Graph-based time–space trade-offs for approximate near neighbors

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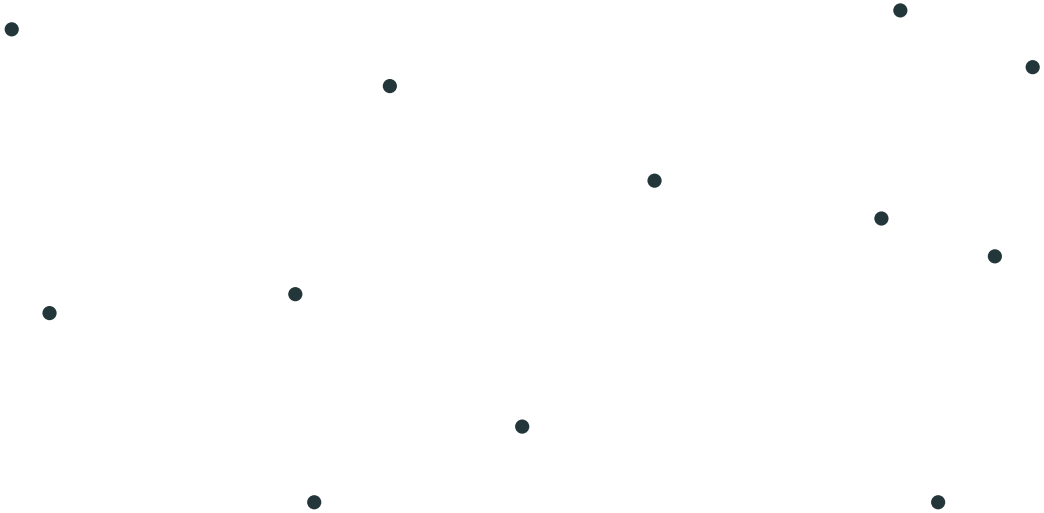
Nearest neighbor searching

Nearest neighbor problem – Problem description



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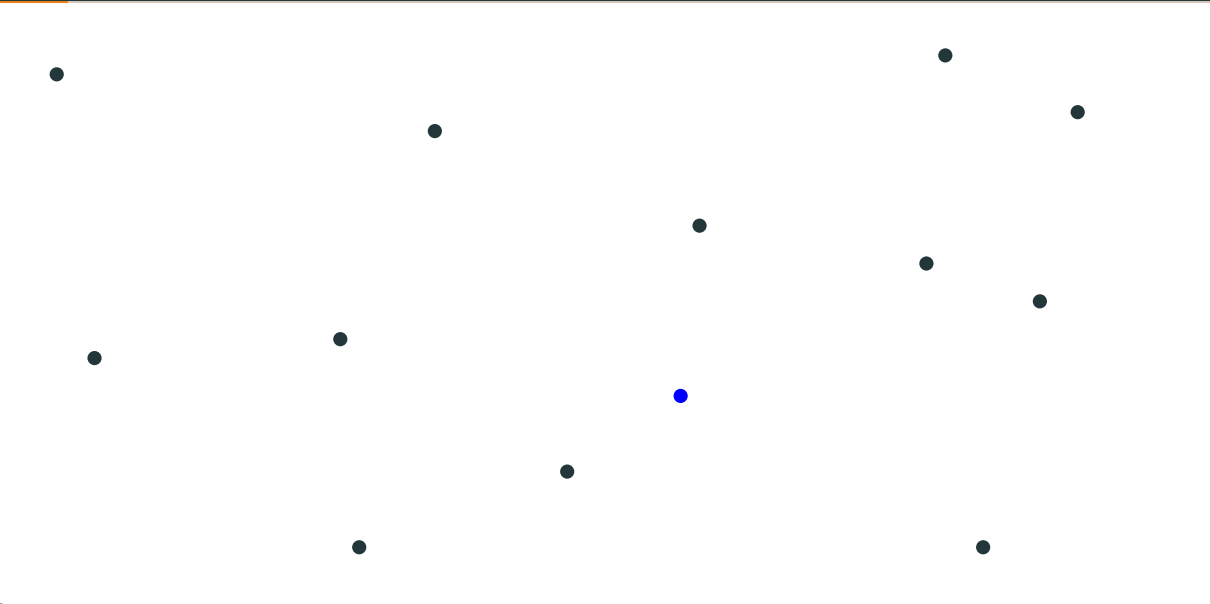

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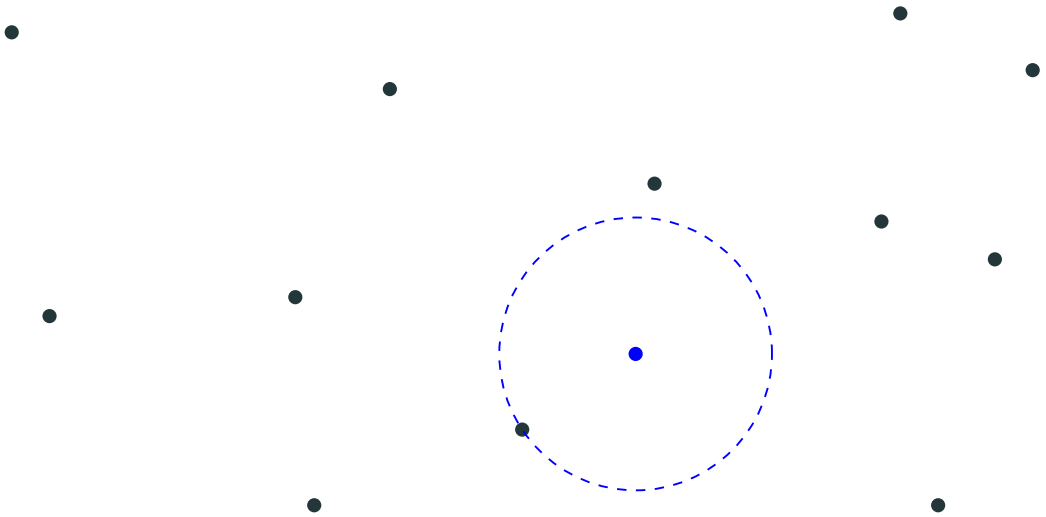
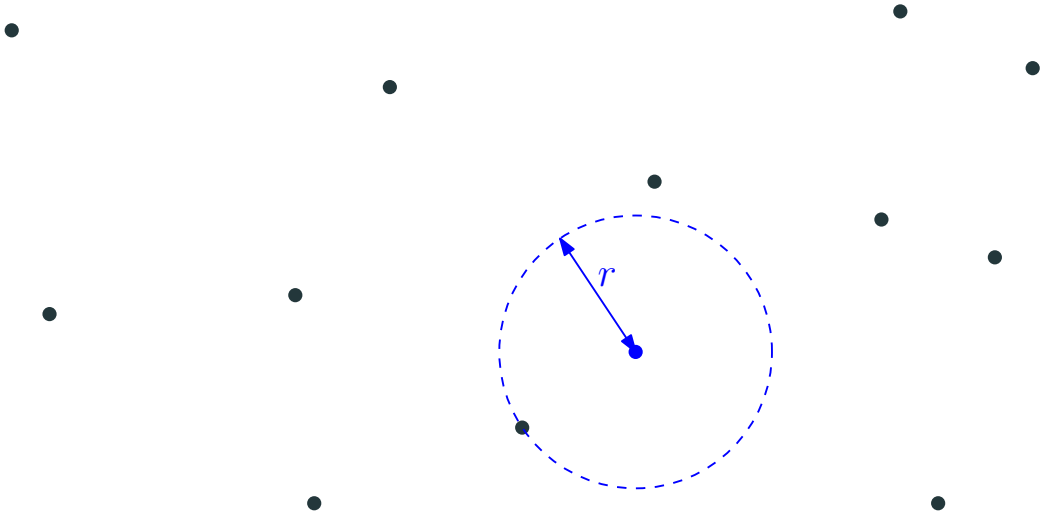


Nearest neighbor problem – Approximate solutions



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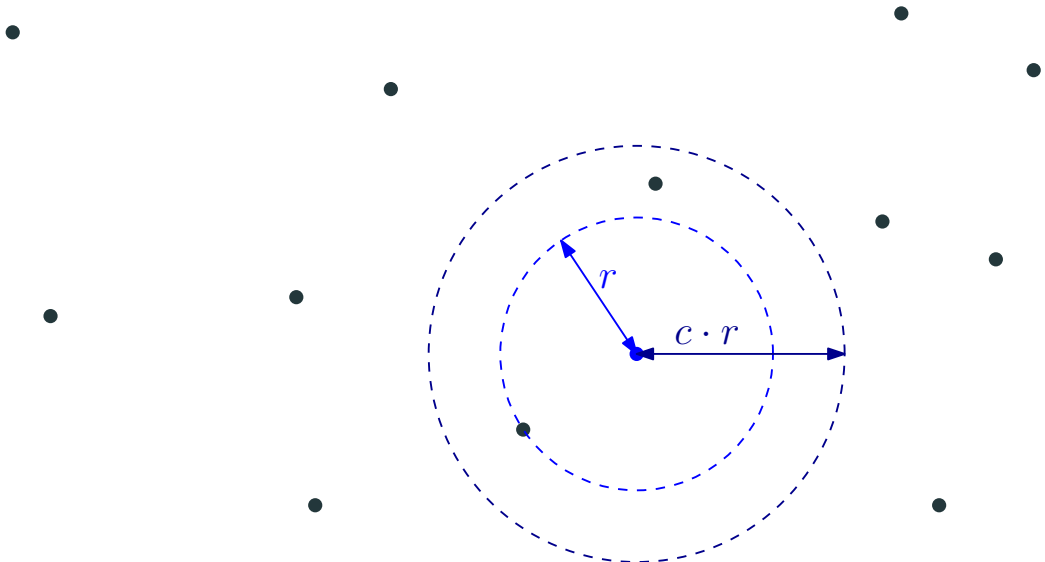
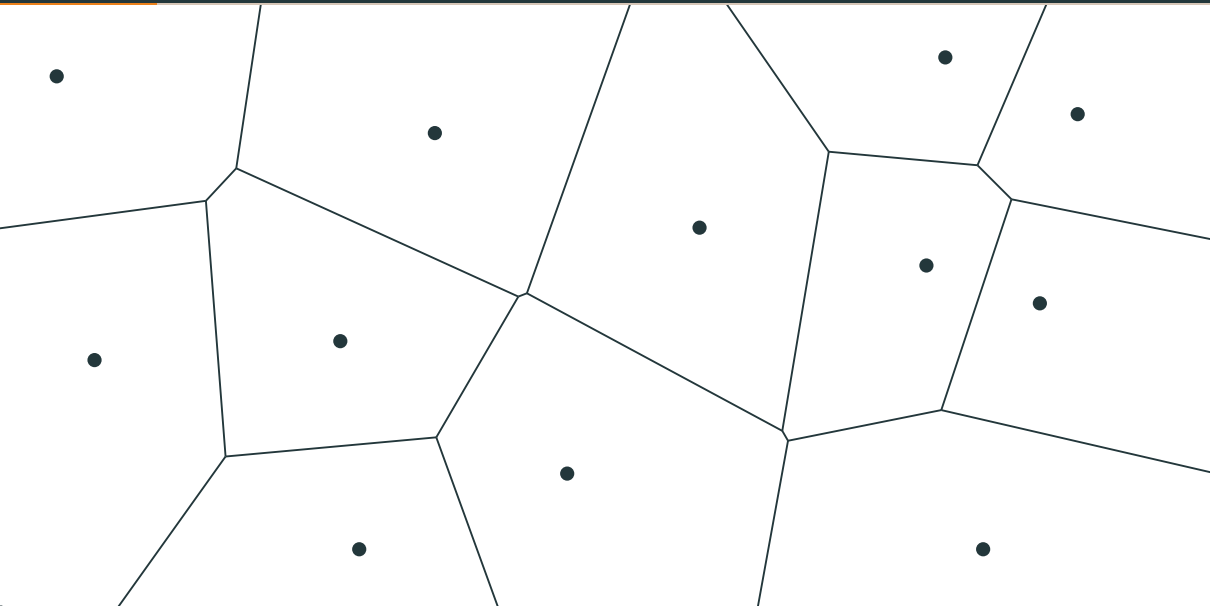


Nearest neighbor problem – Example: Voronoi cells



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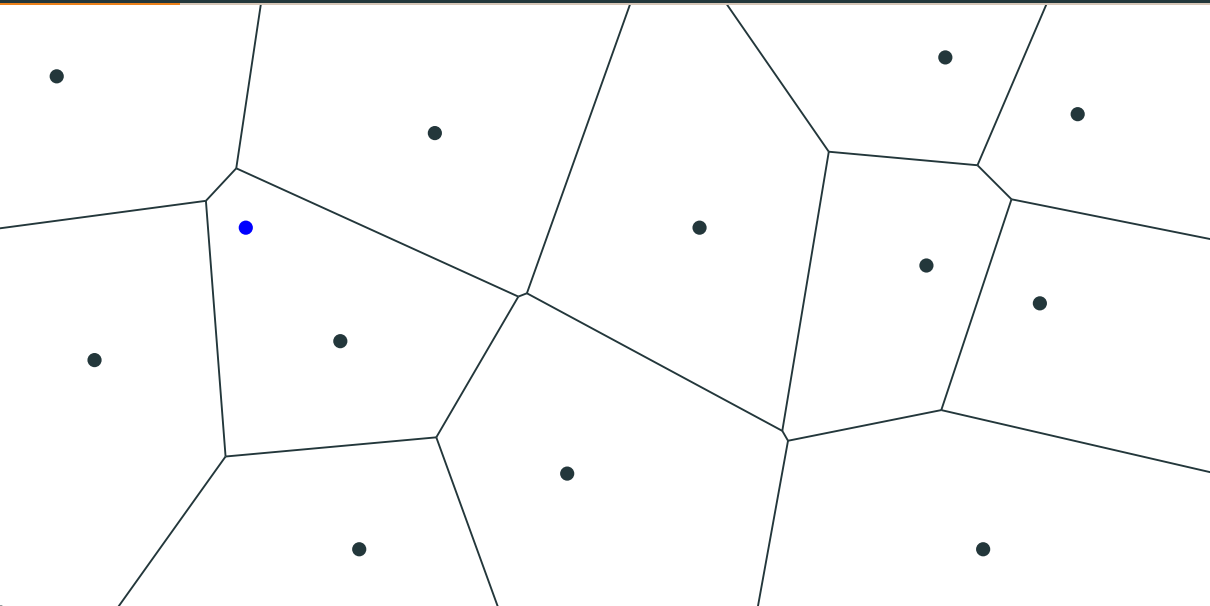

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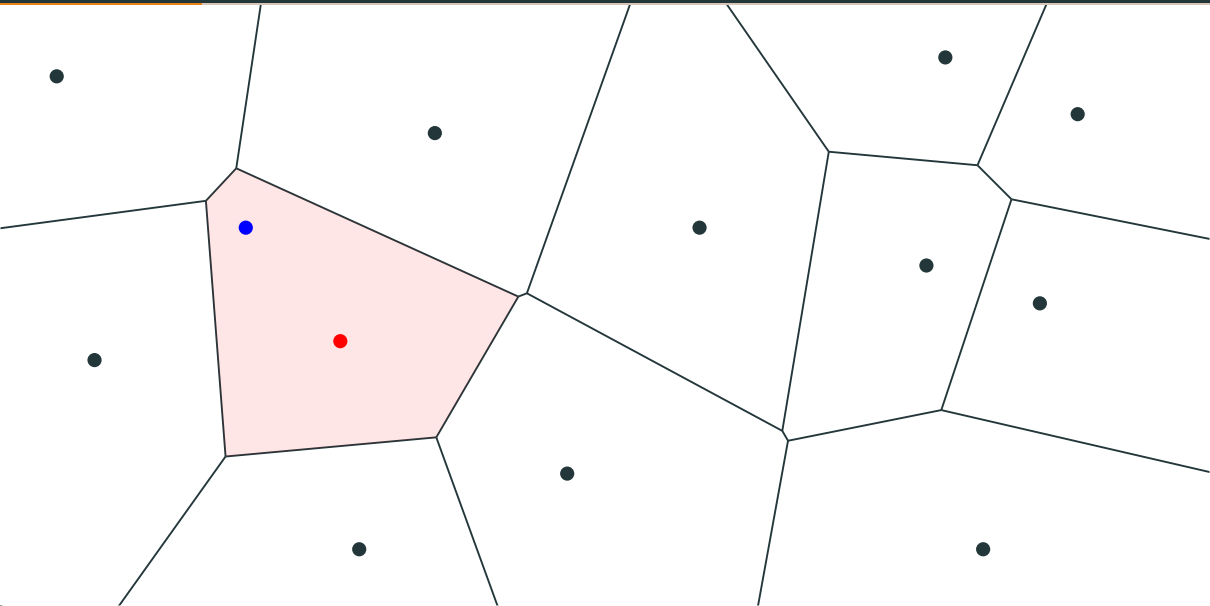
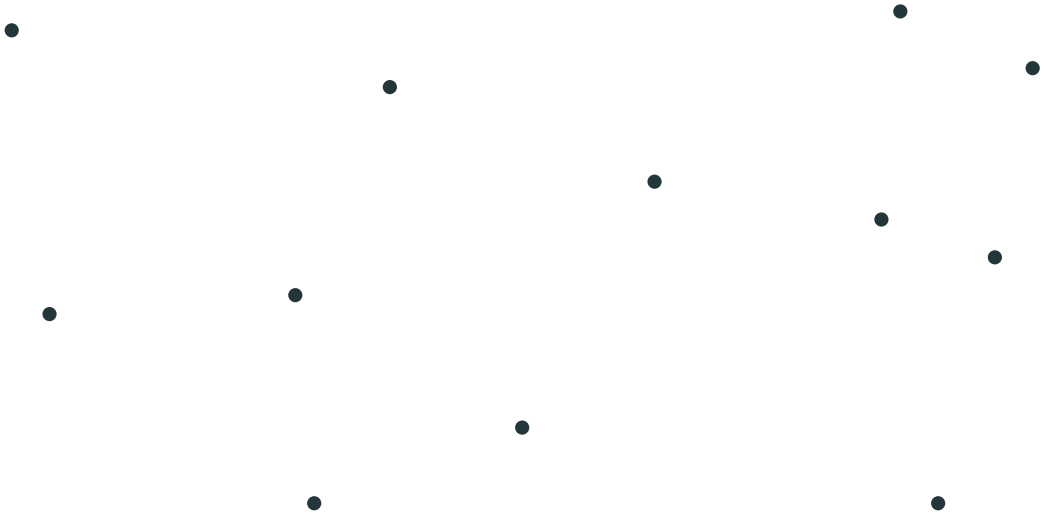


Partition-based methods

Partition-based methods – Data structure



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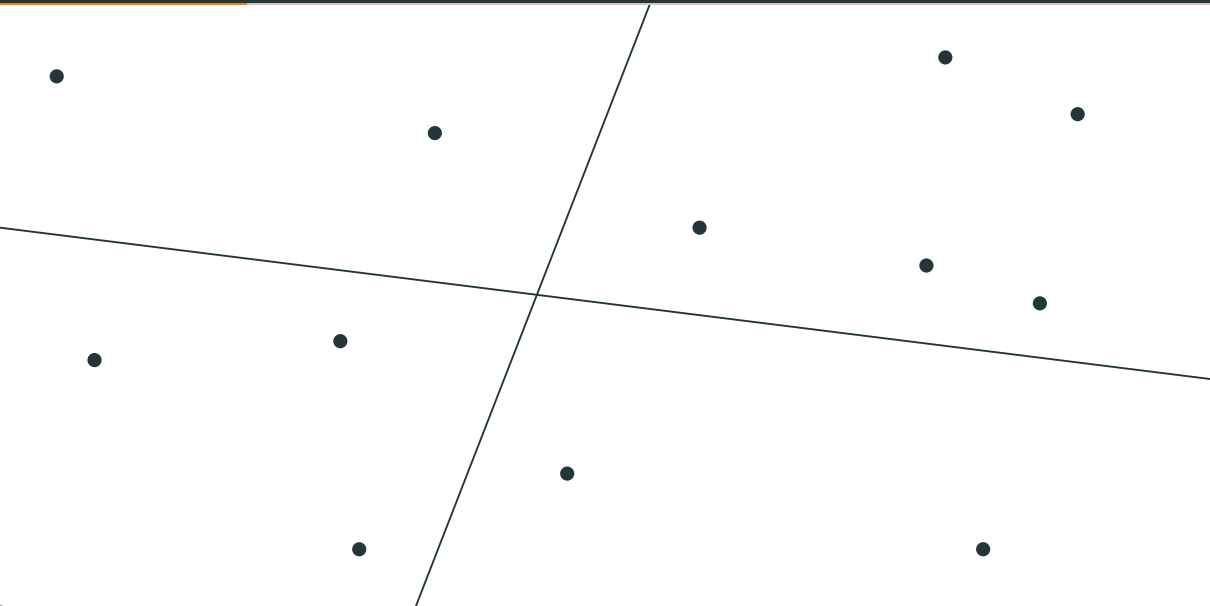
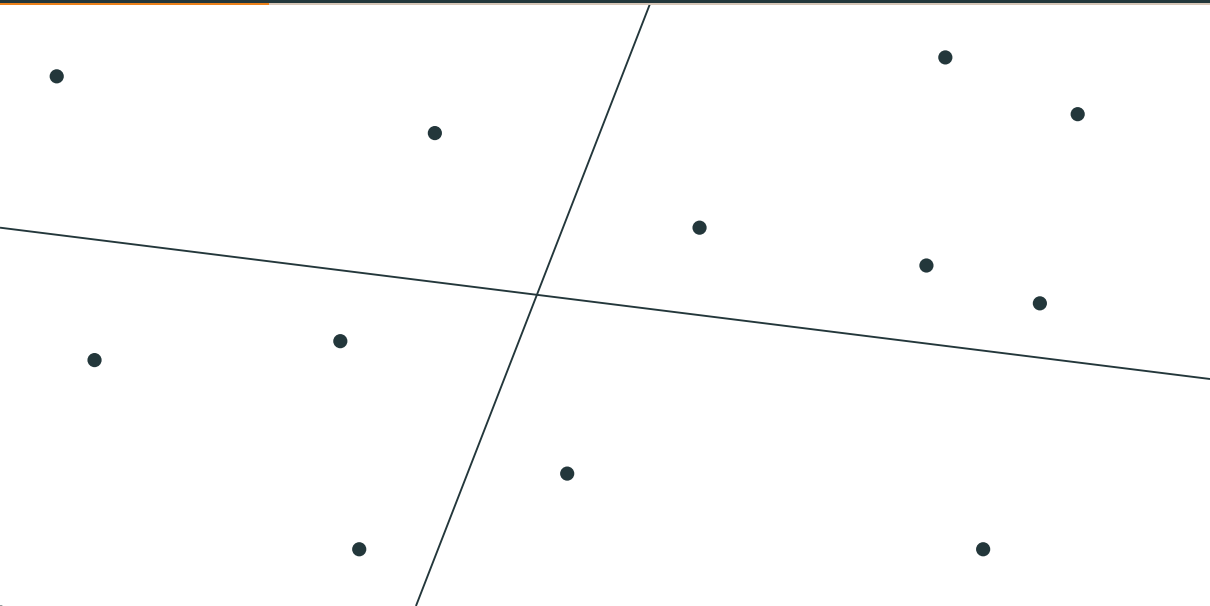


Partition-based methods – Hash table lookups



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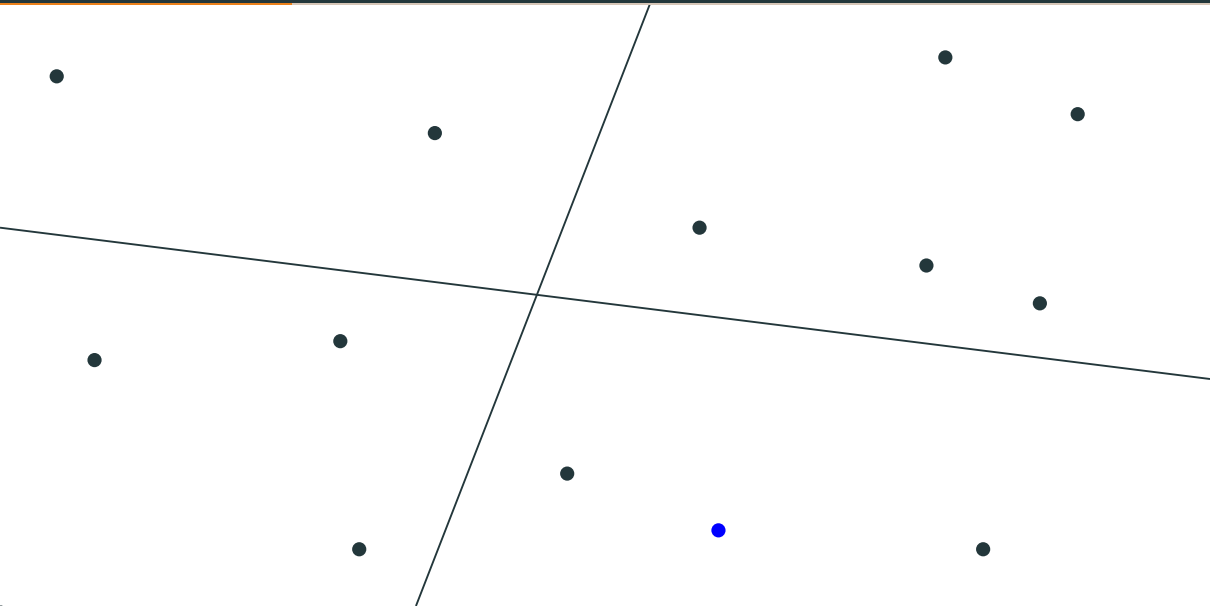

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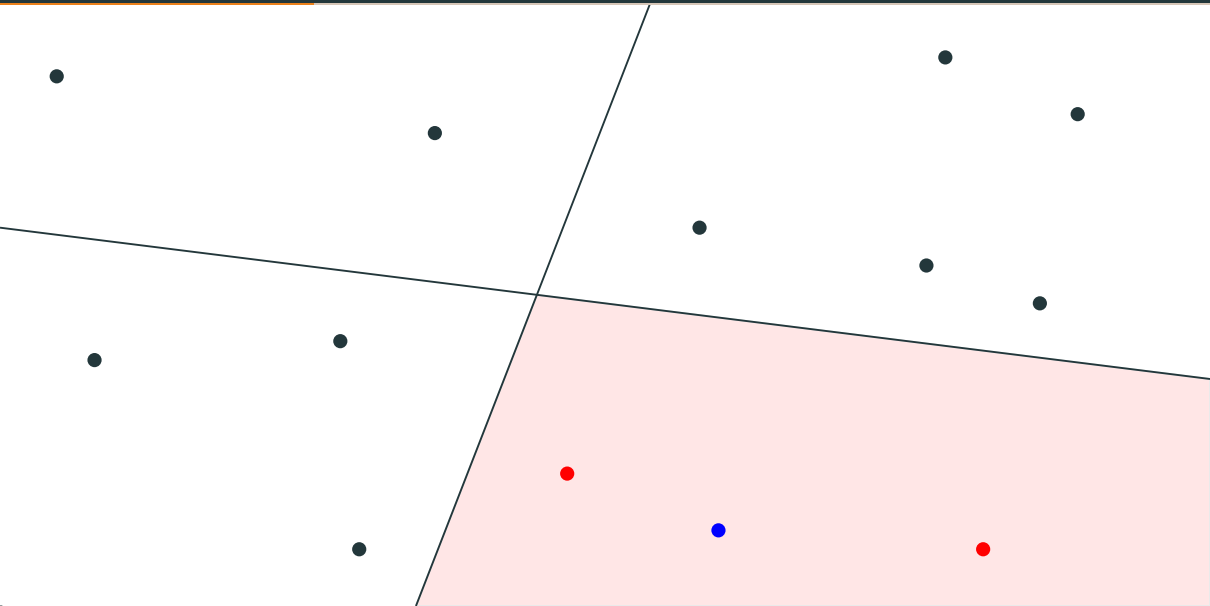

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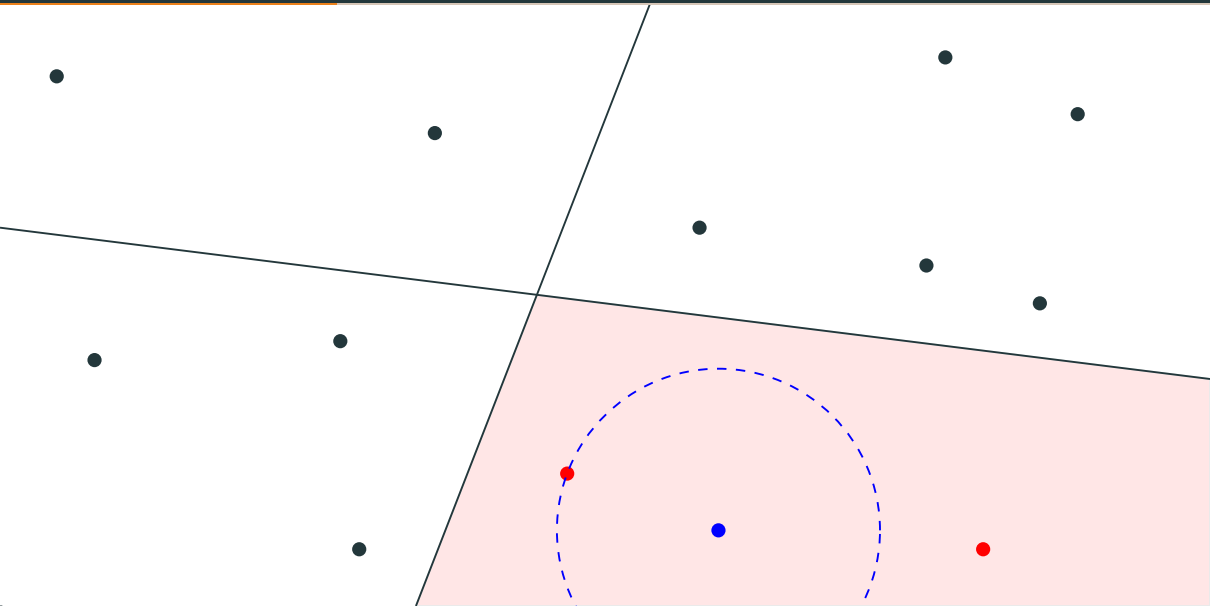

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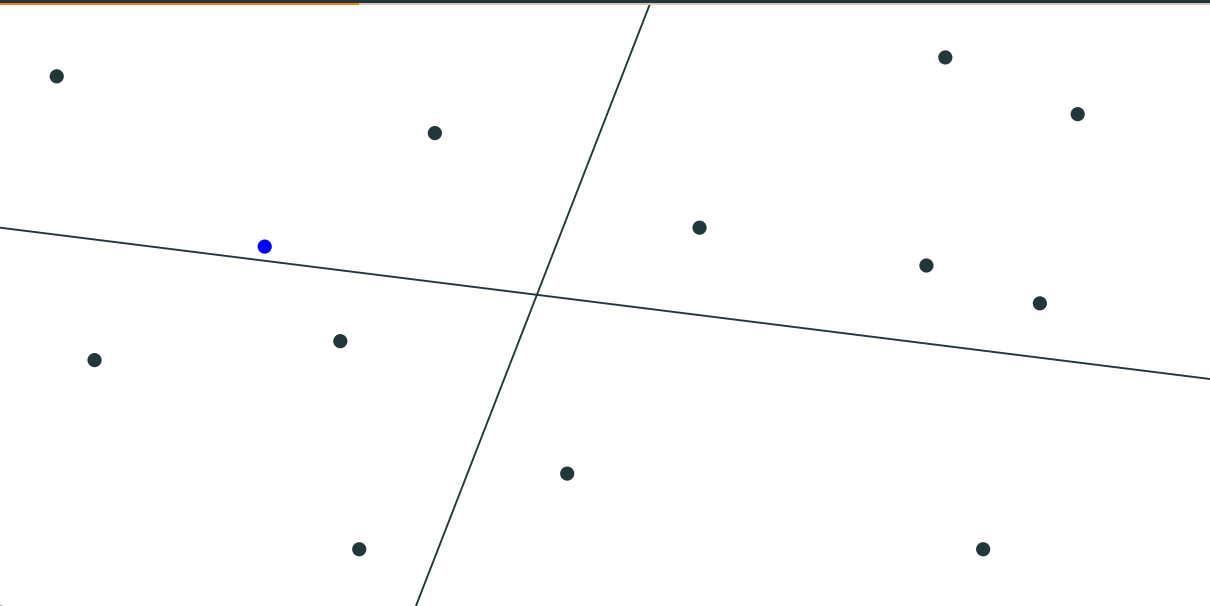

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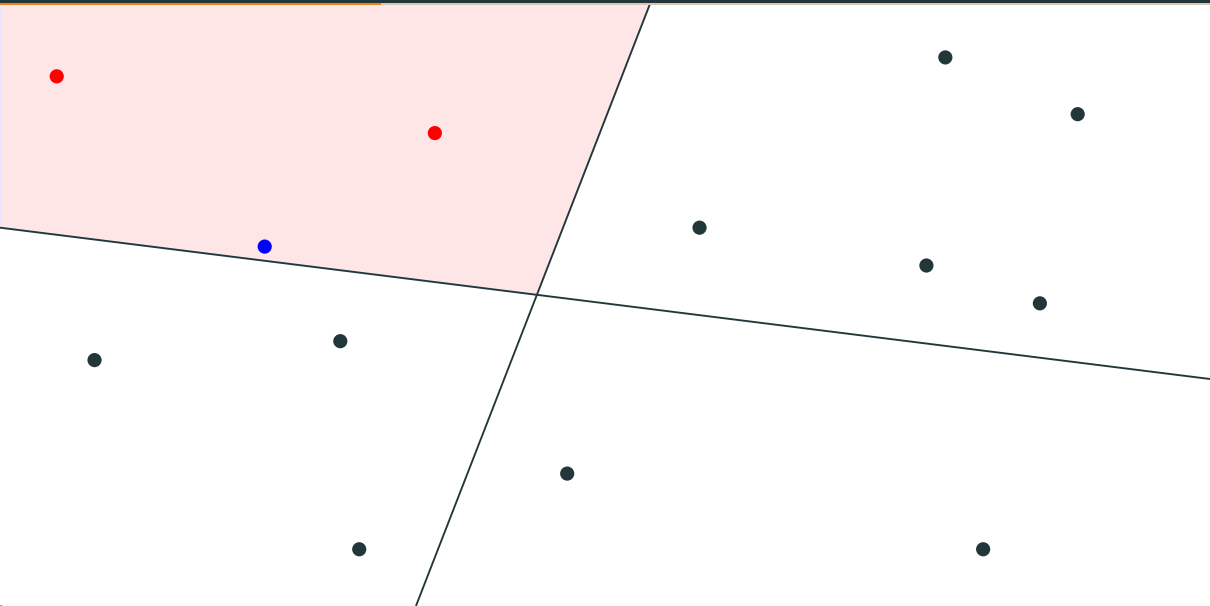

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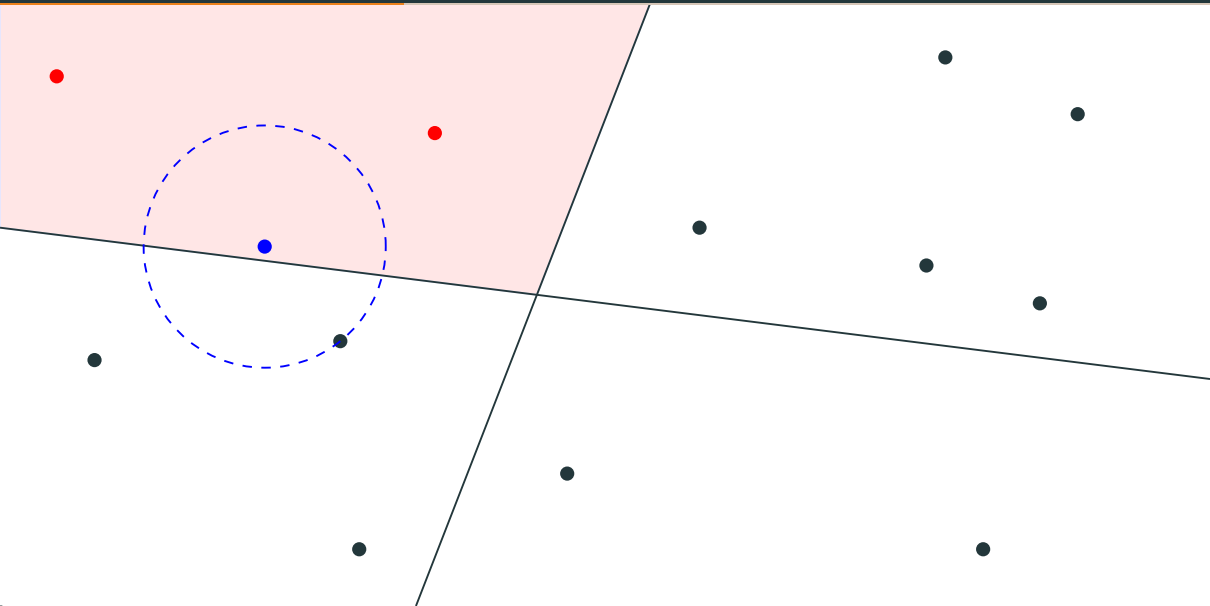

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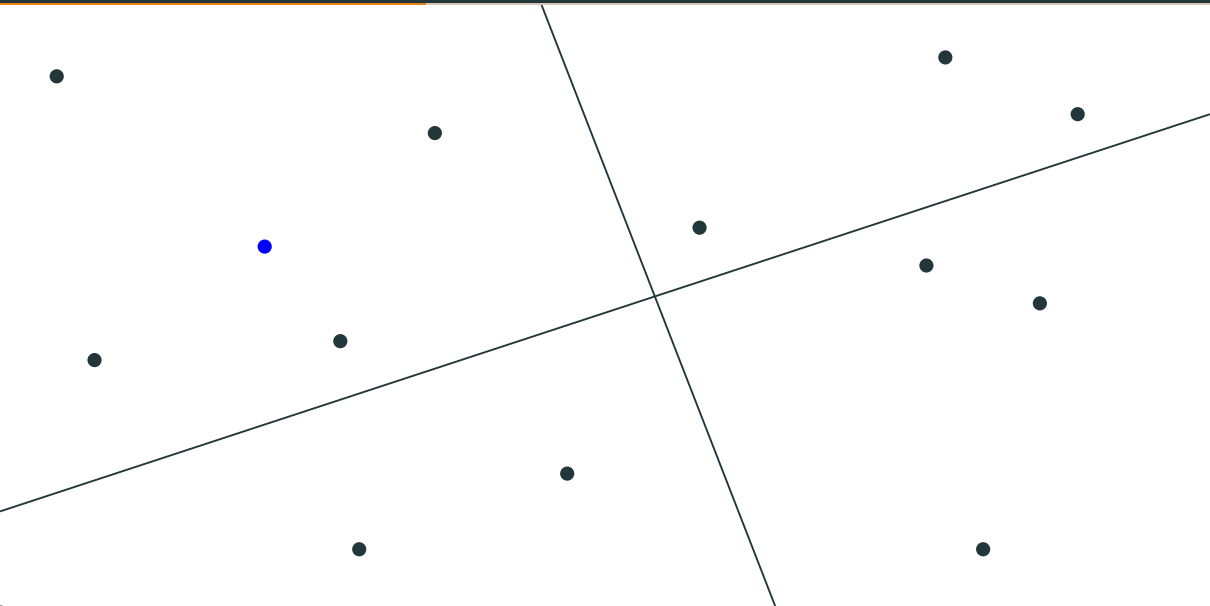

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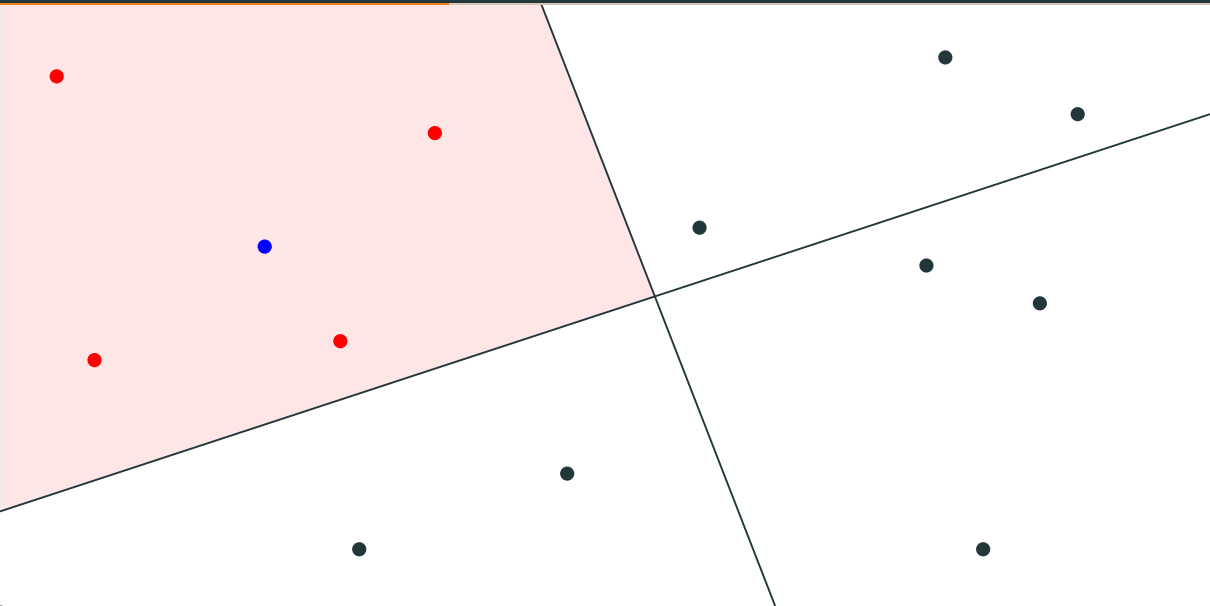

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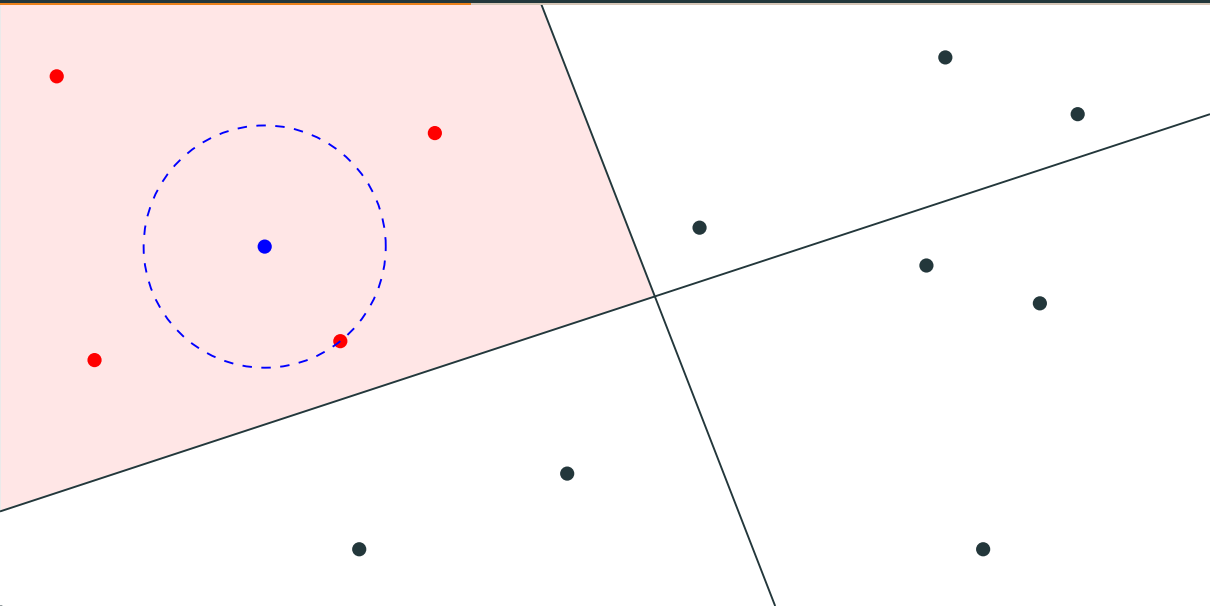

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- Product of bisections; [Cha03]
- Voronoi cells induced by hypercube; [TT07, Laa16]
- Random (overlapping) spheres; [AI06, AINR14]
- Voronoi cells induced by cross-polytopes; [TT07, AIL+15, KW17]
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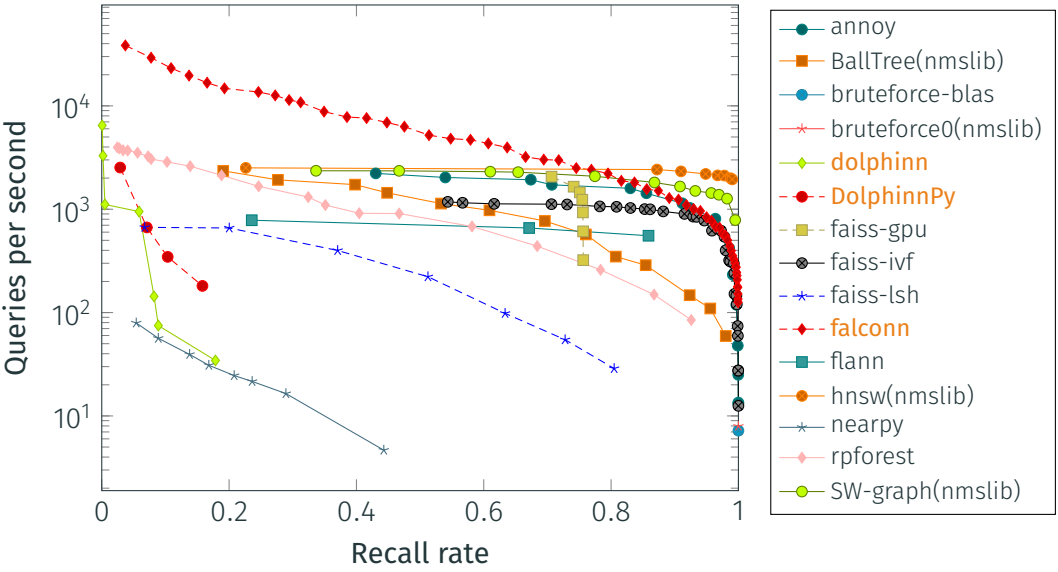
Best techniques are **theoretically optimal** as well as **practical**.

Nearest neighbor methods – Practice (ANN Benchmarks [ABF17])



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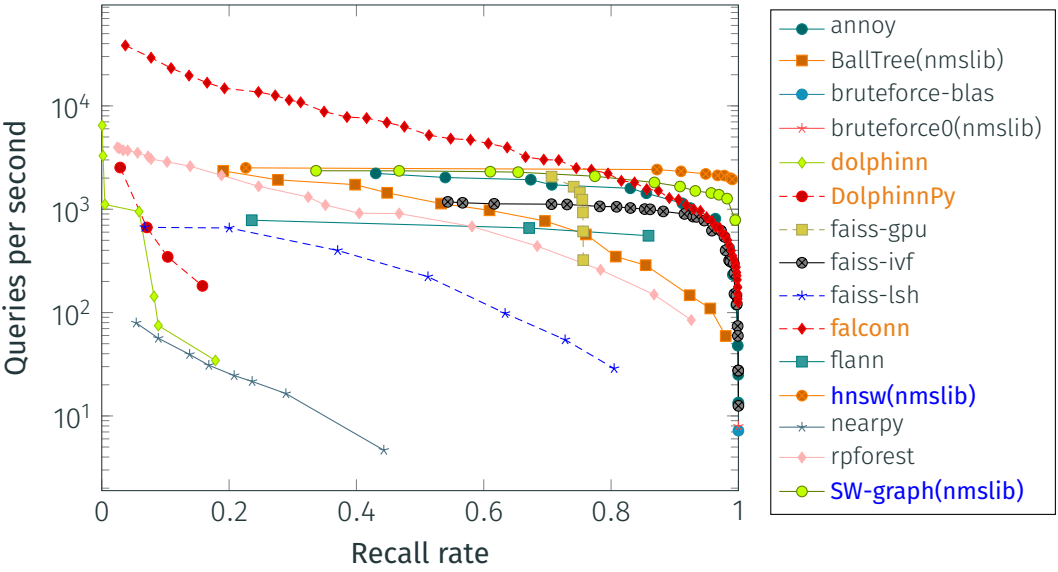

Graph-based methods

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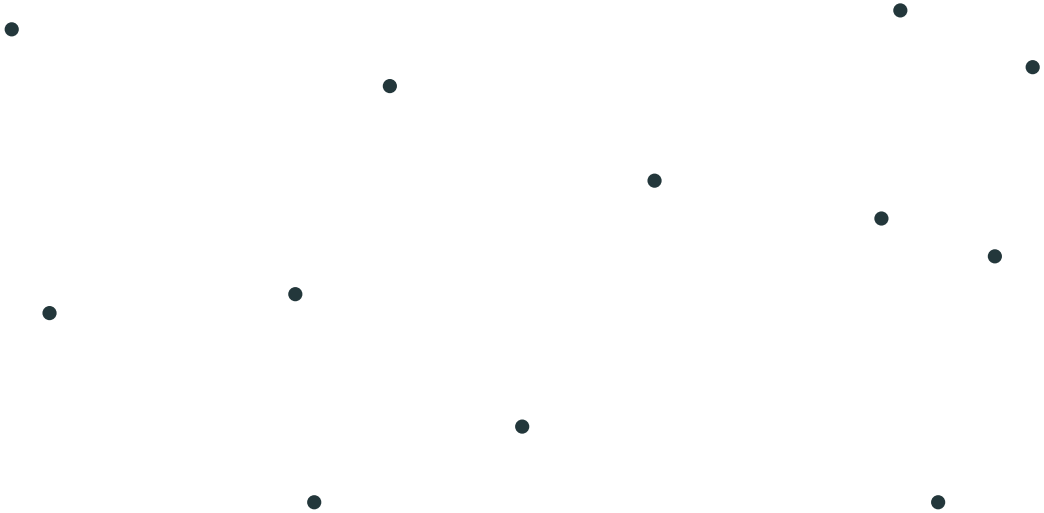

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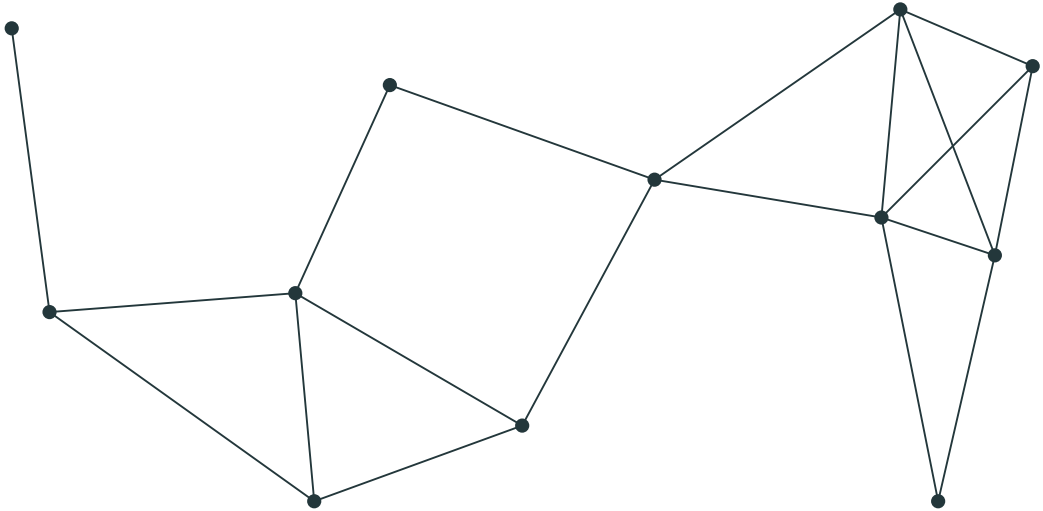
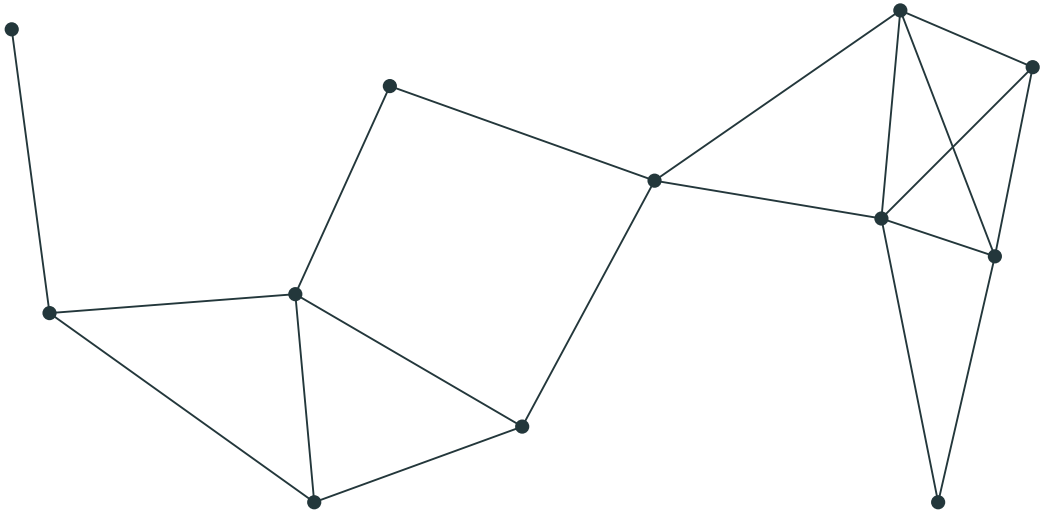


Graph-based methods – Greedy algorithm



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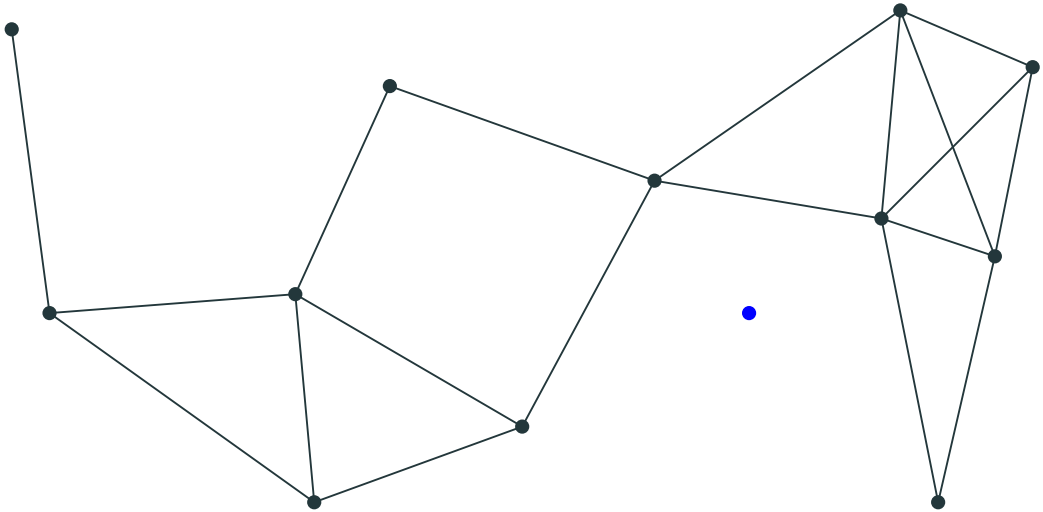

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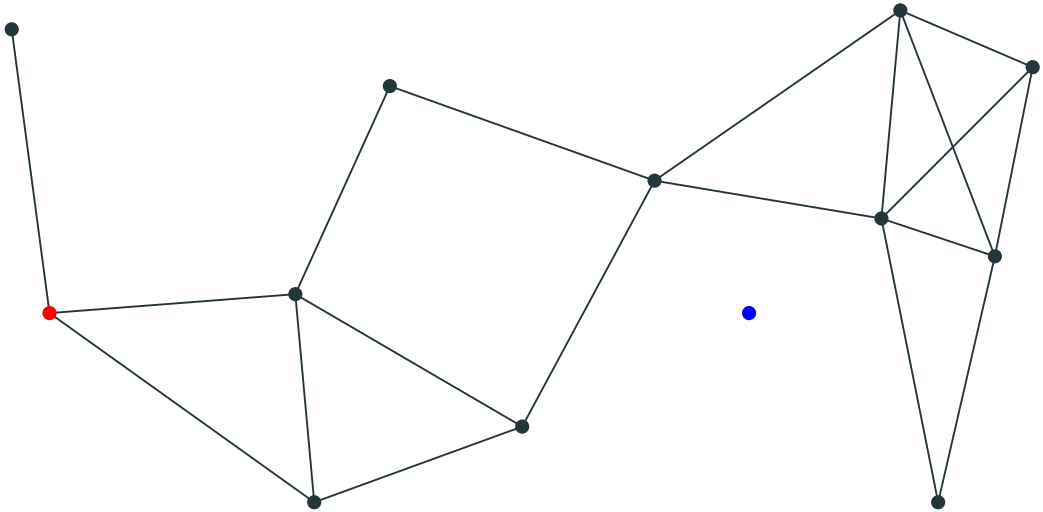

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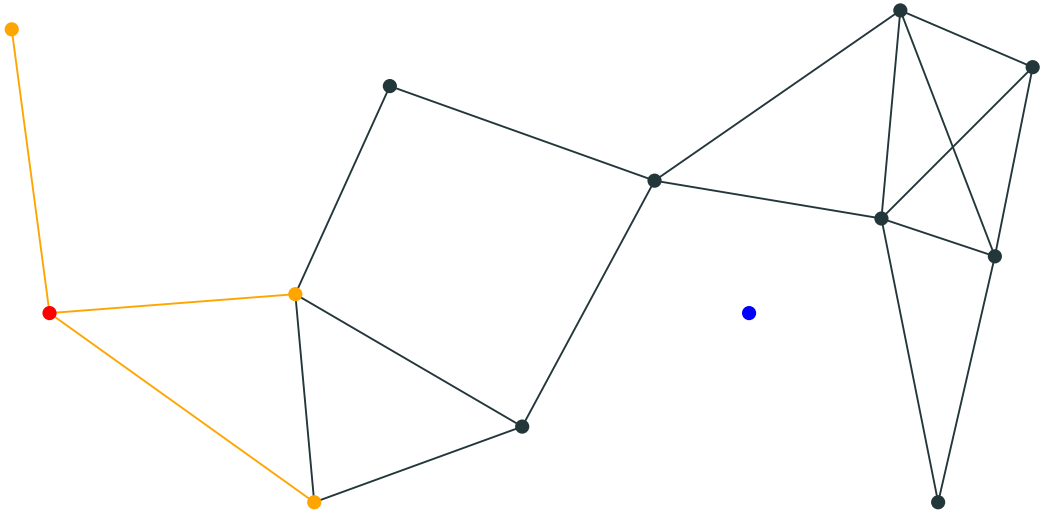

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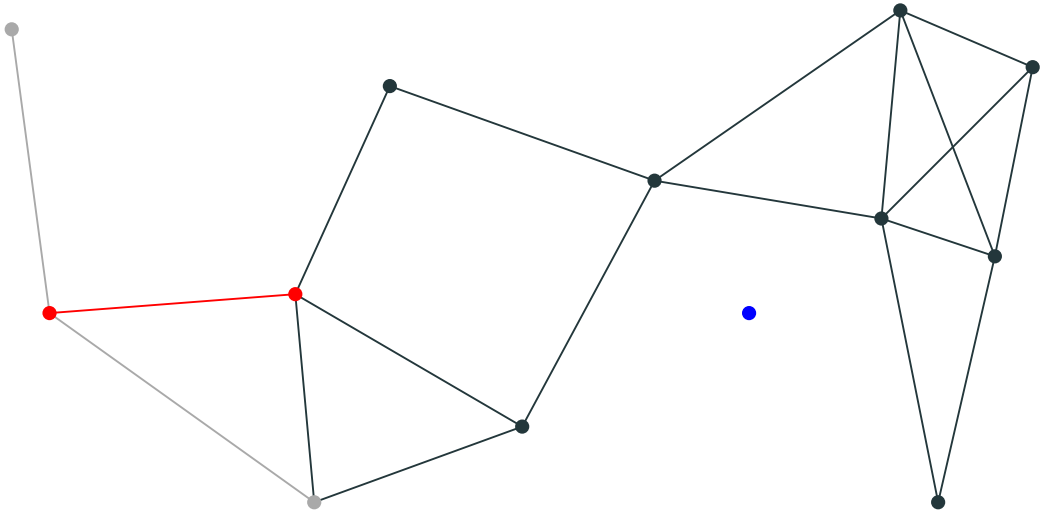

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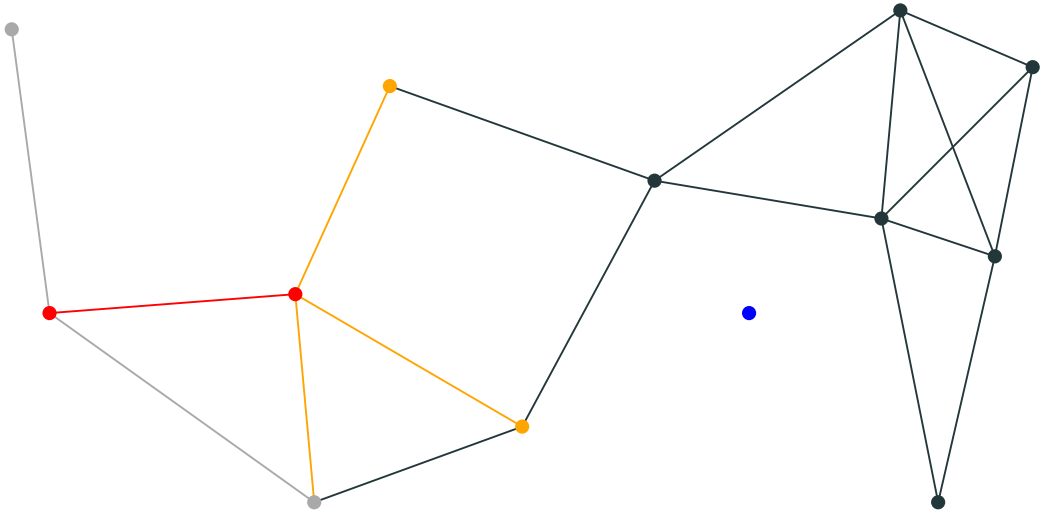

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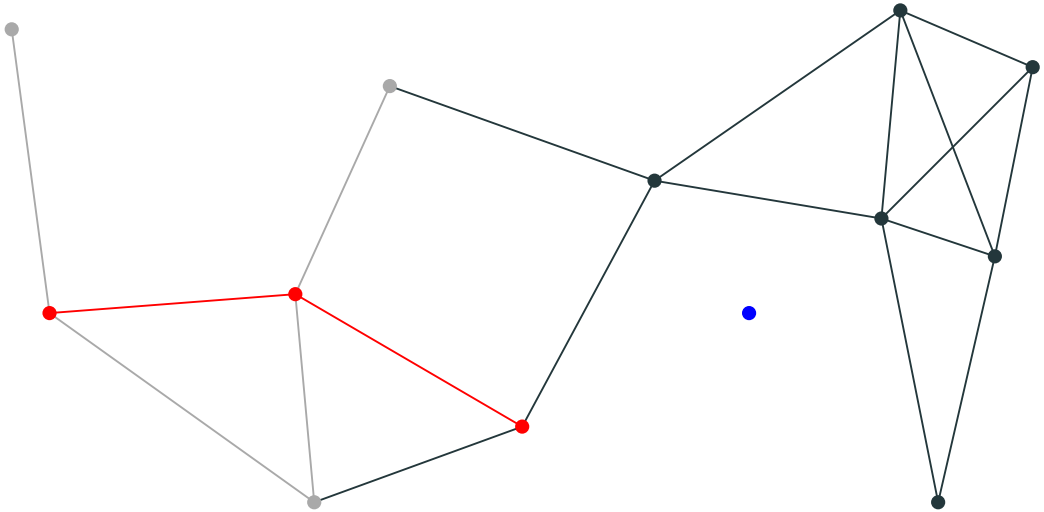

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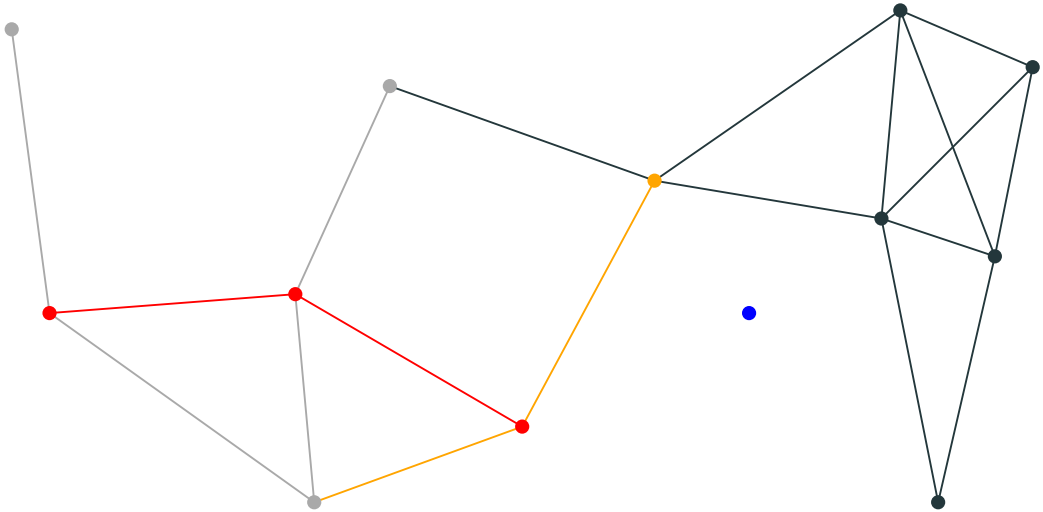

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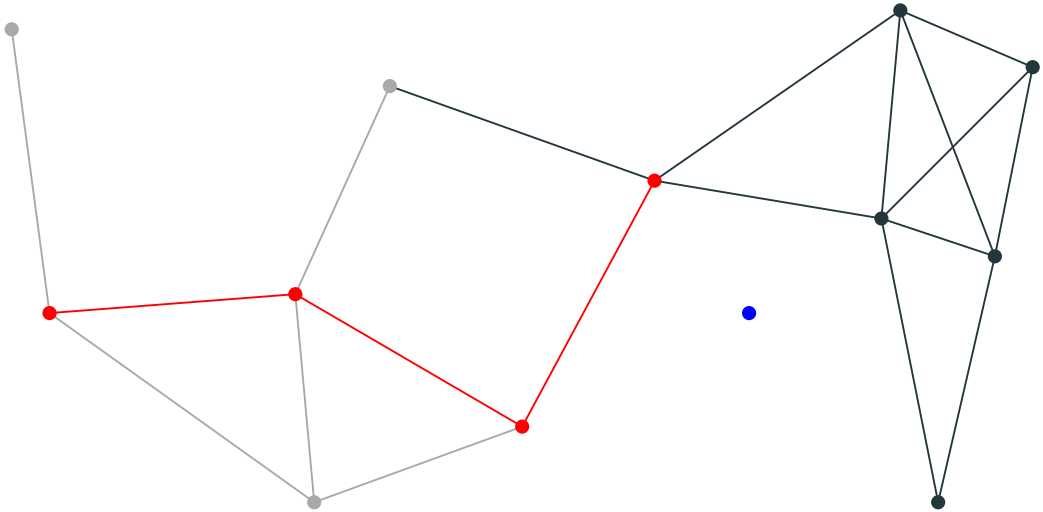

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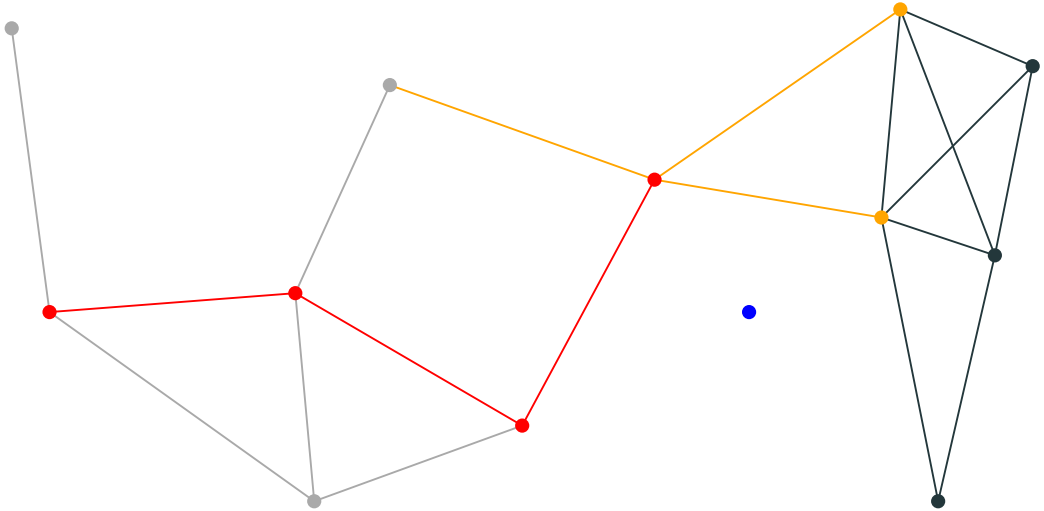

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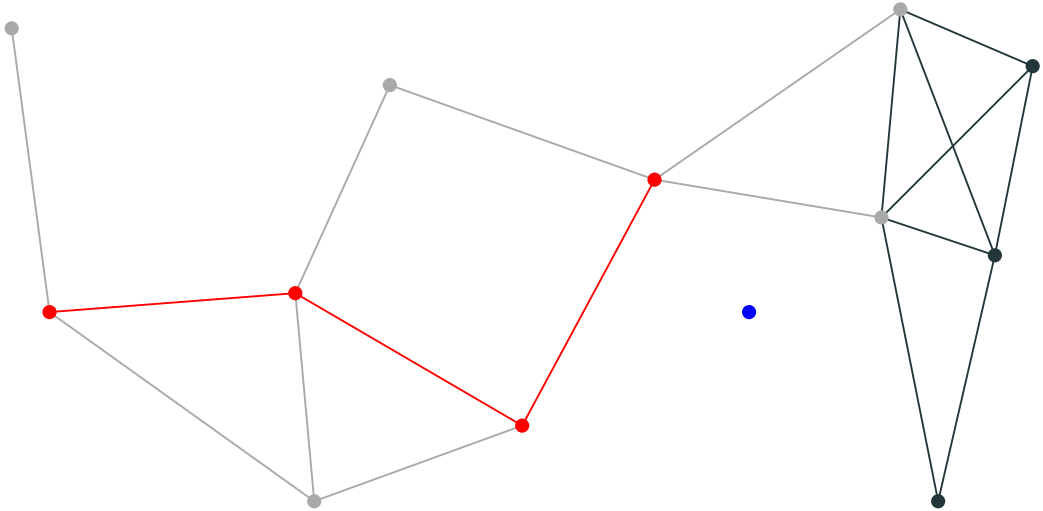

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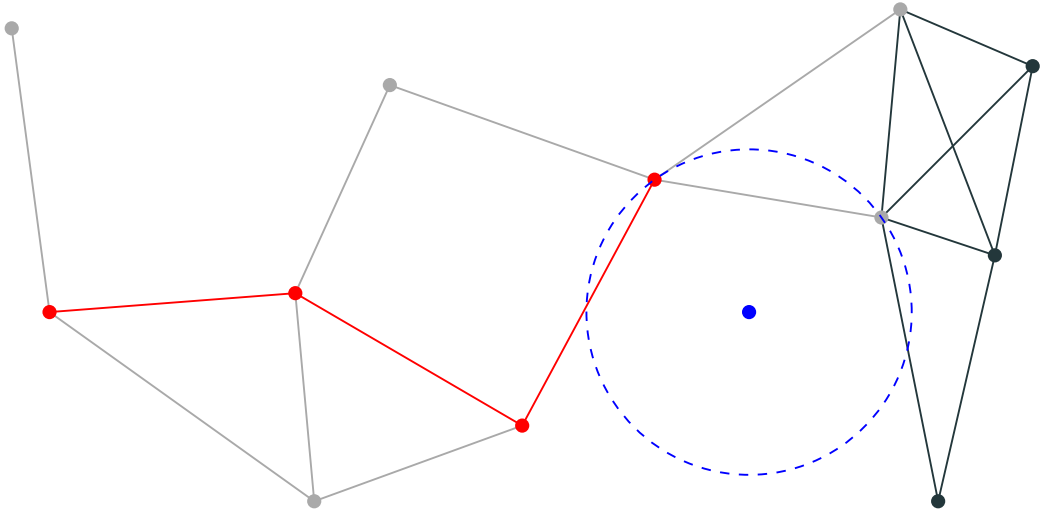

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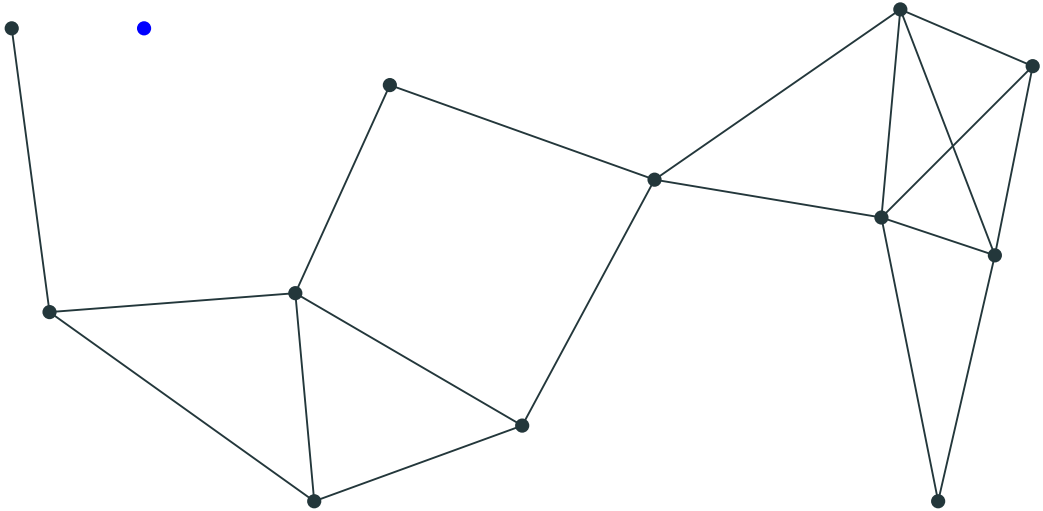

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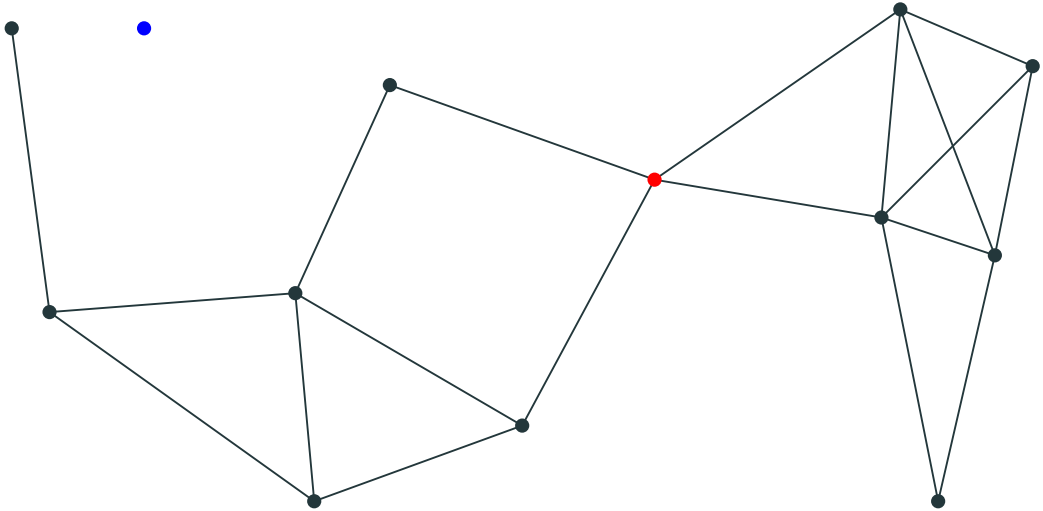

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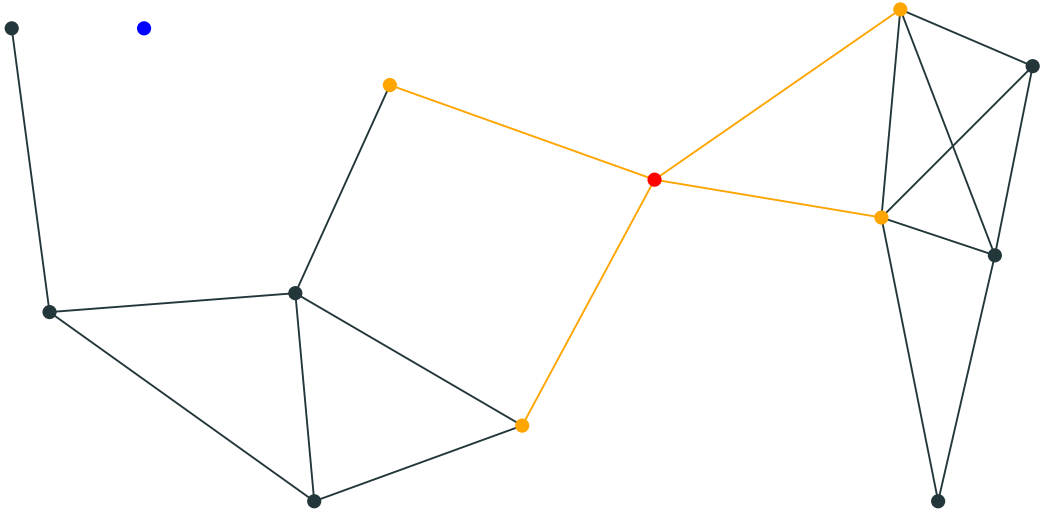

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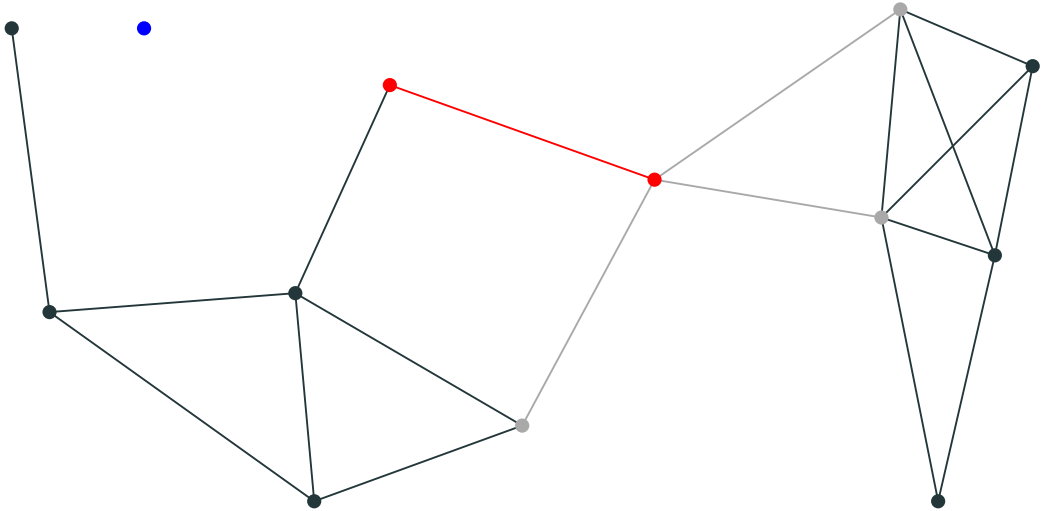

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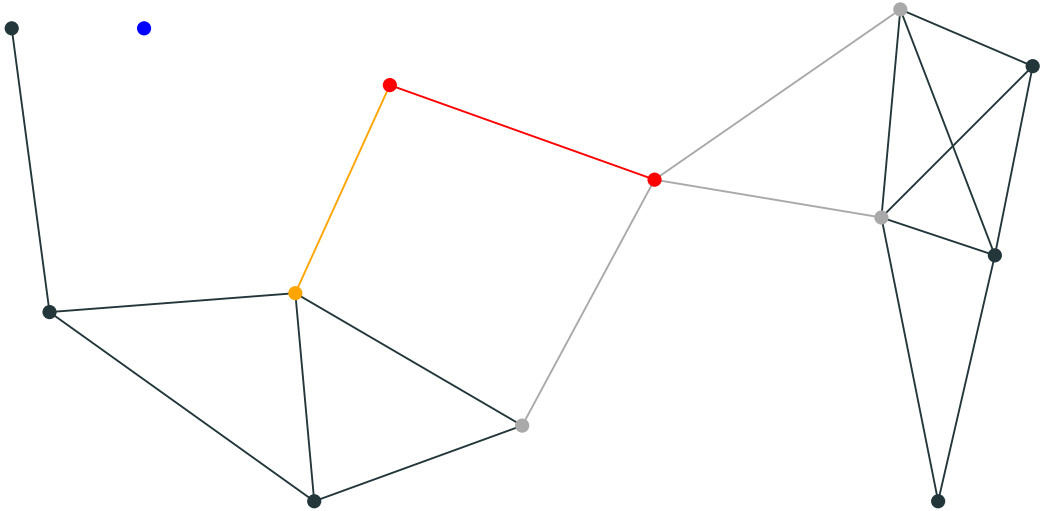

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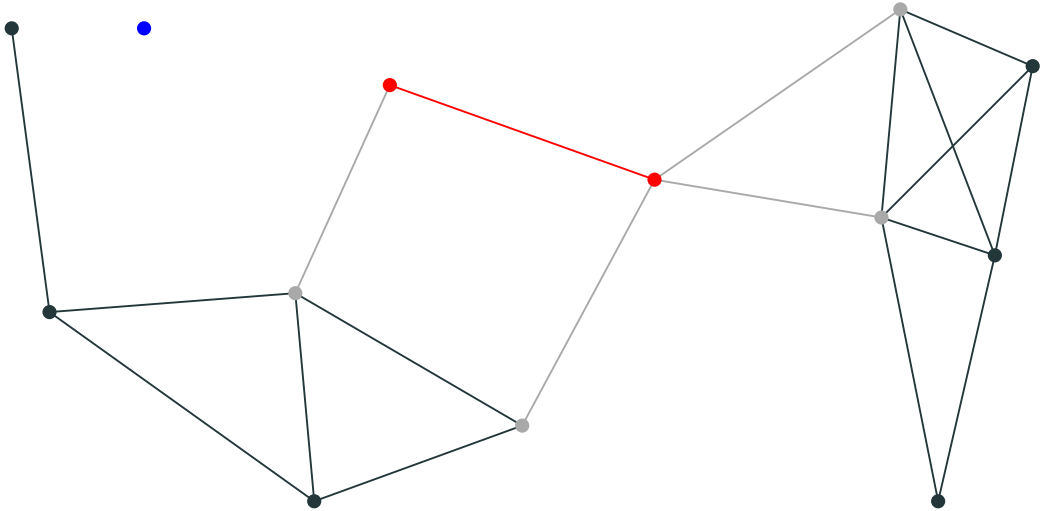

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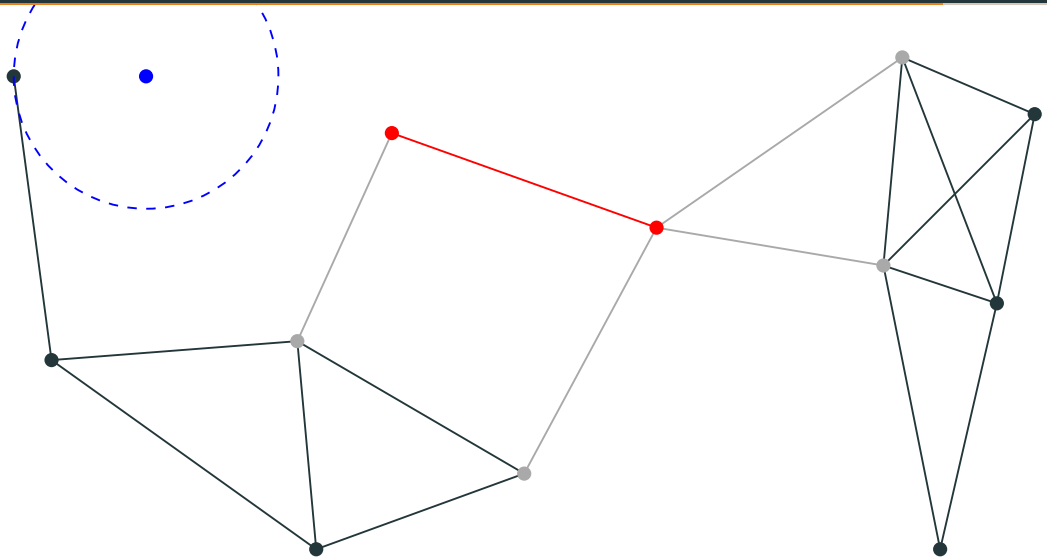

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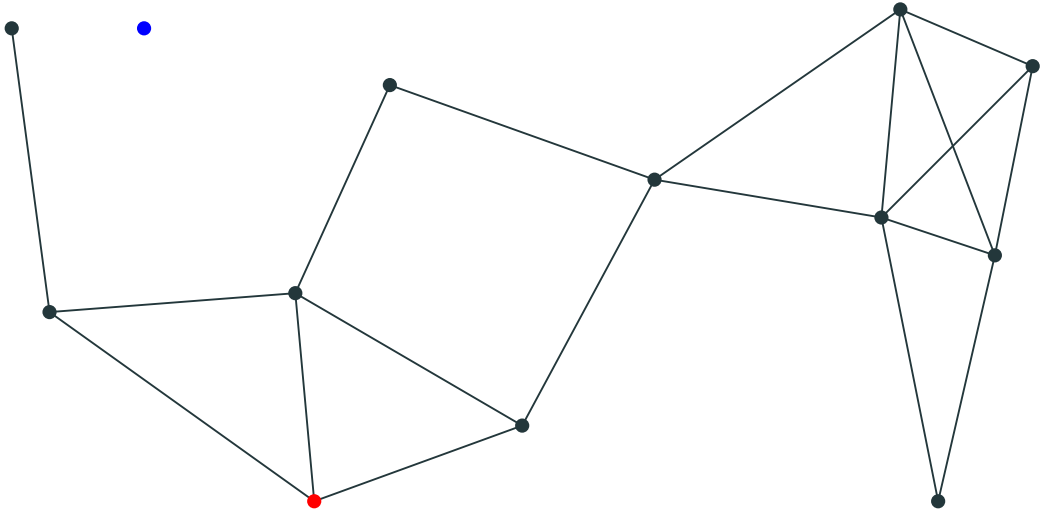

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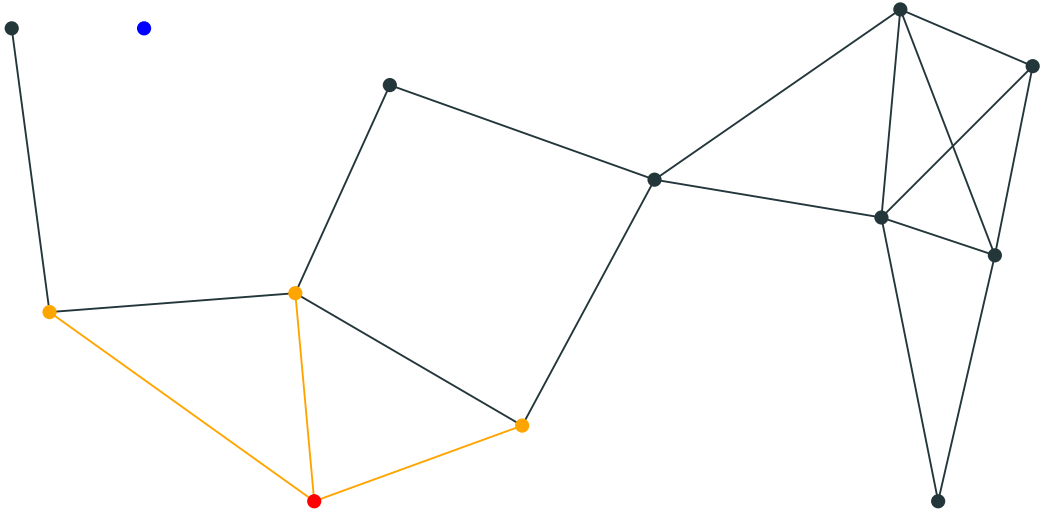

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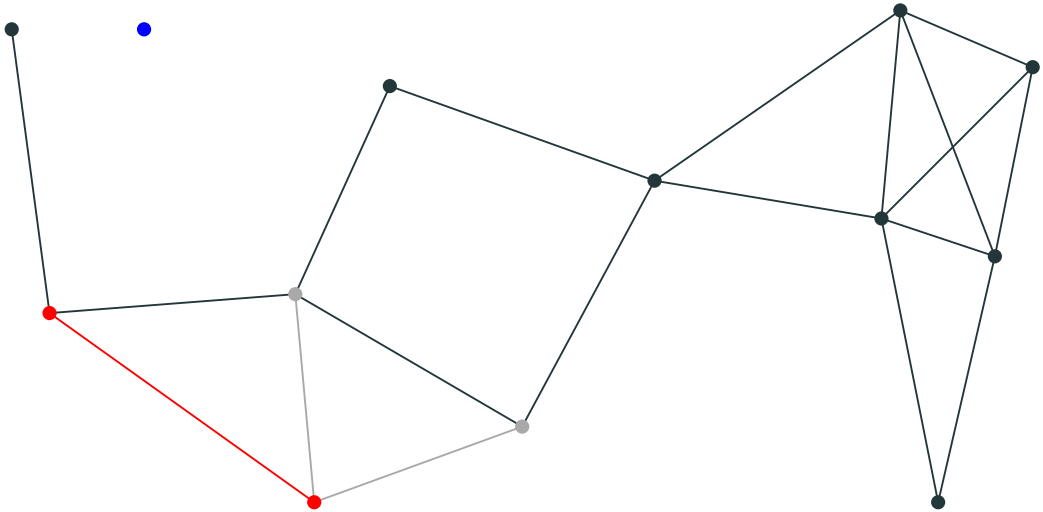

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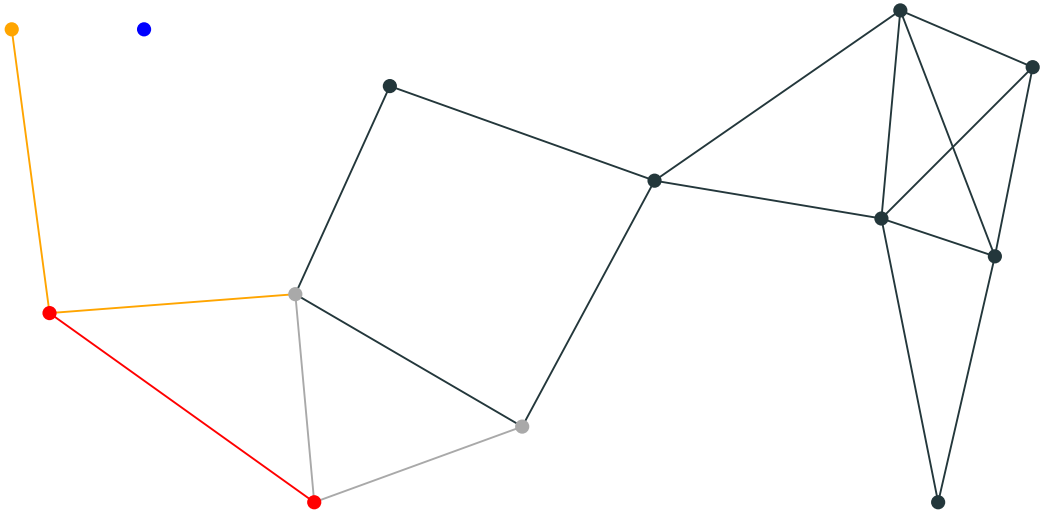

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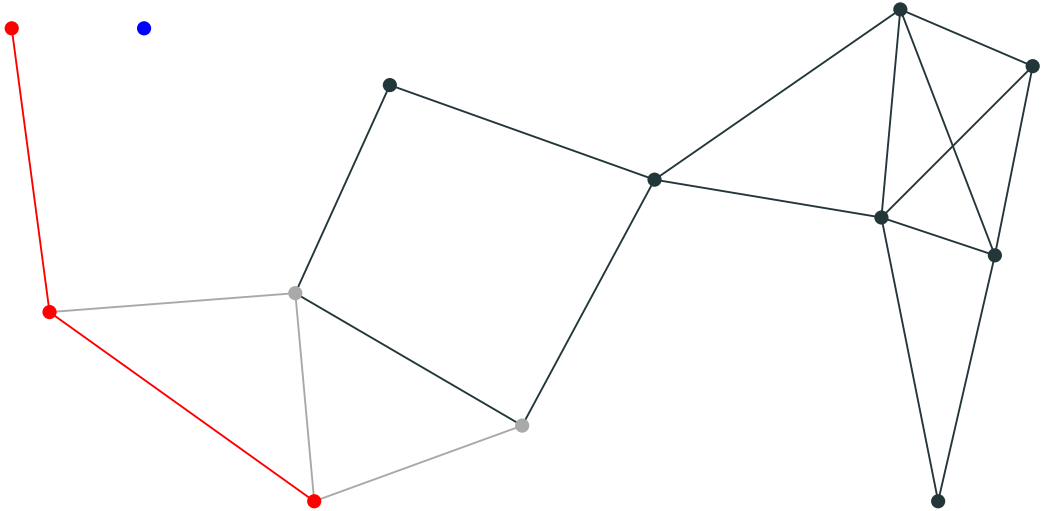

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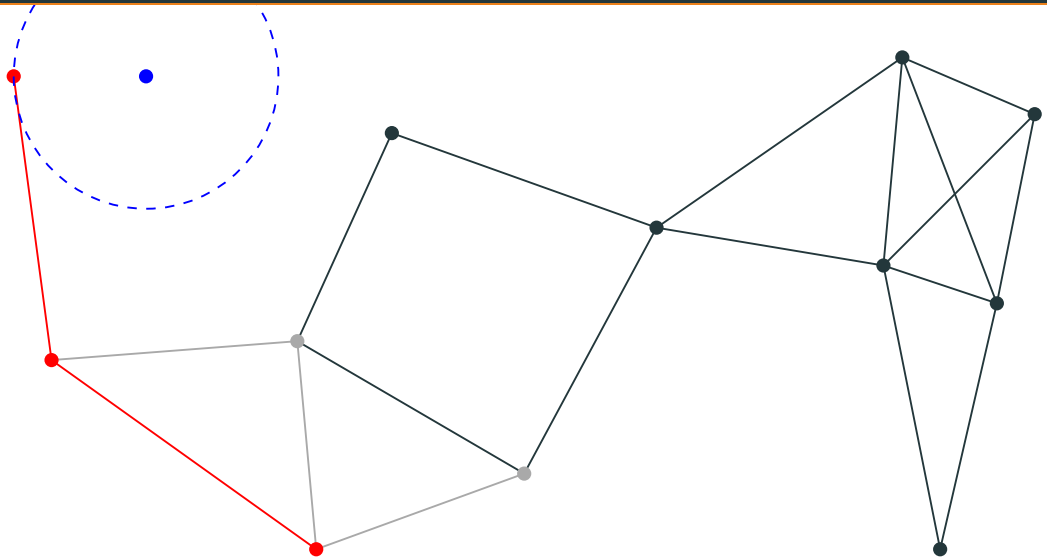

Graph-based methods – Challenges

Main problem: designing the graph.

- Intuitively: connect **near neighbors** for gradual progress;
- Avoiding local minima: add a few **long edges**;
- **Hierarchical graphs**: long edges in upper layers, short edges in bottom layers.

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Graph-based methods – Contributions

Theorem (Main result, informal)

For randomized greedy walks on the near neighbor graph and for "random" data sets, we can solve the approximate nearest neighbor problem on n points with query time $O(n^{\rho_q})$ and space $O(n^{1+\rho_s})$ with $\rho_q, \rho_s \geq 0$ satisfying

$$(2c^2 - 1)\rho_q + 2c^2(c^2 - 1)\sqrt{\rho_s(1 - \rho_s)} \geq c^4.$$

Graph-based methods – Contributions

In the most common regime of $c \approx 1$ (**high recall rate**) and $\rho_s \approx 0$ (near-linear space), this **scales equivalently** as the best partition-based trade-offs: [ALRW17]

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Positive result: greedy algorithm already "optimal" for $c \approx 1$ and $\rho_s \approx 0$.

Graph-based methods – Contributions

In the most common regime of $c \approx 1$ (**high recall rate**) and $\rho_s \approx 0$ (near-linear space), this **scales equivalently** as the best partition-based trade-offs: [ALRW17]

$$\rho_q = 1 - 4(c-1)\sqrt{\rho_s} \cdot (1 + o(1)). \tag{1}$$

Positive result: greedy algorithm already "optimal" for $c \approx 1$ and $\rho_s \approx 0$.

Negative result: (analysis of) this algorithm is not competitive for $c \gg 1$ or $\rho_s \gg 0$.

Graph-based methods – Open problems

Various **open problems** remain:

- Current **analysis** may not be sharp – can it be tightened?
- Does adding **long edges** lead to better theoretical guarantees?
- What can theoretically be said about **hierarchical approaches**?
- Can we obtain **lower bounds** showing limitations of graph-based methods?

Thank you!