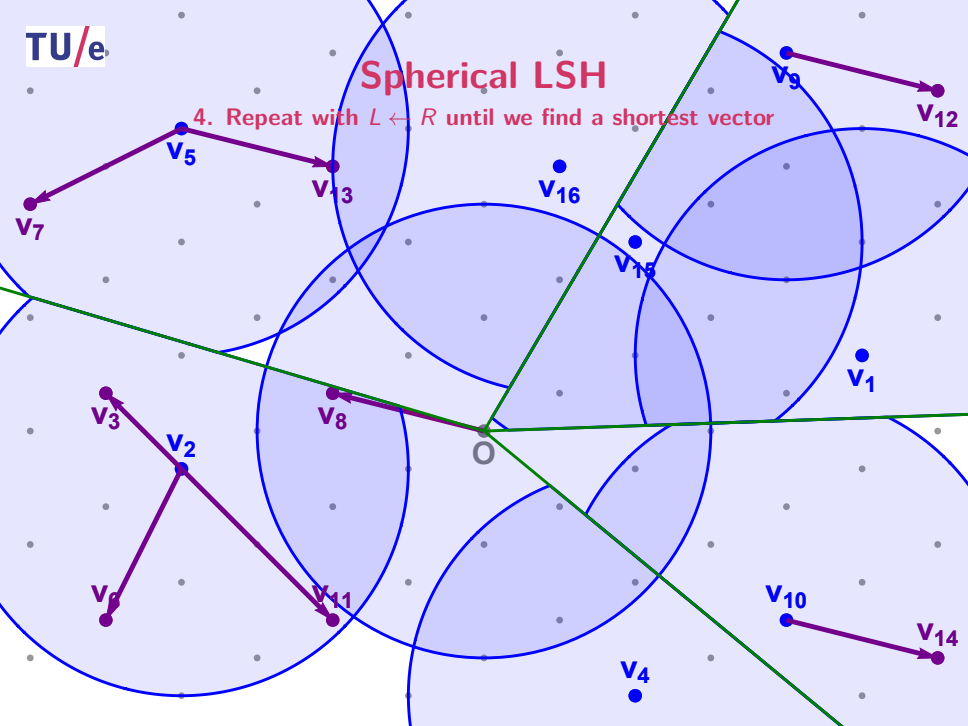
Spherical LSH

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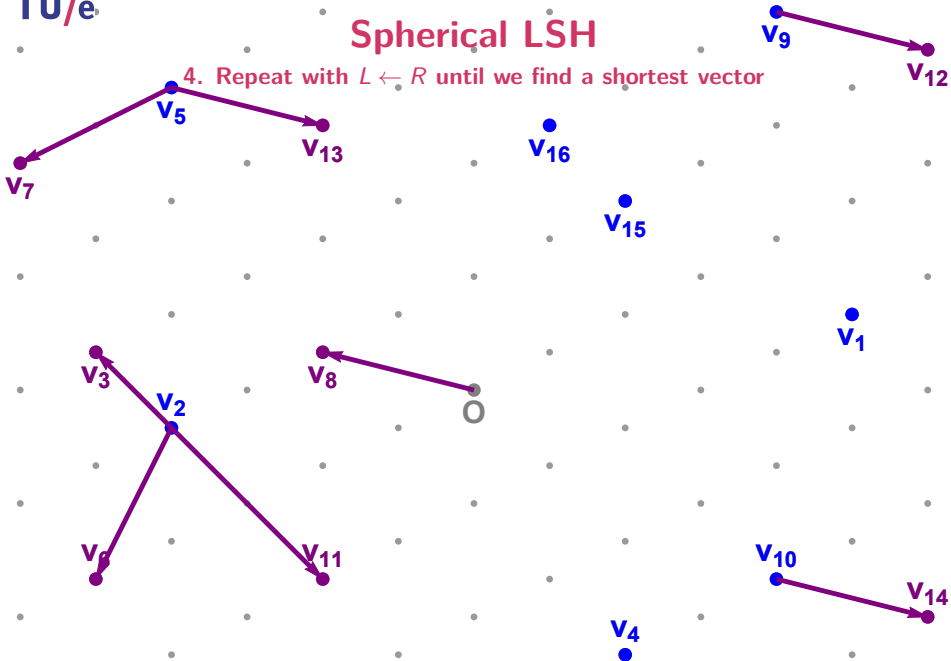
Spherical LSH

4. Repeat with $L \leftarrow R$ until we find a shortest vector



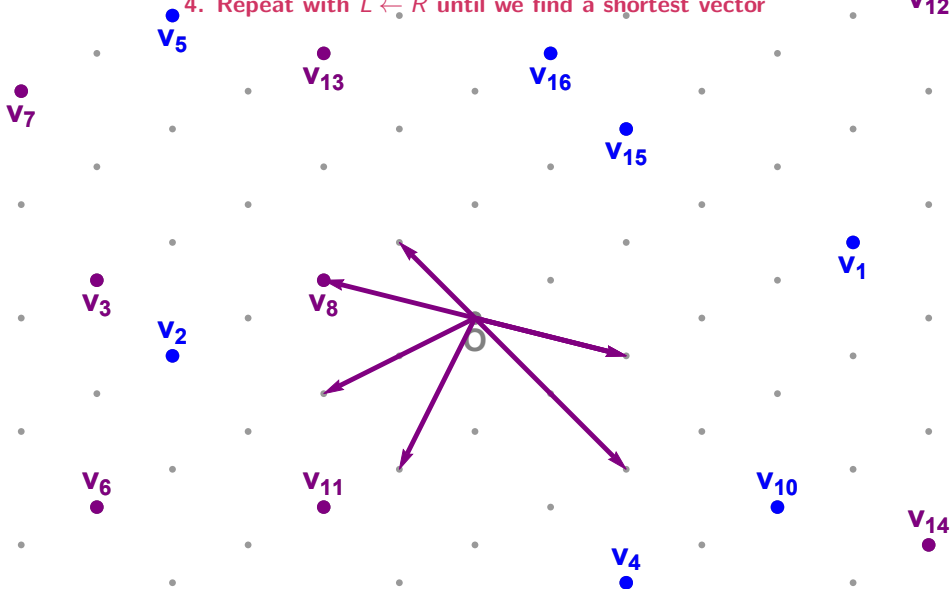
Spherical LSH

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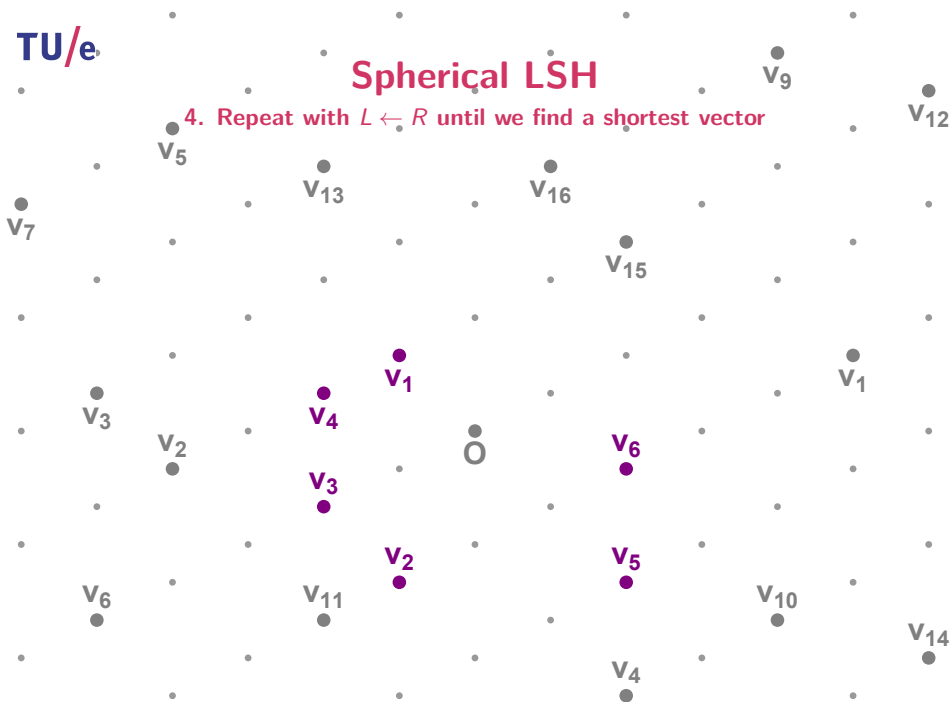
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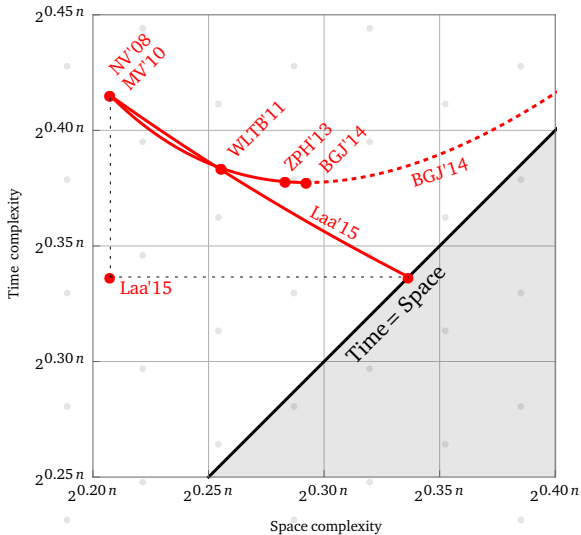
Spherical LSH

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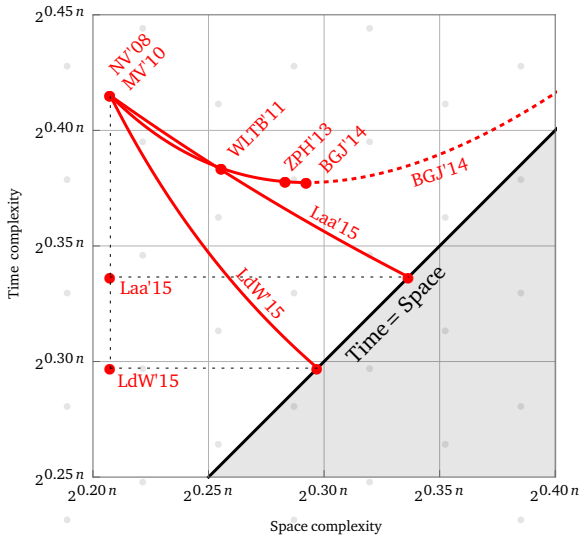
Spherical LSH

Space/time trade-off



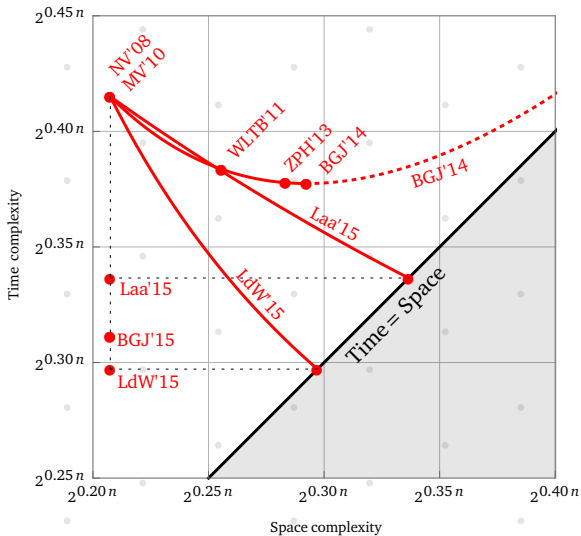
Spherical LSH

Space/time trade-off



May and Ozerov's NNS method

Space/time trade-off



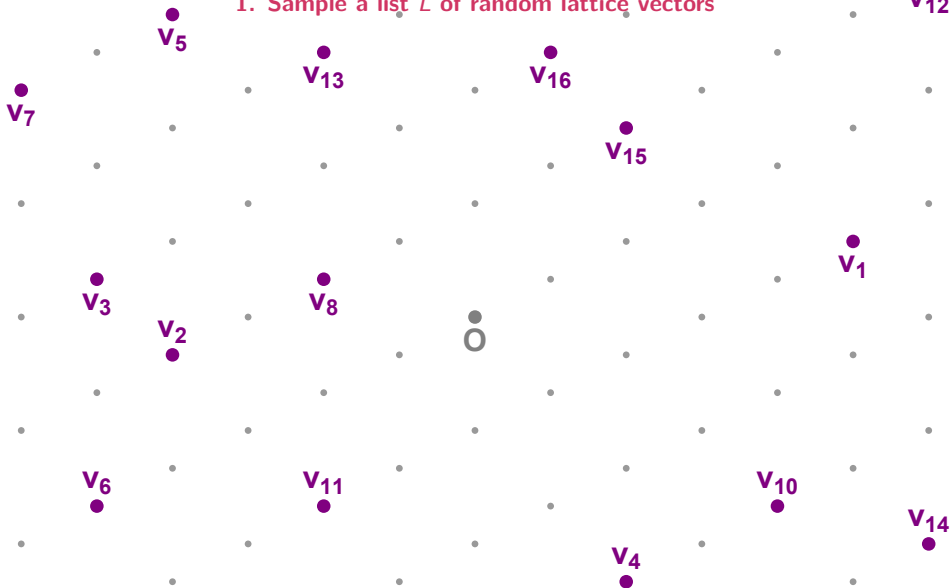
Cross-Polytope LSH

1. Sample a list L of random lattice vectors



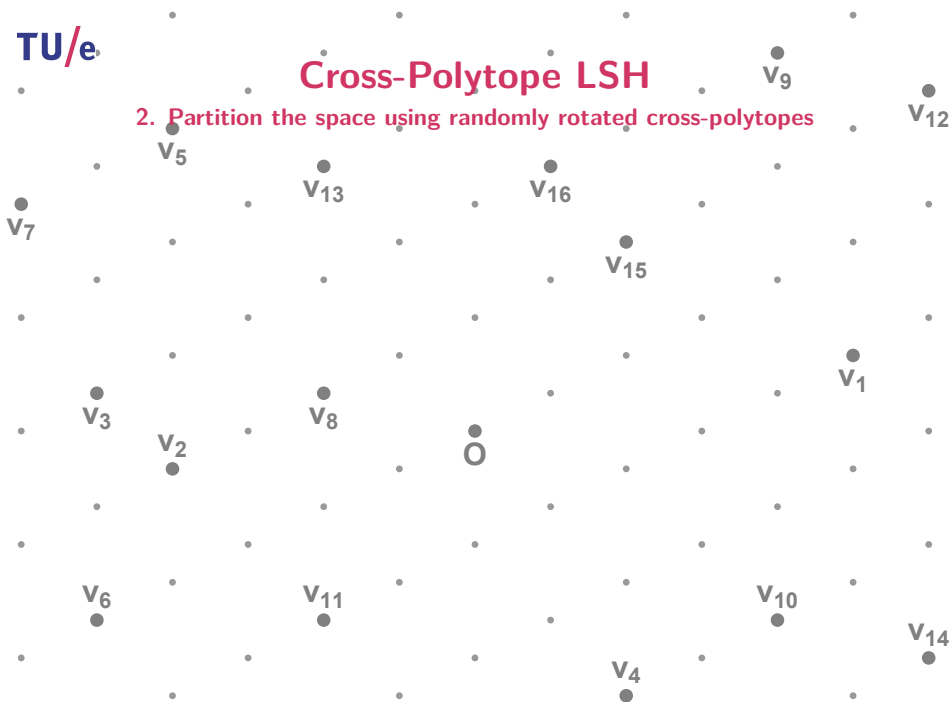
Cross-Polytope LSH

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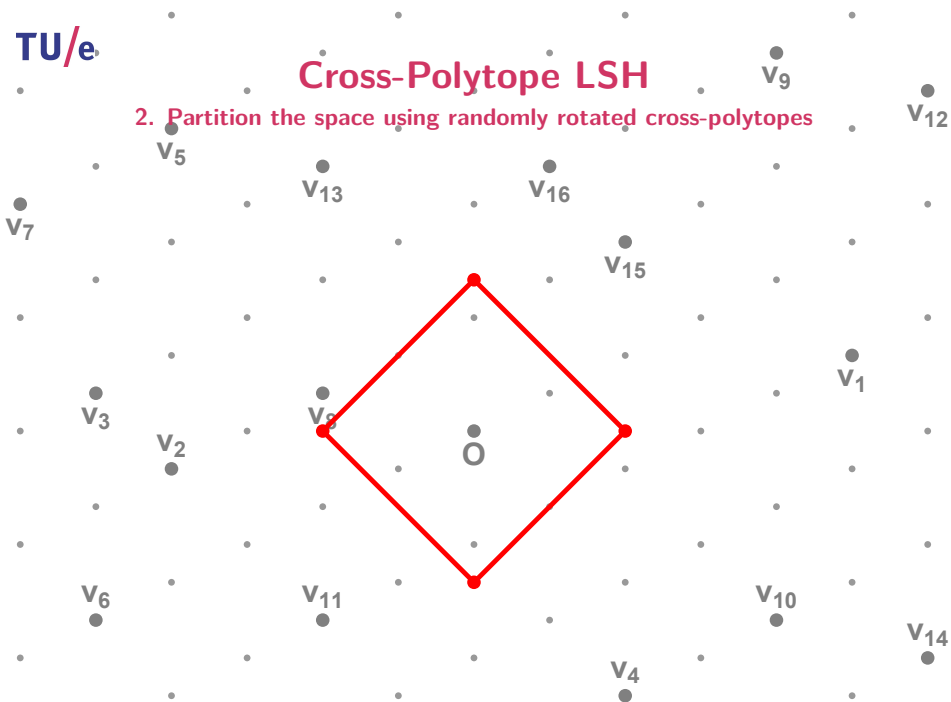
Cross-Polytope LSH

2. Partition the space using randomly rotated cross-polytopes



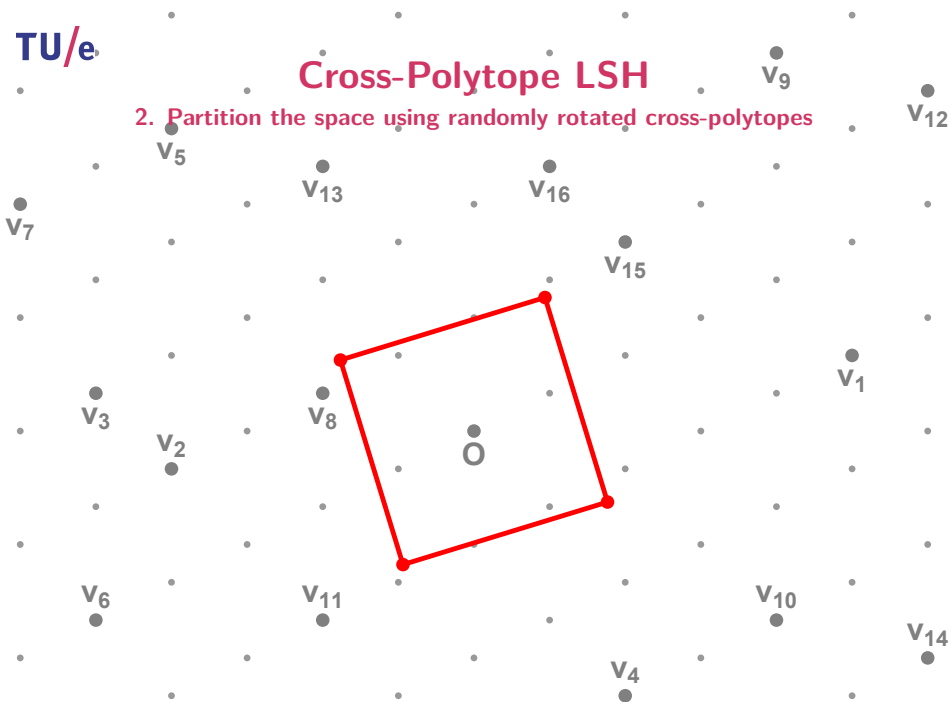
Cross-Polytope LSH

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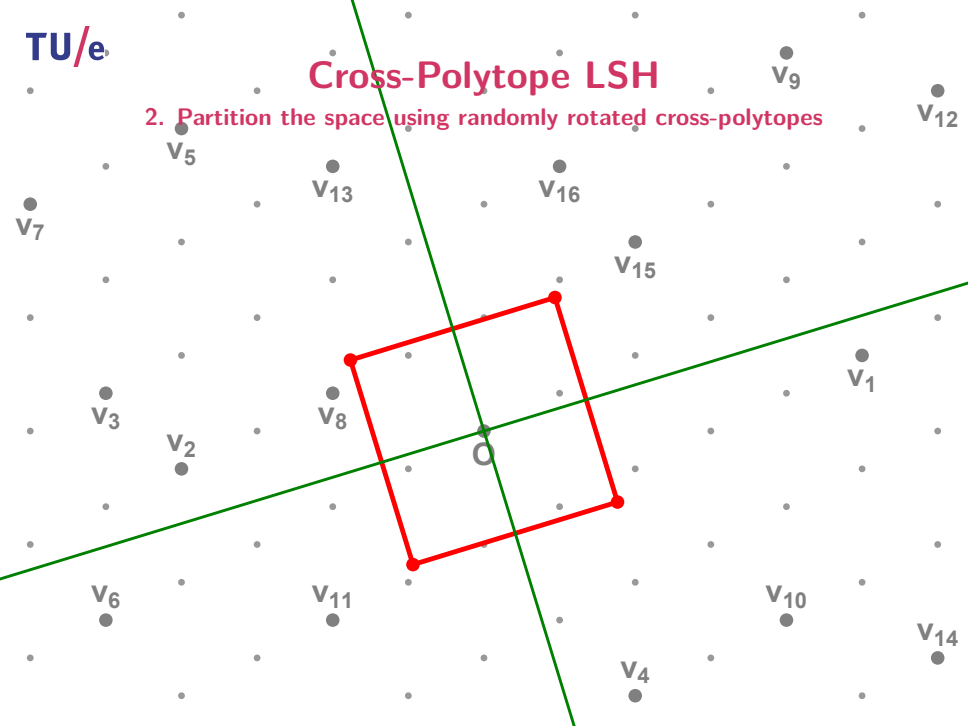
Cross-Polytope LSH

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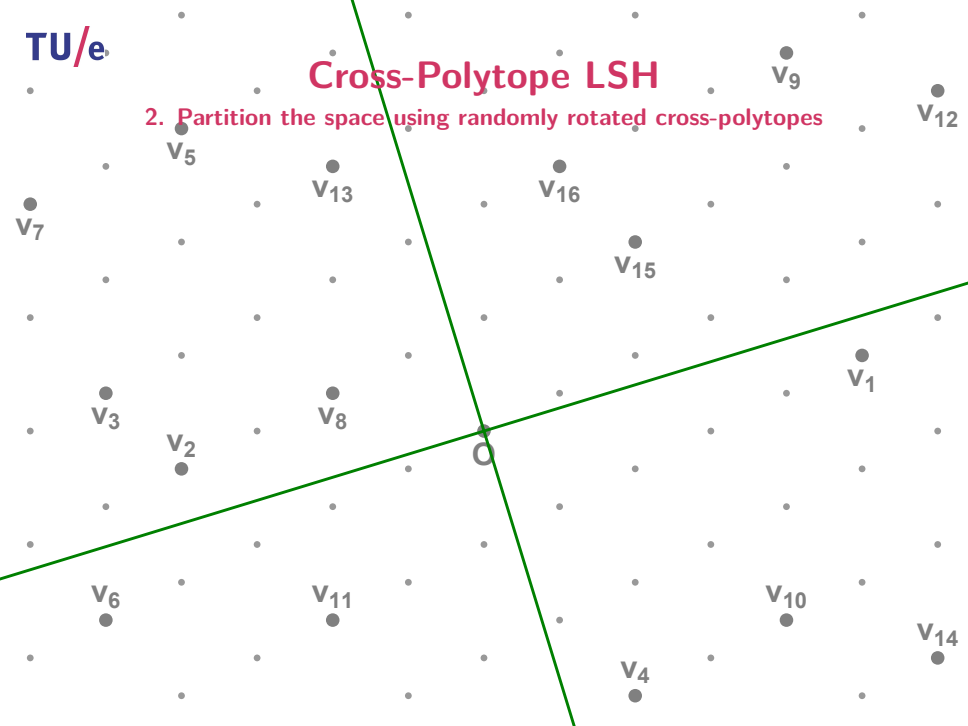
Cross-Polytope LSH

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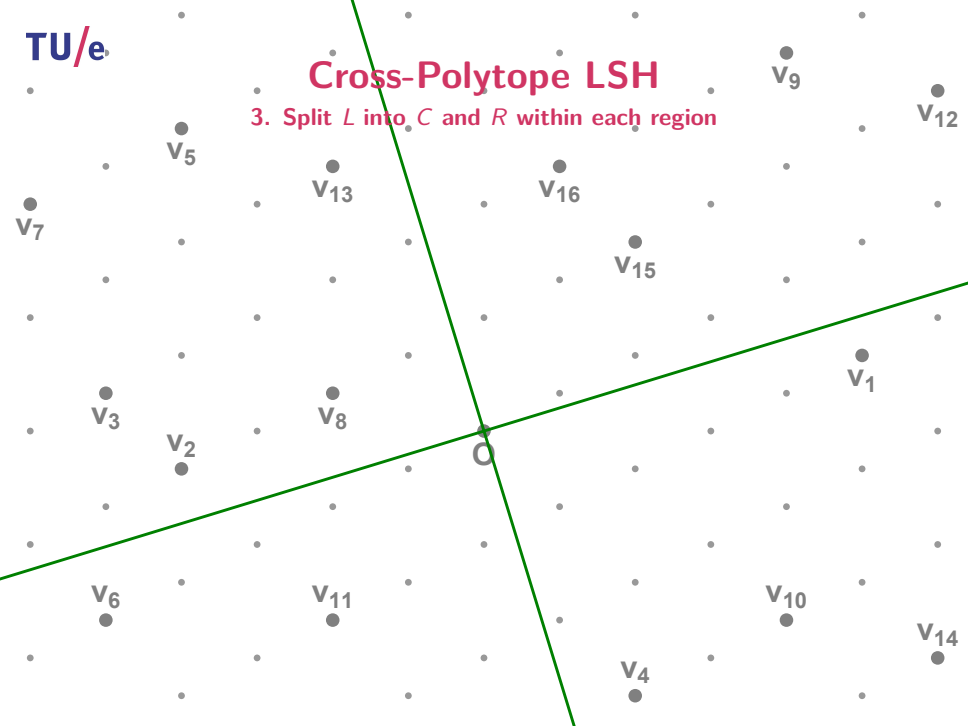
Cross-Polytope LSH

2. Partition the space using randomly rotated cross-polytopes



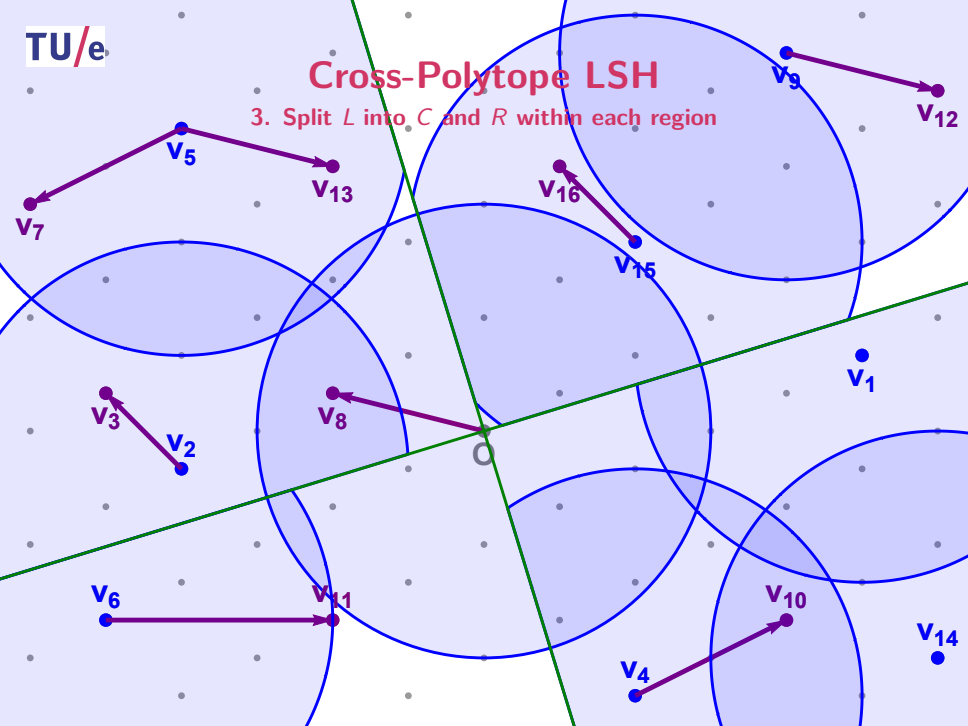
Cross-Polytope LSH

3. Split L into C and R within each region



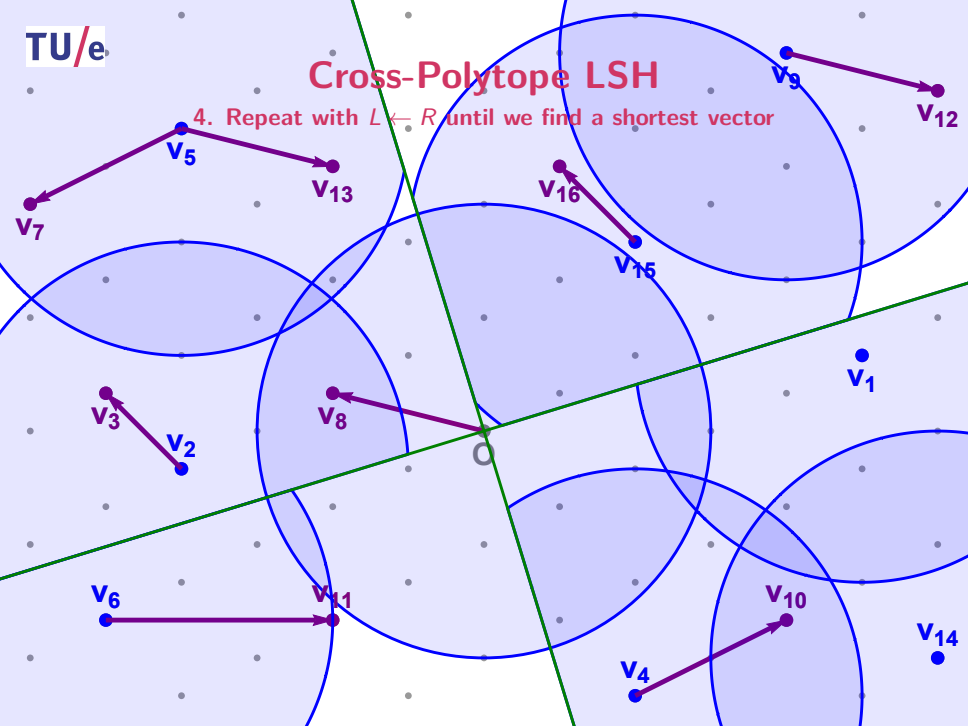
Cross-Polytope LSH

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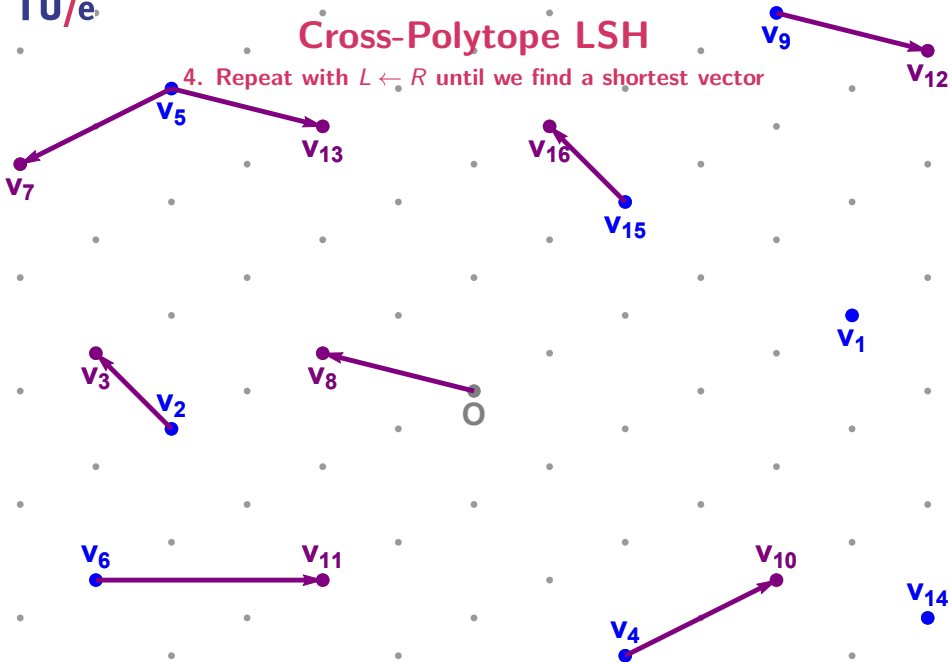
Cross-Polytope LSH

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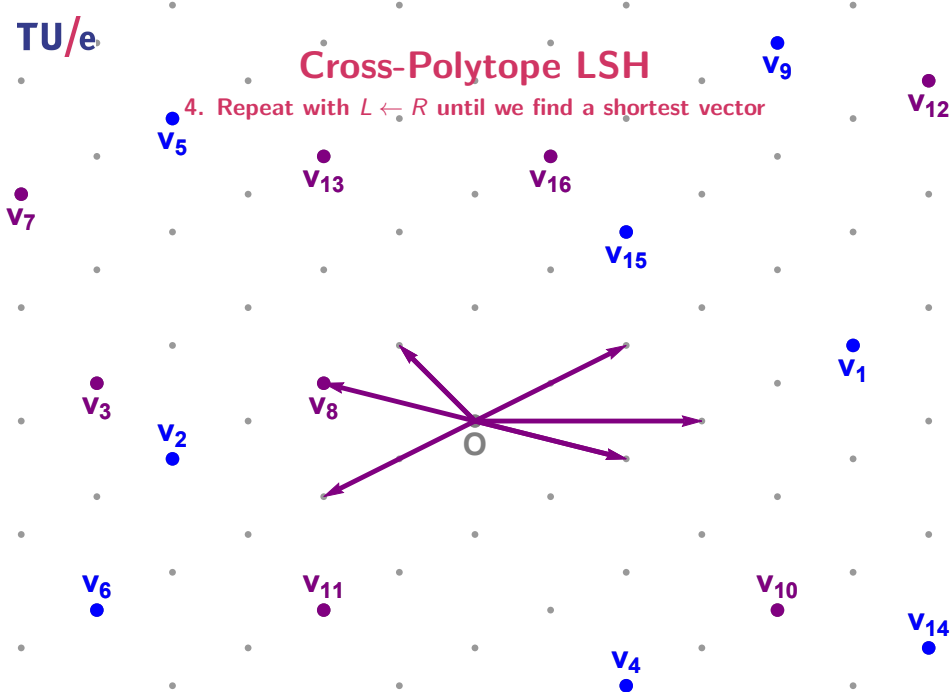
Cross-Polytope LSH

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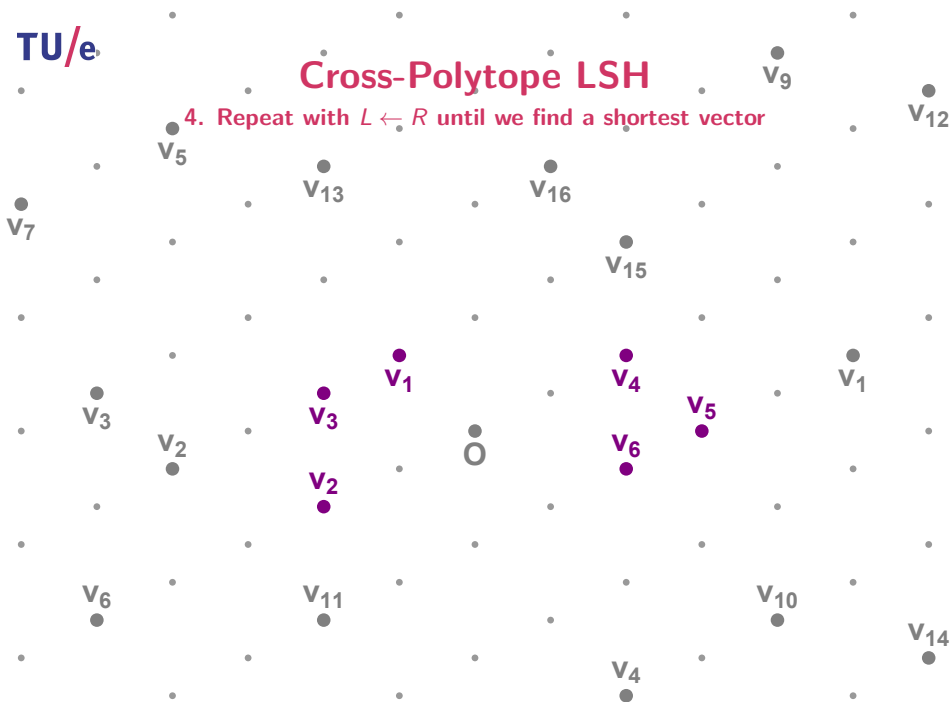
Cross-Polytope LSH

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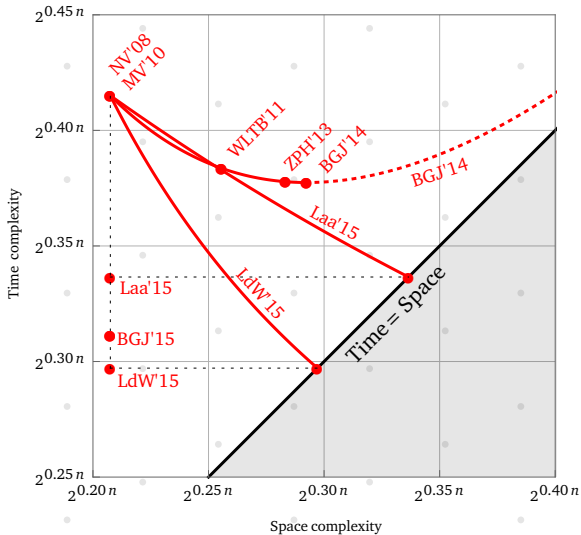
Cross-Polytope LSH

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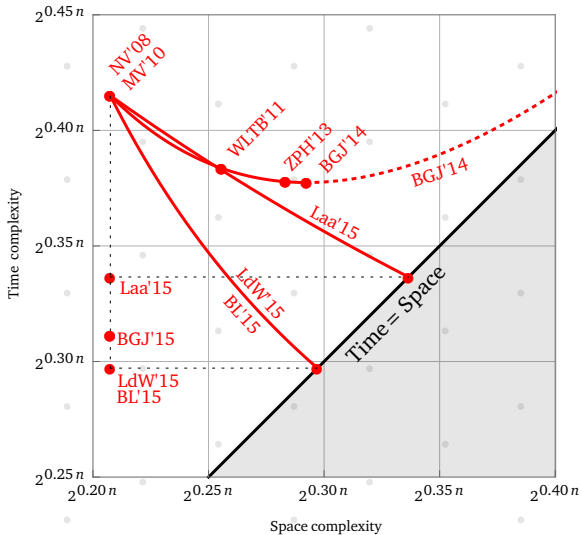
Cross-Polytope LSH

Space/time trade-off



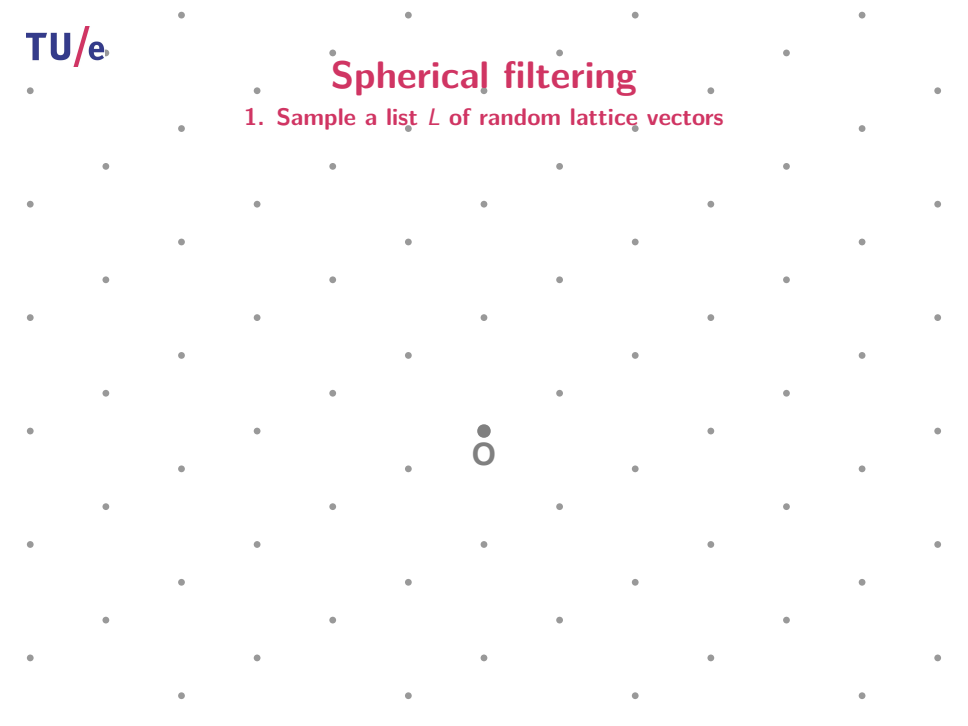
Cross-Polytope LSH

Space/time trade-off



Spherical filtering

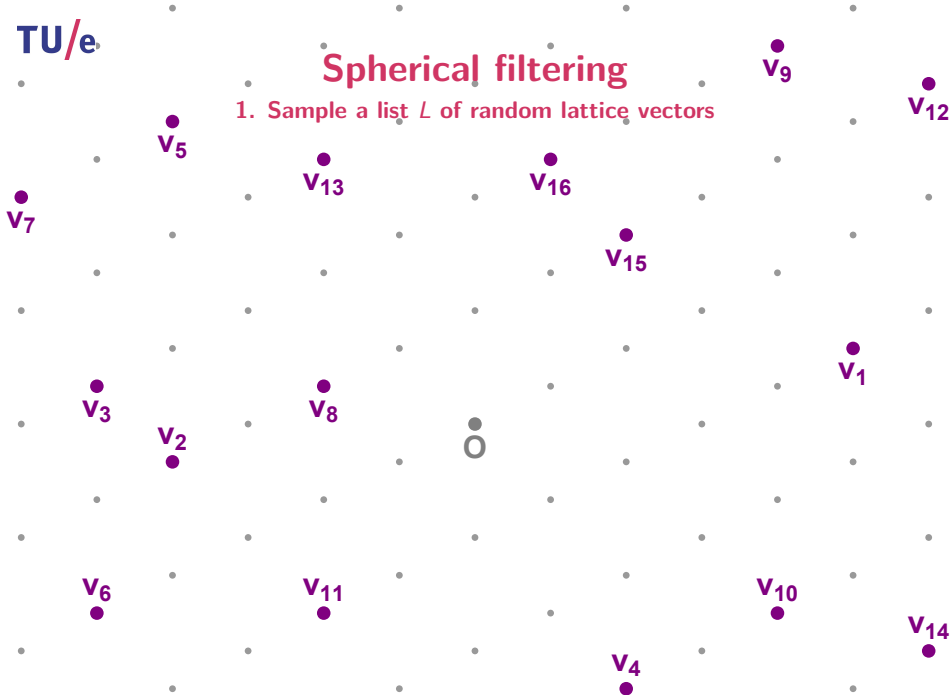
1. Sample a list L of random lattice vectors



O

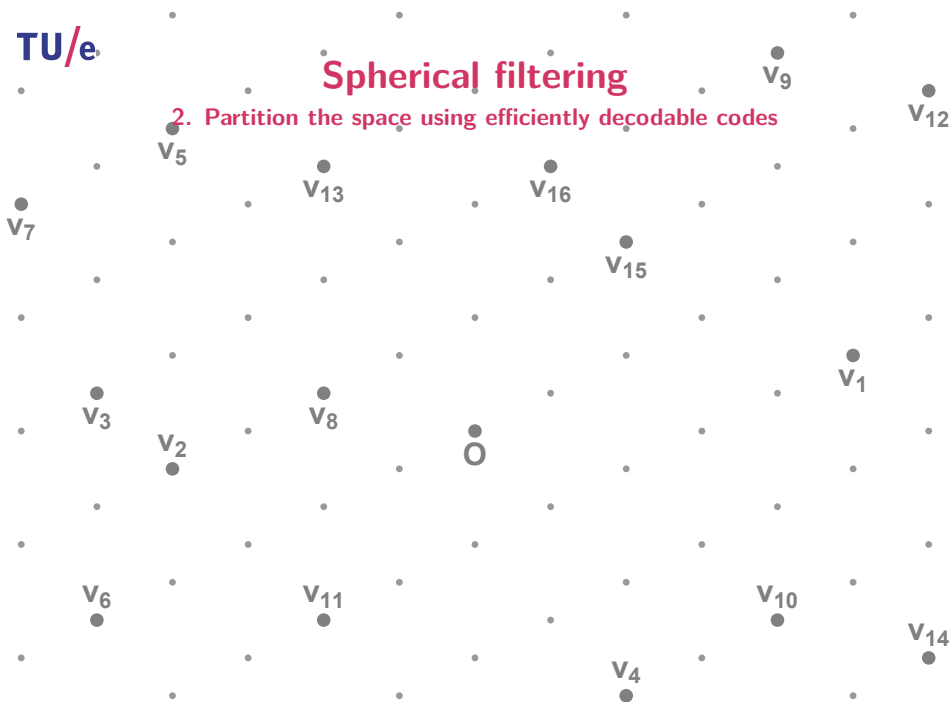
Spherical filtering

1. Sample a list L of random lattice vectors



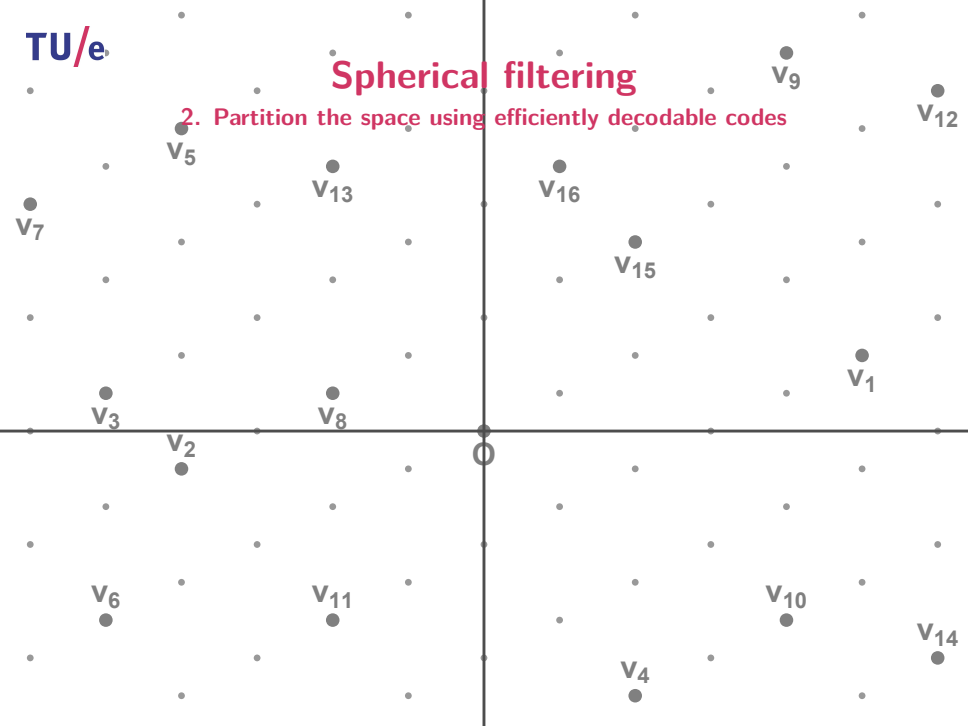
Spherical filtering

2. Partition the space using efficiently decodable codes



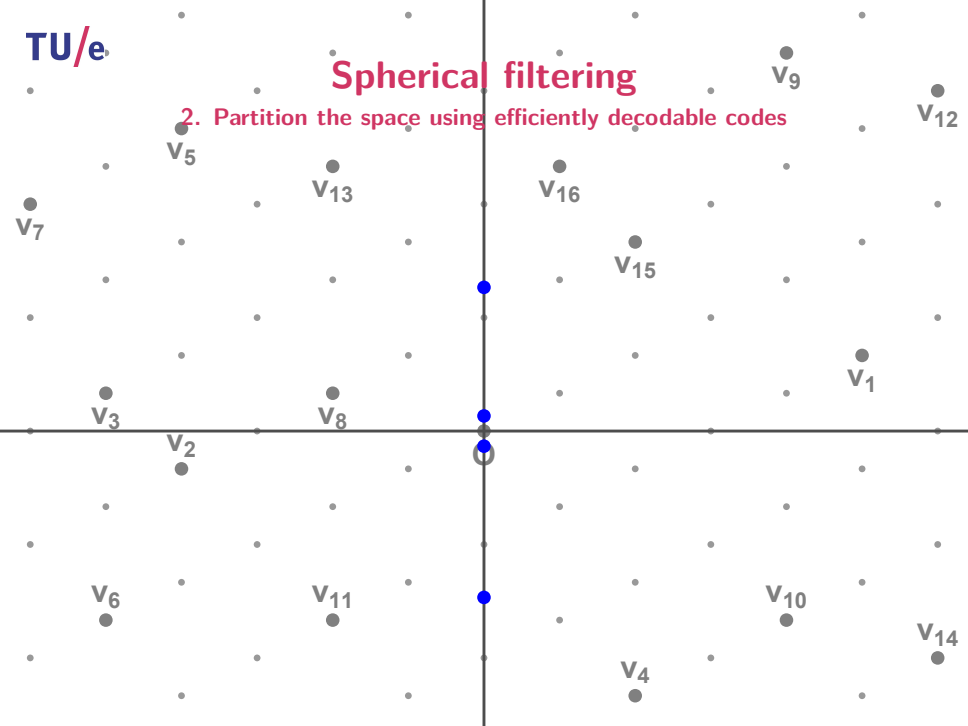
Spherical filtering

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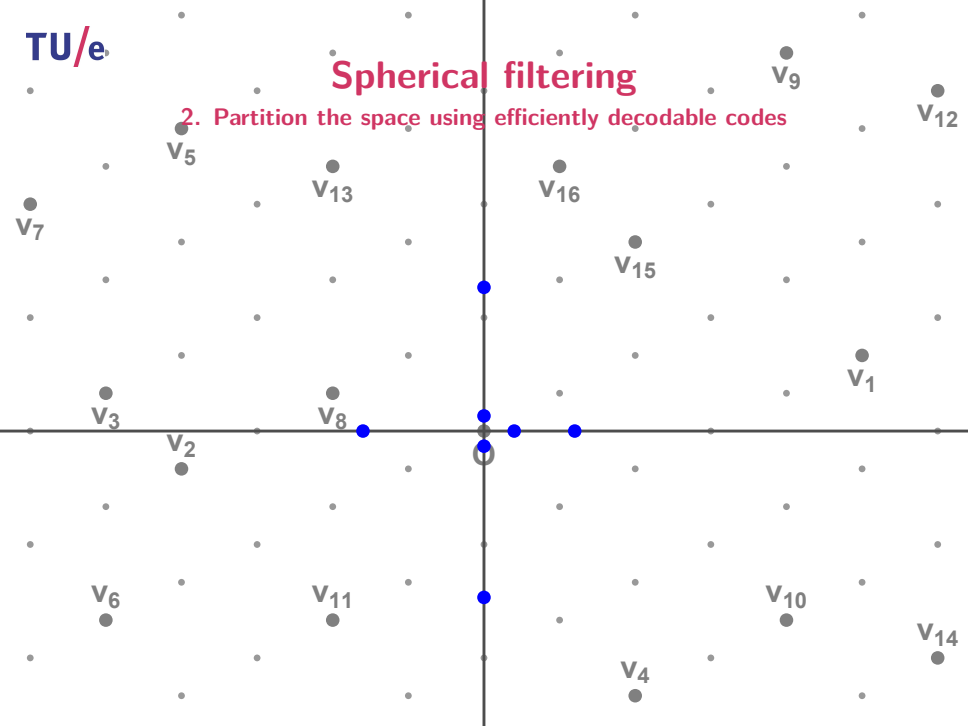
Spherical filtering

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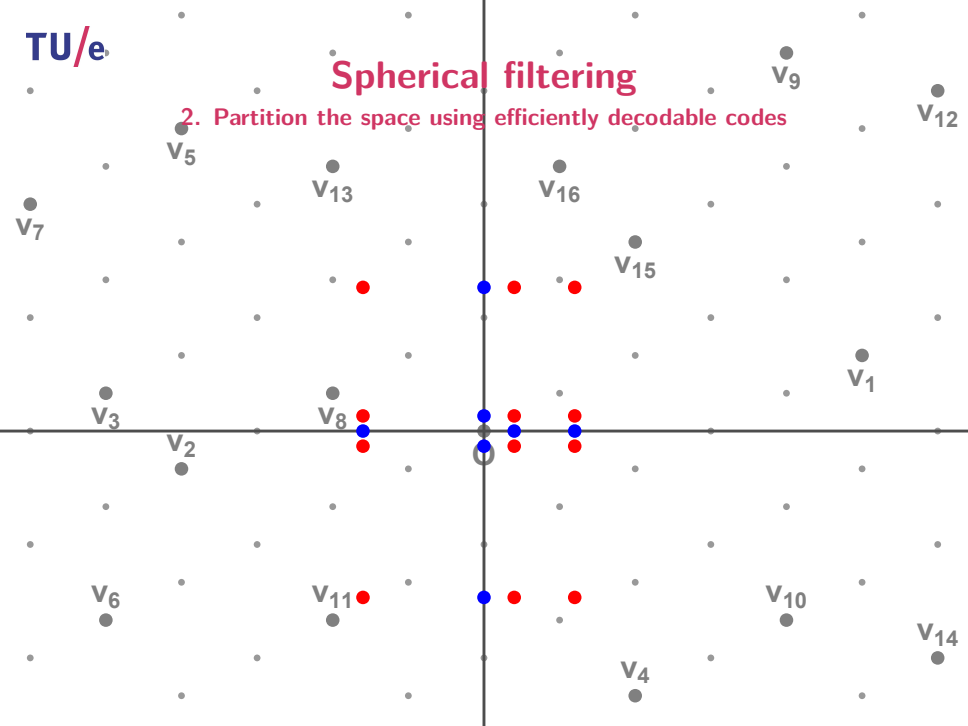
Spherical filtering

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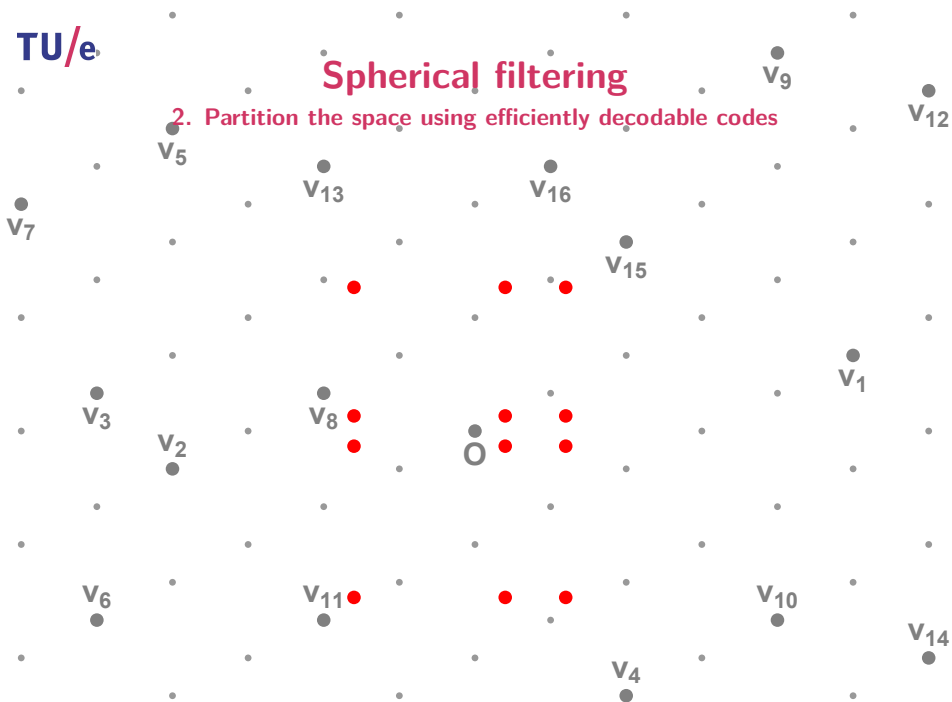
Spherical filtering

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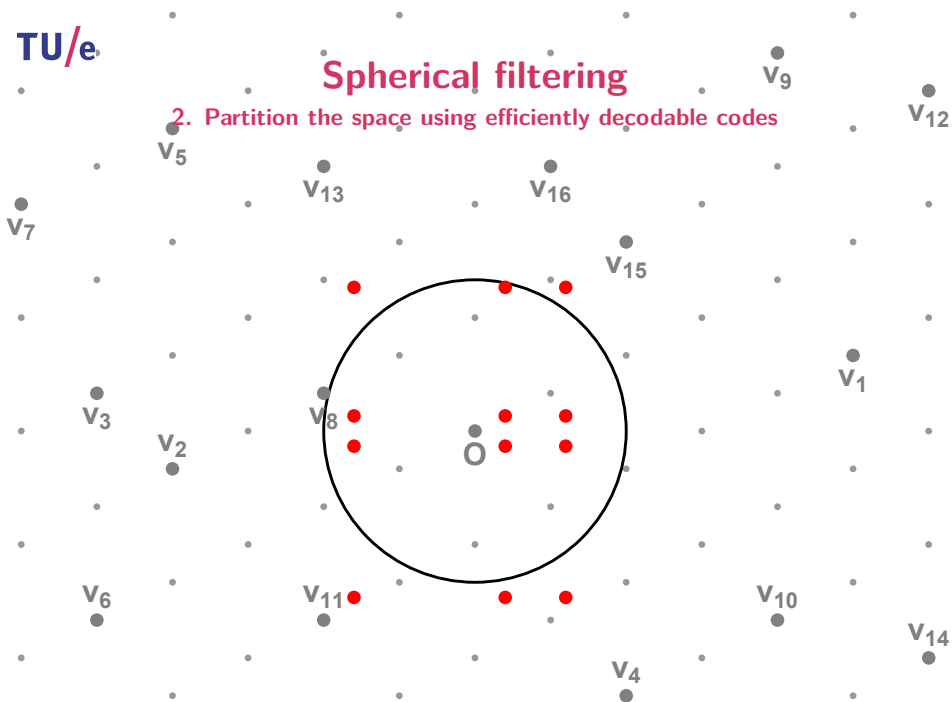
Spherical filtering

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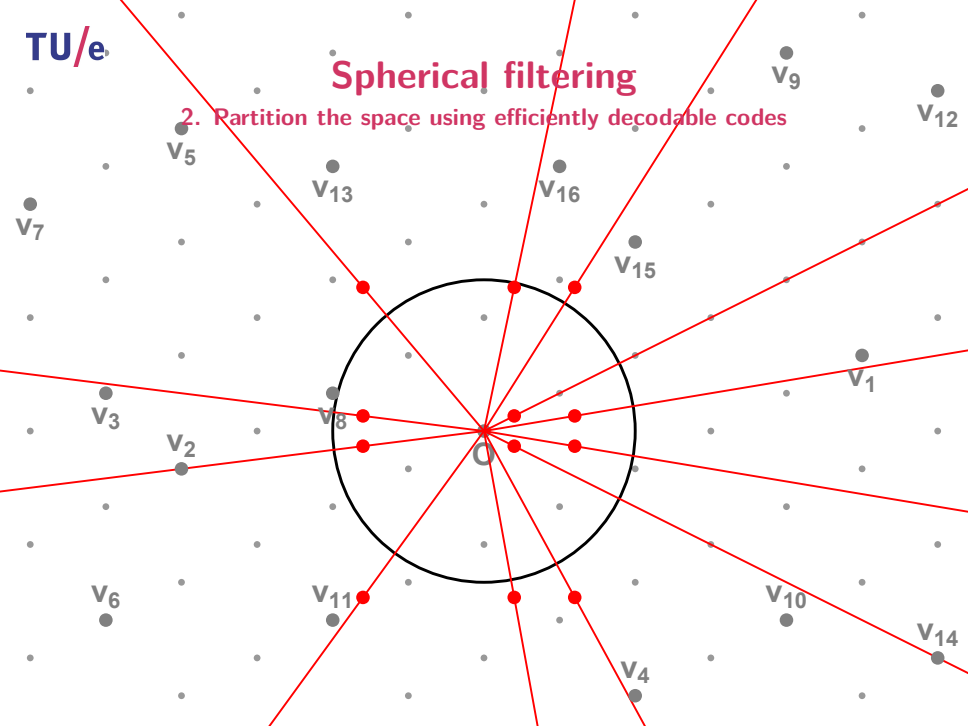
Spherical filtering

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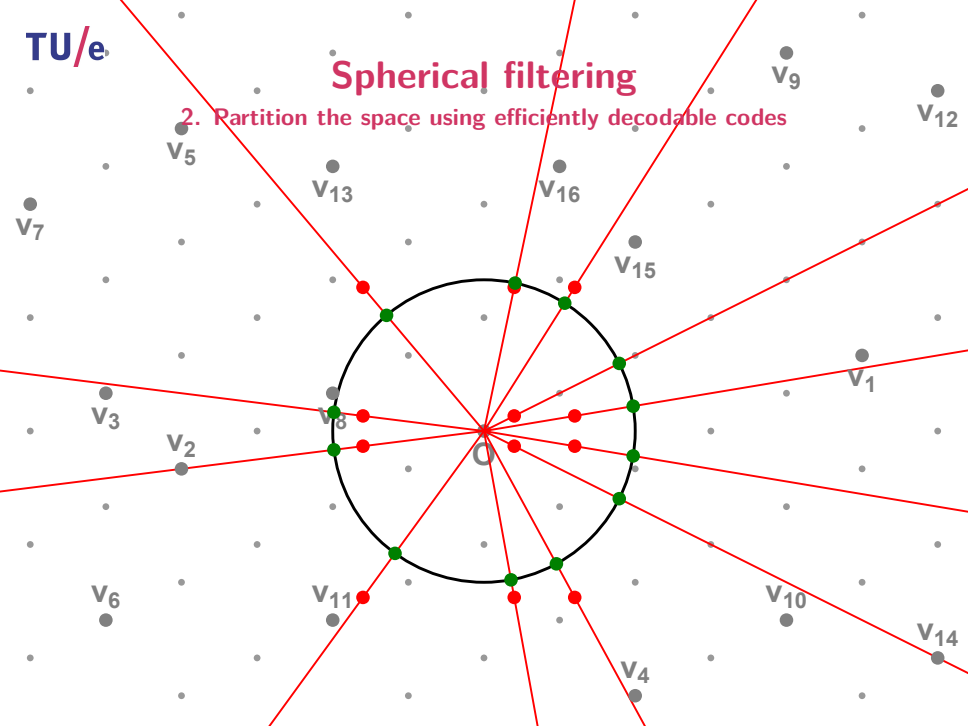
Spherical filtering

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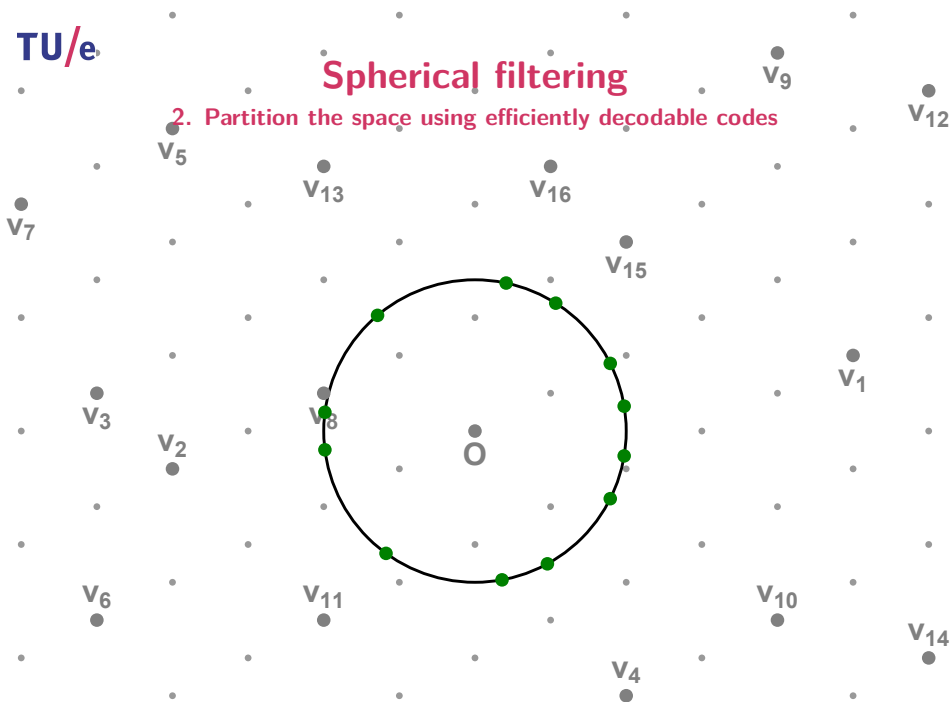
Spherical filtering

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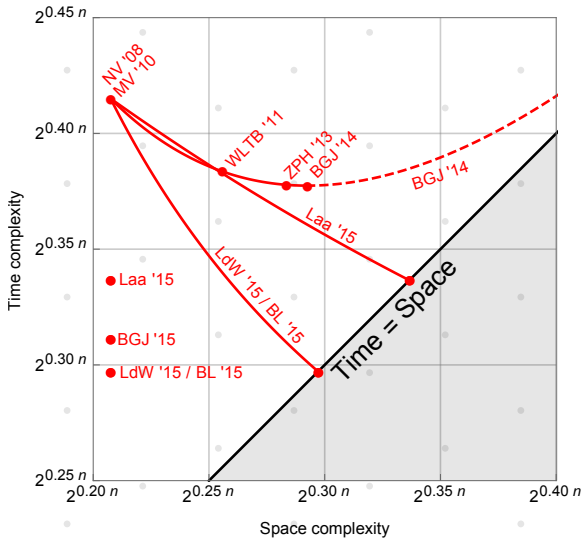
Spherical filtering

2. Partition the space using efficiently decodable codes



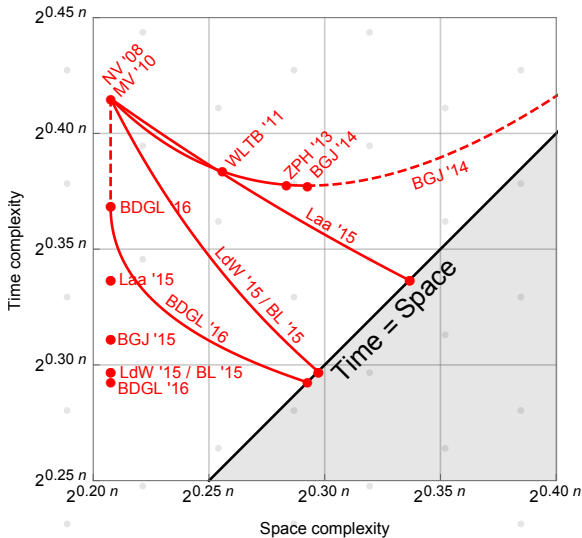
Spherical filtering

Space/time trade-off



Spherical filtering

Space/time trade-off



Questions

[vdP'12]

